

# HFLAV-Tau Spring 2017 report

Sw. Banerjee, University of Louisville, USA  
M. Chrząszcz, IFJ PAN, Kraków, Poland and Universität Zürich, Switzerland  
K. Hayasaka, Niigata University, Japan  
H. Hayashii, Nara Women's University, Japan  
A. Lusiani, Scuola Normale Superiore and INFN Pisa, Italy  
M. Roney, University of Victoria, Canada  
B. Shwartz, Budker Institute of Nuclear Physics, Russia

18 April 2017

## Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Branching fraction fit</b>	<b>2</b>
2.1	Technical implementation of the fit procedure	2
2.2	Fit results	3
2.3	Changes with respect to the previous report	3
2.4	Differences between the HFLAV Spring 2017 fit and the PDG 2016 fit	4
2.5	Branching ratio fit results and experimental inputs	5
2.6	Correlation terms between basis branching fractions uncertainties	13
2.7	Equality constraints	16
<b>3</b>	<b>Tests of lepton universality</b>	<b>19</b>
<b>4</b>	<b>Universality improved <math>B(\tau \rightarrow e\nu\bar{\nu})</math> and <math>R_{\text{had}}</math></b>	<b>20</b>
<b>5</b>	<b><math> V_{us} </math> measurement</b>	<b>21</b>
5.1	$ V_{us} $ from $B(\tau \rightarrow X_s\nu)$	21
5.2	$ V_{us} $ from $B(\tau \rightarrow K\nu)/B(\tau \rightarrow \pi\nu)$	22
5.3	$ V_{us} $ from $\tau$ summary	23
<b>6</b>	<b>Upper limits on <math>\tau</math> lepton-flavour-violating branching fractions</b>	<b>24</b>
<b>7</b>	<b>Combination of upper limits on <math>\tau</math> lepton-flavour-violating branching fractions</b>	<b>26</b>
<b>A</b>	<b>Branching fractions fit measurement list by reference</b>	<b>31</b>
<b>References</b>		<b>37</b>

# 1 Introduction

We present averages of a selection of  $\tau$  lepton quantities with the goal to provide the best tests of the universality of the charged-current weak interaction (Section 3) and of the Cabibbo-Kobayashi-Maskawa (CKM) matrix coefficient  $|V_{us}|$  from  $\tau$  decays (Section 5). We focus on the averages that benefit most from the adoption of the HFLAV methodology [1], namely a global fit of the  $\tau$  branching fractions that best exploits the available experimental information. Since the 2016 edition, the HFLAV-Tau group has collaborated to the determination of the  $\tau$ -lepton branching fractions based on a global fit and to the related mini-review that are included in the “Review of particle physics” [2]. The differences between the PDG 2016 fit and the fit presented here are detailed in Section 2.4.

All relevant published statistical correlations are used, and a selection of measurements, particularly the most precise and the most recent ones, was studied to take into account the significant systematic dependencies from external parameters and common sources of systematic uncertainty.

Finally, we report in Section 6 the latest limits on the lepton-flavour-violating  $\tau$  branching fractions and in Section 7 we determine the combined upper limits for the branching fractions that have multiple experimental results.

The  $\tau$  lepton results are obtained from inputs available through summer 2016 and have been published on the web in 2016 with the label “Summer 2016”. However, there have been minor revisions since then, and we have updated tables and plots in this report with the label “Spring 2017”.

## 2 Branching fraction fit

A global fit of the available experimental measurements is used to determine the  $\tau$  branching fractions, together with their uncertainties and statistical correlations. The  $\tau$  branching fractions provide a test for theory predictions based on the Standard Model (SM) EW and QCD interactions and can be further elaborated to test the EW charged-current universality for leptons, to determine the CKM matrix coefficient  $|V_{us}|$  (and the QCD coupling constant  $\alpha_s$  at the  $\tau$  mass).

The measurements used in the fit are listed in Table 1 and consist of either  $\tau$  decay branching fractions, labelled as  $\Gamma_i$ , or ratios of two  $\tau$  decay branching fractions, labelled as  $\Gamma_i/\Gamma_j$ . A minimum  $\chi^2$  fit is performed for all the measured quantities and for some additional branching fractions and ratios of branching fractions, and all fit results are listed in Table 1. Some fitted quantities are equal to the ratio of two other fitted quantities, as documented with the notation  $\Gamma_i/\Gamma_j$  in Table 1. Some fitted quantities are sums of other fitted quantities, for instance  $\Gamma_8 = B(\tau \rightarrow h^- \nu_\tau)$  is the sum of  $\Gamma_9 = B(\tau \rightarrow \pi^- \nu_\tau)$  and  $\Gamma_{10} = B(\tau \rightarrow K^- \nu_\tau)$ . The symbol  $h$  is used to mean either a  $\pi$  or  $K$ . Section 2.7 lists all equations relating one quantity to the sum of other quantities. In the following, we refer to both types of relations between fitted quantities collectively as constraint equations or constraints. The fit  $\chi^2$  is minimized subject to all the above mentioned constraints, listed in Table 1 and Section 2.7. The fit procedure is equivalent to that employed in the previous HFLAV reports [1, 3, 4].

### 2.1 Technical implementation of the fit procedure

The fit computes the quantities  $q_i$  by minimizing a  $\chi^2$  while respecting a series of equality constraints on the  $q_i$ . The  $\chi^2$  is computed using the measurements  $x_i$  and their covariance matrix  $V_{ij}$  as

$$\chi^2 = (x_i - A_{ik} q_k)^t V_{ij}^{-1} (x_j - A_{jl} q_l), \quad (1)$$

where the model matrix  $A_{ij}$  is used to get the vector of the predicted measurements  $x'_i$  from the vector of the fit parameters  $q_j$  as  $x'_i = A_{ij} q_j$ . In this particular implementation, the measurements are grouped according to the measured quantity, and all quantities with at least one measurement correspond to a fit parameter. Therefore, the matrix  $A_{ij}$  has one row per measurement  $x_i$  and one column per fitted quantity  $q_j$ , with unity coefficients for the rows and column that identify a measurement  $x_i$  of the quantity  $q_j$ . In summary, the  $\chi^2$  given in Eq. (1) is minimized subject to the constraints

$$f_r(q_s) - c_r = 0, \quad (2)$$

where Eq. (2) corresponds to the constraint equations, written as a set of “constraint expressions” that are equated to zero. Using the method of Lagrange multipliers, a set of equations is obtained by taking the derivatives with respect

to the fitted quantities  $q_k$  and the Lagrange multipliers  $\lambda_r$  of the sum of the  $\chi^2$  and the constraint expressions multiplied by the Lagrange multipliers  $\lambda_r$ , one for each constraint:

$$\min \left[ (A_{ik} q_k - x_i)^t V_{ij}^{-1} (A_{jl} q_l - x_j) + 2\lambda_r (f_r(q_s) - c_r) \right] \quad (3)$$

$$(\partial/\partial q_k, \partial/\partial \lambda_r) [\text{expression above}] = 0. \quad (4)$$

Equation (4) defines a set of equations for the vector of the unknowns  $(q_k, \lambda_r)$ , some of which may be non-linear, in case of non-linear constraints. An iterative minimization procedure approximates at each step the non-linear constraint expressions by their first order Taylor expansion around the current values of the fitted quantities,  $\bar{q}_s$ :

$$f_r(q_s) - c_r \simeq f_r(\bar{q}_s) + \frac{\partial f_r(q_s)}{\partial q_s} \Big|_{\bar{q}_s} (q_s - \bar{q}_s) - c_r, \quad (5)$$

which can be written as

$$B_{rs} q_s - c'_r, \quad (6)$$

where  $c'_r$  are the resulting constant known terms, independent of  $q_s$  at first order. After linearization, the differentiation by  $q_k$  and  $\lambda_r$  is trivial and leads to a set of linear equations

$$A_{ki}^t V_{ij}^{-1} A_{jl} q_l + B_{kr}^t \lambda_r = A_{ki}^t V_{ij}^{-1} x_j \quad (7)$$

$$B_{rs} q_s = c'_r, \quad (8)$$

which can be expressed as:

$$F_{ij} u_j = v_i, \quad (9)$$

where  $u_j = (q_k, \lambda_r)$  and  $v_i$  is the vector of the known constant terms running over the index  $k$  and then  $r$  in the right terms of Eq. (7) and Eq. (8). Solving the equation set in Eq. (9) gives the fitted quantities and their covariance matrix, using the measurements and their covariance matrix. The fit procedure starts by computing the linear approximation of the non-linear constraint expressions around the quantities seed values. With an iterative procedure, the unknowns are updated at each step by solving the equations and the equations are then linearized around the updated values, until the RMS average of relative variation of the fitted unknowns is reduced below  $10^{-12}$ .

## 2.2 Fit results

The fit output consists of 135 fitted quantities that correspond to either branching fractions or ratios of branching fractions. The fitted quantities values and uncertainties are listed in Table 1. The off-diagonal statistical correlation terms between a subset of 47 “basis quantities” are listed in Section 2.6. All the remaining statistical correlation terms can be obtained using the constraint equations listed in Table 1 and Section 2.7.

The fit has  $\chi^2/\text{d.o.f.} = 137/123$ , corresponding to a confidence level  $\text{CL} = 17.84\%$ . We use a total of 170 measurements to fit the above mentioned 135 quantities subjected to 88 constraints. Although the unitarity constraint is not applied, the fit is statistically consistent with unitarity, where the residual is  $\Gamma_{998} = 1 - \Gamma_{\text{All}} = (0.0355 \pm 0.1031) \cdot 10^{-2}$ .

A scale factor of 5.44 (as in the three previous reports [1, 3, 4]) has been applied to the published uncertainties of the two severely inconsistent measurements of  $\Gamma_{96} = \tau \rightarrow KKK\nu$  by *BABAR* and *Belle*. The scale factor has been determined using the PDG procedure, *i.e.*, to the proper size in order to obtain a reduced  $\chi^2$  equal to 1 when fitting just the two  $\Gamma_{96}$  measurements.

For several old results, for historical reasons, the table reports the total error (statistical plus systematic) in the position of the statistical error and zero in the position of the systematic error. Since the fit depends only on the total errors, the results are unaffected.

## 2.3 Changes with respect to the previous report

The following changes have been introduced with respect to the previous HFLAV report [4].

Two old preliminary results have been removed:

- $\Gamma_{35} = B(\tau \rightarrow \pi K_S \nu)$ , *BABAR* [5],
- $\Gamma_{40} = B(\tau \rightarrow \pi K_S \pi^0 \nu)$ , *BABAR* [6].

They were announced in 2008 and 2009 but have not been published.

In the 2014 report, for several *BABAR* and *Belle* experimental results we used more precise numerical values than the published ones, using internal information from the Collaborations. We revert to the published figures in this report, as the improvements in the fit results were negligible. In so doing, we use in this report the same values that are used in the PDG 2016 fit.

The *Belle* result on  $\tau^- \rightarrow K_S^0 (\text{particles})^- \nu_\tau$  [7] has been discarded, because it was determined that the published information does not permit a reliable determination of the correlations with the other results in the same paper. The correlations estimated for the HFLAV 2014 report were inconsistent. As a result, both the covariance matrix of the *Belle* results and the overall correlation matrix for the branching ratio fit results were non-positive-definite. It has been found that the inconsistency had negligible impact on lepton universality tests and on the  $|V_{us}|$  measurements.

The ALEPH result on  $\Gamma_{46} (\tau^- \rightarrow \pi^- K^0 \bar{K}^0 \nu_\tau)$  [8] has been removed from the fit inputs, since it is simply the sum of twice  $\Gamma_{47} = \pi^- K_S^0 K_S^0 \nu_\tau$  and  $\Gamma_{48} = \pi^- K_S^0 K_L^0 \nu_\tau$  from the same paper, hence 100% correlated with them.

Several minor corrections have been applied to the constraints. The list of constraints included in the following fully documents the changes when compared with the same list in the 2014 edition. In some cases the relation equating one decay mode to a sum of modes included some minor terms that did not match the mode definitions. In other cases, the sum included modes with overlapping components. The effects on the 2014 fit results have been found to be modest with respect to the quoted uncertainties. For instance, the definition of the total branching fraction has been updated as follows:

$$\begin{aligned} \Gamma_{\text{All}} = & \Gamma_3 + \Gamma_5 + \Gamma_9 + \Gamma_{10} + \Gamma_{14} + \Gamma_{16} + \Gamma_{20} + \Gamma_{23} + \Gamma_{27} + \Gamma_{28} + \Gamma_{30} + \Gamma_{35} + \Gamma_{37} + \Gamma_{40} + \Gamma_{42} + \Gamma_{47} \cdot (1 + ((\Gamma_{<K^0|K_L>} \cdot \\ & \Gamma_{<\bar{K}^0|K_L>} / (\Gamma_{<K^0|K_S>} \cdot \Gamma_{<\bar{K}^0|K_S>}))) + \Gamma_{48} + \Gamma_{62} + \Gamma_{70} + \Gamma_{77} + \Gamma_{811} + \Gamma_{812} + \Gamma_{93} + \Gamma_{94} + \Gamma_{832} + \Gamma_{833} + \Gamma_{126} + \\ & \Gamma_{128} + \Gamma_{802} + \Gamma_{803} + \Gamma_{800} + \Gamma_{151} + \Gamma_{130} + \Gamma_{132} + \Gamma_{44} + \Gamma_{53} + \Gamma_{50} \cdot (1 + ((\Gamma_{<K^0|K_L>} \cdot \Gamma_{<\bar{K}^0|K_L>} / (\Gamma_{<K^0|K_S>} \cdot \\ & \Gamma_{<\bar{K}^0|K_S>}))) + \Gamma_{51} + \Gamma_{167} \cdot (\Gamma_{\phi \rightarrow K+K-} + \Gamma_{\phi \rightarrow K_S K_L}) + \Gamma_{152} + \Gamma_{920} + \Gamma_{821} + \Gamma_{822} + \Gamma_{831} + \Gamma_{136} + \Gamma_{945} + \Gamma_{805} . \end{aligned}$$

In the 2014 definition, the term  $\Gamma_{78} = h^- h^- h^+ 3\pi^0 \nu_\tau$  included the contributions of  $\Gamma_{50} = \pi^- \pi^0 K_S^0 K_S^0 \nu_\tau$  and  $\Gamma_{132} = \pi^- \bar{K}^0 \eta \nu_\tau$ , which were already included explicitly in  $\Gamma_{\text{All}}$ . In the present definition,  $\Gamma_{78}$  has been replaced with modes whose sum corresponds to

$$\Gamma_{810} = 2\pi^- \pi^+ 3\pi^0 \nu_\tau \text{ (ex. } K^0\text{)} .$$

As in 2014, the total  $\tau$  branching fraction  $\Gamma_{\text{All}}$  definition includes two modes that have overlapping final states, to a minor extent, which we consider negligible:

$$\begin{aligned} \Gamma_{50} &= \pi^- \pi^0 K_S^0 K_S^0 \nu_\tau \\ \Gamma_{132} &= \pi^- \bar{K}^0 \eta \nu_\tau . \end{aligned}$$

Finally, we updated to the PDG 2015 results [9] all the parameters corresponding to the measurements' systematic biases and uncertainties and all the parameters appearing in the constraint equations in Section 2.7 and Table 1.

## 2.4 Differences between the HFLAV Spring 2017 fit and the PDG 2016 fit

As is standard for the PDG branching fraction fits, the PDG 2016  $\tau$  branching fraction fit is unitarity constrained, while the HFLAV 2016 fit is unconstrained.

The HFLAV-Tau fit uses the ALEPH measurements of branching fractions defined according to the final state content of "hadrons" and kaons, where a "hadron" corresponds to either a pion or a kaon, since this set of results is closer to the actual experimental measurements and facilitates a more comprehensive treatment of the experimental results correlations [1]. The PDG 2016 fit on the other hand continues to use – as in the past editions – the ALEPH measurements of modes with pions and kaons, which correspond to the final set of published measurements of the collaboration. It is planned eventually to update the PDG fit to use the same ALEPH measurement set that is used by HFLAV.

The HFLAV Spring 2017 fit, as in 2014, uses the ALEPH estimate for  $\Gamma_{805} = B(\tau \rightarrow a_1^- (\rightarrow \pi^-\gamma)\nu_\tau)$ , which is not a direct measurement. The PDG 2016 fit uses the PDG average of  $B(a_1 \rightarrow \pi\gamma)$  as a parameter and defines  $\Gamma_{805} = B(a_1 \rightarrow \pi\gamma) \times B(\tau \rightarrow 3\pi\nu)$ . As a consequence, the PDG fit procedure does not take into account the large uncertainty on  $B(a_1 \rightarrow \pi\gamma)$ , resulting in an underestimated fit uncertainty on  $\Gamma_{805}$ . Therefore, in this case an appropriate correction has to be applied after the fit.

## 2.5 Branching ratio fit results and experimental inputs

Table 1 reports the  $\tau$  branching ratio fit results and experimental inputs.

Table 1: HFLAV Spring 2017 branching fractions fit results.

$\tau$ lepton branching fraction	Fit value / Exp.	HFLAV Fit / Ref.
$\Gamma_1 = (\text{particles})^- \geq 0 \text{ neutrals} \geq 0 K^0 \nu_\tau$	$0.8519 \pm 0.0011$	HFLAV Spring 2017 fit
$\Gamma_2 = (\text{particles})^- \geq 0 \text{ neutrals} \geq 0 K_L^0 \nu_\tau$	$0.8453 \pm 0.0010$	HFLAV Spring 2017 fit
$\Gamma_3 = \mu^- \bar{\nu}_\mu \nu_\tau$	$0.17392 \pm 0.00040$	HFLAV Spring 2017 fit
$0.17319 \pm 0.00077 \pm 0.00000$	ALEPH	[10]
$0.17325 \pm 0.00095 \pm 0.00077$	DELPHI	[11]
$0.17342 \pm 0.00110 \pm 0.00067$	L3	[12]
$0.17340 \pm 0.00090 \pm 0.00060$	OPAL	[13]
$\frac{\Gamma_3}{\Gamma_5} = \frac{\mu^- \bar{\nu}_\mu \nu_\tau}{e^- \bar{\nu}_e \nu_\tau}$	$0.9762 \pm 0.0028$	HFLAV Spring 2017 fit
$0.9970 \pm 0.0350 \pm 0.0400$	ARGUS	[14]
$0.9796 \pm 0.0016 \pm 0.0036$	<i>BABAR</i>	[15]
$0.9777 \pm 0.0063 \pm 0.0087$	CLEO	[16]
$\Gamma_5 = e^- \bar{\nu}_e \nu_\tau$	$0.17816 \pm 0.00041$	HFLAV Spring 2017 fit
$0.17837 \pm 0.00080 \pm 0.00000$	ALEPH	[10]
$0.17760 \pm 0.00060 \pm 0.00170$	CLEO	[16]
$0.17877 \pm 0.00109 \pm 0.00110$	DELPHI	[11]
$0.17806 \pm 0.00104 \pm 0.00076$	L3	[12]
$0.17810 \pm 0.00090 \pm 0.00060$	OPAL	[17]
$\Gamma_7 = h^- \geq 0 K_L^0 \nu_\tau$	$0.12023 \pm 0.00054$	HFLAV Spring 2017 fit
$0.12400 \pm 0.00700 \pm 0.00700$	DELPHI	[18]
$0.12470 \pm 0.00260 \pm 0.00430$	L3	[19]
$0.12100 \pm 0.00700 \pm 0.00500$	OPAL	[20]
$\Gamma_8 = h^- \nu_\tau$	$0.11506 \pm 0.00054$	HFLAV Spring 2017 fit
$0.11524 \pm 0.00105 \pm 0.00000$	ALEPH	[10]
$0.11520 \pm 0.00050 \pm 0.00120$	CLEO	[16]
$0.11571 \pm 0.00120 \pm 0.00114$	DELPHI	[21]
$0.11980 \pm 0.00130 \pm 0.00160$	OPAL	[22]
$\frac{\Gamma_8}{\Gamma_5} = \frac{h^- \nu_\tau}{e^- \bar{\nu}_e \nu_\tau}$	$0.6458 \pm 0.0033$	HFLAV Spring 2017 fit
$\Gamma_9 = \pi^- \nu_\tau$	$0.10810 \pm 0.00053$	HFLAV Spring 2017 fit
$\frac{\Gamma_9}{\Gamma_5} = \frac{\pi^- \nu_\tau}{e^- \bar{\nu}_e \nu_\tau}$	$0.6068 \pm 0.0032$	HFLAV Spring 2017 fit
$0.5945 \pm 0.0014 \pm 0.0061$	<i>BABAR</i>	[15]
$\Gamma_{10} = K^- \nu_\tau$	$(0.6960 \pm 0.0096) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.6960 \pm 0.0287 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[23]

**Table 1 – continued from previous page**

$\tau$ lepton branching fraction	Fit value / Exp.	HFLAV Fit / Ref.
$(0.6600 \pm 0.0700 \pm 0.0900) \cdot 10^{-2}$	CLEO	[24]
$(0.8500 \pm 0.1800 \pm 0.0000) \cdot 10^{-2}$	DELPHI	[25]
$(0.6580 \pm 0.0270 \pm 0.0290) \cdot 10^{-2}$	OPAL	[26]
$\frac{\Gamma_{10}}{\Gamma_5} = \frac{K^- \nu_\tau}{e^- \bar{\nu}_e \nu_\tau}$	$(3.906 \pm 0.054) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(3.882 \pm 0.032 \pm 0.057) \cdot 10^{-2}$	<i>BABAR</i>	[15]
$\frac{\Gamma_{10}}{\Gamma_9} = \frac{K^- \nu_\tau}{\pi^- \nu_\tau}$	$(6.438 \pm 0.094) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{11} = h^- \geq 1 \text{ neutrals } \nu_\tau$	$0.36973 \pm 0.00097$	HFLAV Spring 2017 fit
$\Gamma_{12} = h^- \geq 1 \pi^0 \nu_\tau \text{ (ex. } K^0)$	$0.36475 \pm 0.00097$	HFLAV Spring 2017 fit
$\Gamma_{13} = h^- \pi^0 \nu_\tau$	$0.25935 \pm 0.00091$	HFLAV Spring 2017 fit
$0.25924 \pm 0.00129 \pm 0.00000$	ALEPH	[10]
$0.25670 \pm 0.00010 \pm 0.00390$	Belle	[27]
$0.25870 \pm 0.00120 \pm 0.00420$	CLEO	[28]
$0.25740 \pm 0.00201 \pm 0.00138$	DELPHI	[21]
$0.25050 \pm 0.00350 \pm 0.00500$	L3	[19]
$0.25890 \pm 0.00170 \pm 0.00290$	OPAL	[22]
$\Gamma_{14} = \pi^- \pi^0 \nu_\tau$	$0.25502 \pm 0.00092$	HFLAV Spring 2017 fit
$\Gamma_{16} = K^- \pi^0 \nu_\tau$	$(0.4327 \pm 0.0149) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.4440 \pm 0.0354 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[23]
$(0.4160 \pm 0.0030 \pm 0.0180) \cdot 10^{-2}$	<i>BABAR</i>	[29]
$(0.5100 \pm 0.1000 \pm 0.0700) \cdot 10^{-2}$	CLEO	[24]
$(0.4710 \pm 0.0590 \pm 0.0230) \cdot 10^{-2}$	OPAL	[30]
$\Gamma_{17} = h^- \geq 2 \pi^0 \nu_\tau$	$0.10775 \pm 0.00095$	HFLAV Spring 2017 fit
$0.09910 \pm 0.00310 \pm 0.00270$	OPAL	[22]
$\Gamma_{18} = h^- 2\pi^0 \nu_\tau$	$(9.458 \pm 0.097) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{19} = h^- 2\pi^0 \nu_\tau \text{ (ex. } K^0)$	$(9.306 \pm 0.097) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(9.295 \pm 0.122 \pm 0.000) \cdot 10^{-2}$	ALEPH	[10]
$(9.498 \pm 0.320 \pm 0.275) \cdot 10^{-2}$	DELPHI	[21]
$(8.880 \pm 0.370 \pm 0.420) \cdot 10^{-2}$	L3	[19]
$\frac{\Gamma_{19}}{\Gamma_{13}} = \frac{h^- 2\pi^0 \nu_\tau \text{ (ex. } K^0)}{h^- \pi^0 \nu_\tau}$	$0.3588 \pm 0.0044$	HFLAV Spring 2017 fit
$0.3420 \pm 0.0060 \pm 0.0160$	CLEO	[31]
$\Gamma_{20} = \pi^- 2\pi^0 \nu_\tau \text{ (ex. } K^0)$	$(9.242 \pm 0.100) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{23} = K^- 2\pi^0 \nu_\tau \text{ (ex. } K^0)$	$(0.0640 \pm 0.0220) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.0560 \pm 0.0250 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[23]
$(0.0900 \pm 0.1000 \pm 0.0300) \cdot 10^{-2}$	CLEO	[24]
$\Gamma_{24} = h^- \geq 3 \pi^0 \nu_\tau$	$(1.318 \pm 0.065) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{25} = h^- \geq 3 \pi^0 \nu_\tau \text{ (ex. } K^0)$	$(1.233 \pm 0.065) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(1.403 \pm 0.214 \pm 0.224) \cdot 10^{-2}$	DELPHI	[21]
$\Gamma_{26} = h^- 3\pi^0 \nu_\tau$	$(1.158 \pm 0.072) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(1.082 \pm 0.093 \pm 0.000) \cdot 10^{-2}$	ALEPH	[10]

**Table 1 – continued from previous page**

$\tau$ lepton branching fraction	Fit value / Exp.	HFLAV Fit / Ref.
$(1.700 \pm 0.240 \pm 0.380) \cdot 10^{-2}$	L3	[19]
$\frac{\Gamma_{26}}{\Gamma_{13}} = \frac{h^- 3\pi^0 \nu_\tau}{h^- \pi^0 \nu_\tau}$	$(4.465 \pm 0.277) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(4.400 \pm 0.300 \pm 0.500) \cdot 10^{-2}$	CLEO	[31]
$\Gamma_{27} = \pi^- 3\pi^0 \nu_\tau$ (ex. $K^0$ )	$(1.029 \pm 0.075) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{28} = K^- 3\pi^0 \nu_\tau$ (ex. $K^0, \eta$ )	$(4.283 \pm 2.161) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$(3.700 \pm 2.371 \pm 0.000) \cdot 10^{-4}$	ALEPH	[23]
$\Gamma_{29} = h^- 4\pi^0 \nu_\tau$ (ex. $K^0$ )	$(0.1568 \pm 0.0391) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.1600 \pm 0.0500 \pm 0.0500) \cdot 10^{-2}$	CLEO	[31]
$\Gamma_{30} = h^- 4\pi^0 \nu_\tau$ (ex. $K^0, \eta$ )	$(0.1099 \pm 0.0391) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.1120 \pm 0.0509 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[10]
$\Gamma_{31} = K^- \geq 0 \pi^0 \geq 0 K^0 \geq 0 \gamma \nu_\tau$	$(1.545 \pm 0.030) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(1.700 \pm 0.120 \pm 0.190) \cdot 10^{-2}$	CLEO	[24]
$(1.540 \pm 0.240 \pm 0.000) \cdot 10^{-2}$	DELPHI	[25]
$(1.528 \pm 0.039 \pm 0.040) \cdot 10^{-2}$	OPAL	[26]
$\Gamma_{32} = K^- \geq 1 (\pi^0 \text{ or } K^0 \text{ or } \gamma) \nu_\tau$	$(0.8528 \pm 0.0286) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{33} = K_S^0 (\text{particles})^- \nu_\tau$	$(0.9372 \pm 0.0292) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.9700 \pm 0.0849 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[8]
$(0.9700 \pm 0.0900 \pm 0.0600) \cdot 10^{-2}$	OPAL	[32]
$\Gamma_{34} = h^- \bar{K}^0 \nu_\tau$	$(0.9865 \pm 0.0139) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.8550 \pm 0.0360 \pm 0.0730) \cdot 10^{-2}$	CLEO	[33]
$\Gamma_{35} = \pi^- \bar{K}^0 \nu_\tau$	$(0.8386 \pm 0.0141) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.9280 \pm 0.0564 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[23]
$(0.8320 \pm 0.0025 \pm 0.0150) \cdot 10^{-2}$	Belle	[7]
$(0.9500 \pm 0.1500 \pm 0.0600) \cdot 10^{-2}$	L3	[34]
$(0.9330 \pm 0.0680 \pm 0.0490) \cdot 10^{-2}$	OPAL	[35]
$\Gamma_{37} = K^- K^0 \nu_\tau$	$(0.1479 \pm 0.0053) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.1580 \pm 0.0453 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[8]
$(0.1620 \pm 0.0237 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[23]
$(0.1480 \pm 0.0013 \pm 0.0055) \cdot 10^{-2}$	Belle	[7]
$(0.1510 \pm 0.0210 \pm 0.0220) \cdot 10^{-2}$	CLEO	[33]
$\Gamma_{38} = K^- K^0 \geq 0 \pi^0 \nu_\tau$	$(0.2982 \pm 0.0079) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.3300 \pm 0.0550 \pm 0.0390) \cdot 10^{-2}$	OPAL	[35]
$\Gamma_{39} = h^- \bar{K}^0 \pi^0 \nu_\tau$	$(0.5314 \pm 0.0134) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.5620 \pm 0.0500 \pm 0.0480) \cdot 10^{-2}$	CLEO	[33]
$\Gamma_{40} = \pi^- \bar{K}^0 \pi^0 \nu_\tau$	$(0.3812 \pm 0.0129) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.2940 \pm 0.0818 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[8]
$(0.3470 \pm 0.0646 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[23]
$(0.3860 \pm 0.0031 \pm 0.0135) \cdot 10^{-2}$	Belle	[7]
$(0.4100 \pm 0.1200 \pm 0.0300) \cdot 10^{-2}$	L3	[34]
$\Gamma_{42} = K^- \pi^0 K^0 \nu_\tau$	$(0.1502 \pm 0.0071) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.1520 \pm 0.0789 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[8]

**Table 1 – continued from previous page**

$\tau$ lepton branching fraction	Fit value / Exp.	HFLAV Fit / Ref.
$(0.1430 \pm 0.0291 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[23]
$(0.1496 \pm 0.0019 \pm 0.0073) \cdot 10^{-2}$	Belle	[7]
$(0.1450 \pm 0.0360 \pm 0.0200) \cdot 10^{-2}$	CLEO	[33]
$\Gamma_{43} = \pi^- \bar{K}^0 \geq 1 \pi^0 \nu_\tau$	$(0.4046 \pm 0.0260) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.3240 \pm 0.0740 \pm 0.0660) \cdot 10^{-2}$	OPAL	[35]
$\Gamma_{44} = \pi^- \bar{K}^0 \pi^0 \pi^0 \nu_\tau$ (ex. $K^0$ )	$(2.340 \pm 2.306) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$(2.600 \pm 2.400 \pm 0.000) \cdot 10^{-4}$	ALEPH	[36]
$\Gamma_{46} = \pi^- K^0 \bar{K}^0 \nu_\tau$	$(0.1513 \pm 0.0247) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{47} = \pi^- K_S^0 K_S^0 \nu_\tau$	$(2.332 \pm 0.065) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$(2.600 \pm 1.118 \pm 0.000) \cdot 10^{-4}$	ALEPH	[8]
$(2.310 \pm 0.040 \pm 0.080) \cdot 10^{-4}$	BABAR	[37]
$(2.330 \pm 0.033 \pm 0.093) \cdot 10^{-4}$	Belle	[7]
$(2.300 \pm 0.500 \pm 0.300) \cdot 10^{-4}$	CLEO	[33]
$\Gamma_{48} = \pi^- K_S^0 K_L^0 \nu_\tau$	$(0.1047 \pm 0.0247) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.1010 \pm 0.0264 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[8]
$\Gamma_{49} = \pi^- K^0 \bar{K}^0 \pi^0 \nu_\tau$	$(3.540 \pm 1.193) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$\Gamma_{50} = \pi^- \pi^0 K_S^0 K_S^0 \nu_\tau$	$(1.815 \pm 0.207) \cdot 10^{-5}$	HFLAV Spring 2017 fit
$(1.600 \pm 0.200 \pm 0.220) \cdot 10^{-5}$	BABAR	[37]
$(2.000 \pm 0.216 \pm 0.202) \cdot 10^{-5}$	Belle	[7]
$\Gamma_{51} = \pi^- \pi^0 K_S^0 K_L^0 \nu_\tau$	$(3.177 \pm 1.192) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$(3.100 \pm 1.100 \pm 0.500) \cdot 10^{-4}$	ALEPH	[8]
$\Gamma_{53} = \bar{K}^0 h^- h^- h^+ \nu_\tau$	$(2.218 \pm 2.024) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$(2.300 \pm 2.025 \pm 0.000) \cdot 10^{-4}$	ALEPH	[8]
$\Gamma_{54} = h^- h^- h^+ \geq 0 \text{ neutrals} \geq 0 K_L^0 \nu_\tau$	$0.15215 \pm 0.00061$	HFLAV Spring 2017 fit
$0.15000 \pm 0.00400 \pm 0.00300$	CELLO	[38]
$0.14400 \pm 0.00600 \pm 0.00300$	L3	[39]
$0.15100 \pm 0.00800 \pm 0.00600$	TPC	[40]
$\Gamma_{55} = h^- h^- h^+ \geq 0 \text{ neutrals} \nu_\tau$ (ex. $K^0$ )	$0.14567 \pm 0.00057$	HFLAV Spring 2017 fit
$0.14556 \pm 0.00105 \pm 0.00076$	L3	[41]
$0.14960 \pm 0.00090 \pm 0.00220$	OPAL	[42]
$\Gamma_{56} = h^- h^- h^+ \nu_\tau$	$(9.780 \pm 0.054) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{57} = h^- h^- h^+ \nu_\tau$ (ex. $K^0$ )	$(9.439 \pm 0.053) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(9.510 \pm 0.070 \pm 0.200) \cdot 10^{-2}$	CLEO	[43]
$(9.317 \pm 0.090 \pm 0.082) \cdot 10^{-2}$	DELPHI	[21]
$\frac{\Gamma_{57}}{\Gamma_{55}} = \frac{h^- h^- h^+ \nu_\tau \text{ (ex. } K^0\text{)}}{h^- h^- h^+ \geq 0 \text{ neutrals} \nu_\tau \text{ (ex. } K^0\text{)}}$	$0.6480 \pm 0.0030$	HFLAV Spring 2017 fit
$0.6600 \pm 0.0040 \pm 0.0140$	OPAL	[42]
$\Gamma_{58} = h^- h^- h^+ \nu_\tau$ (ex. $K^0, \omega$ )	$(9.408 \pm 0.053) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(9.469 \pm 0.096 \pm 0.000) \cdot 10^{-2}$	ALEPH	[10]
$\Gamma_{59} = \pi^- \pi^+ \pi^- \nu_\tau$	$(9.290 \pm 0.052) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{60} = \pi^- \pi^+ \pi^- \nu_\tau$ (ex. $K^0$ )	$(9.000 \pm 0.051) \cdot 10^{-2}$	HFLAV Spring 2017 fit

**Table 1 – continued from previous page**

$\tau$ lepton branching fraction	Fit value / Exp.	HFLAV Fit / Ref.
$(8.830 \pm 0.010 \pm 0.130) \cdot 10^{-2}$	<i>BABAR</i>	[44]
$(8.420 \pm 0.000^{+0.260}_{-0.250}) \cdot 10^{-2}$	<i>Belle</i>	[45]
$(9.130 \pm 0.050 \pm 0.460) \cdot 10^{-2}$	<i>CLEO3</i>	[46]
$\Gamma_{62} = \pi^- \pi^- \pi^+ \nu_\tau$ (ex. $K^0, \omega$ )	$(8.970 \pm 0.052) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{63} = h^- h^- h^+ \geq 1$ neutrals $\nu_\tau$	$(5.325 \pm 0.050) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{64} = h^- h^- h^+ \geq 1 \pi^0 \nu_\tau$ (ex. $K^0$ )	$(5.120 \pm 0.049) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{65} = h^- h^- h^+ \pi^0 \nu_\tau$	$(4.790 \pm 0.052) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{66} = h^- h^- h^+ \pi^0 \nu_\tau$ (ex. $K^0$ )	$(4.606 \pm 0.051) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(4.734 \pm 0.077 \pm 0.000) \cdot 10^{-2}$	<i>ALEPH</i>	[10]
$(4.230 \pm 0.060 \pm 0.220) \cdot 10^{-2}$	<i>CLEO</i>	[43]
$(4.545 \pm 0.106 \pm 0.103) \cdot 10^{-2}$	<i>DELPHI</i>	[21]
$\Gamma_{67} = h^- h^- h^+ \pi^0 \nu_\tau$ (ex. $K^0, \omega$ )	$(2.820 \pm 0.070) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{68} = \pi^- \pi^+ \pi^- \pi^0 \nu_\tau$	$(4.651 \pm 0.053) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{69} = \pi^- \pi^+ \pi^- \pi^0 \nu_\tau$ (ex. $K^0$ )	$(4.519 \pm 0.052) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(4.190 \pm 0.100 \pm 0.210) \cdot 10^{-2}$	<i>CLEO</i>	[47]
$\Gamma_{70} = \pi^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. $K^0, \omega$ )	$(2.769 \pm 0.071) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{74} = h^- h^- h^+ \geq 2 \pi^0 \nu_\tau$ (ex. $K^0$ )	$(0.5135 \pm 0.0312) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.5610 \pm 0.0680 \pm 0.0950) \cdot 10^{-2}$	<i>DELPHI</i>	[21]
$\Gamma_{75} = h^- h^- h^+ 2\pi^0 \nu_\tau$	$(0.5024 \pm 0.0310) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{76} = h^- h^- h^+ 2\pi^0 \nu_\tau$ (ex. $K^0$ )	$(0.4925 \pm 0.0310) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.4350 \pm 0.0461 \pm 0.0000) \cdot 10^{-2}$	<i>ALEPH</i>	[10]
$\frac{\Gamma_{76}}{\Gamma_{54}} = \frac{h^- h^- h^+ 2\pi^0 \nu_\tau}{h^- h^- h^+ \geq 0 \text{ neutrals} \geq 0 K_L^0 \nu_\tau}$	$(3.237 \pm 0.202) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(3.400 \pm 0.200 \pm 0.300) \cdot 10^{-2}$	<i>CLEO</i>	[48]
$\Gamma_{77} = h^- h^- h^+ 2\pi^0 \nu_\tau$ (ex. $K^0, \omega, \eta$ )	$(9.759 \pm 3.550) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$\Gamma_{78} = h^- h^- h^+ 3\pi^0 \nu_\tau$	$(2.107 \pm 0.299) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$(2.200 \pm 0.300 \pm 0.400) \cdot 10^{-4}$	<i>CLEO</i>	[49]
$\Gamma_{79} = K^- h^- h^+ \geq 0$ neutrals $\nu_\tau$	$(0.6297 \pm 0.0141) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{80} = K^- \pi^- h^+ \nu_\tau$ (ex. $K^0$ )	$(0.4363 \pm 0.0073) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\frac{\Gamma_{80}}{\Gamma_{60}} = \frac{K^- \pi^- h^+ \nu_\tau}{\pi^- \pi^+ \pi^- \nu_\tau}$ (ex. $K^0$ )	$(4.847 \pm 0.080) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(5.440 \pm 0.210 \pm 0.530) \cdot 10^{-2}$	<i>CLEO</i>	[50]
$\Gamma_{81} = K^- \pi^- h^+ \pi^0 \nu_\tau$ (ex. $K^0$ )	$(8.726 \pm 1.177) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$\frac{\Gamma_{81}}{\Gamma_{69}} = \frac{K^- \pi^- h^+ \pi^0 \nu_\tau}{\pi^- \pi^+ \pi^- \pi^0 \nu_\tau}$ (ex. $K^0$ )	$(1.931 \pm 0.266) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(2.610 \pm 0.450 \pm 0.420) \cdot 10^{-2}$	<i>CLEO</i>	[50]
$\Gamma_{82} = K^- \pi^- \pi^+ \geq 0$ neutrals $\nu_\tau$	$(0.4780 \pm 0.0137) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.5800^{+0.1500}_{-0.1300} \pm 0.1200) \cdot 10^{-2}$	<i>TPC</i>	[51]
$\Gamma_{83} = K^- \pi^- \pi^+ \geq 0 \pi^0 \nu_\tau$ (ex. $K^0$ )	$(0.3741 \pm 0.0135) \cdot 10^{-2}$	HFLAV Spring 2017 fit

**Table 1 – continued from previous page**

$\tau$ lepton branching fraction	Fit value / Exp.	HFLAV Fit / Ref.
$\Gamma_{84} = K^- \pi^- \pi^+ \nu_\tau$	$(0.3441 \pm 0.0070) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{85} = K^- \pi^+ \pi^- \nu_\tau$ (ex. $K^0$ ) $(0.2140 \pm 0.0470 \pm 0.0000) \cdot 10^{-2}$ $(0.2730 \pm 0.0020 \pm 0.0090) \cdot 10^{-2}$ $(0.3300 \pm 0.0010^{+0.0160}_{-0.0170}) \cdot 10^{-2}$ $(0.3840 \pm 0.0140 \pm 0.0380) \cdot 10^{-2}$ $(0.4150 \pm 0.0530 \pm 0.0400) \cdot 10^{-2}$	$(0.2929 \pm 0.0067) \cdot 10^{-2}$ ALEPH <i>BABAR</i> Belle CLEO3 OPAL	HFLAV Spring 2017 fit [52] [44] [45] [46] [30]
$\frac{\Gamma_{85}}{\Gamma_{60}} = \frac{K^- \pi^+ \pi^- \nu_\tau}{\pi^- \pi^+ \pi^- \nu_\tau}$ (ex. $K^0$ )	$(3.254 \pm 0.074) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{87} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$	$(0.1331 \pm 0.0119) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{88} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. $K^0$ ) $(6.100 \pm 4.295 \pm 0.000) \cdot 10^{-4}$ $(7.400 \pm 0.800 \pm 1.100) \cdot 10^{-4}$	$(8.115 \pm 1.168) \cdot 10^{-4}$ ALEPH CLEO3	HFLAV Spring 2017 fit [52] [53]
$\Gamma_{89} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. $K^0, \eta$ )	$(7.761 \pm 1.168) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$\Gamma_{92} = \pi^- K^- K^+ \geq 0$ neutrals $\nu_\tau$ $(0.1590 \pm 0.0530 \pm 0.0200) \cdot 10^{-2}$ $(0.1500^{+0.0900}_{-0.0700} \pm 0.0300) \cdot 10^{-2}$	$(0.1495 \pm 0.0033) \cdot 10^{-2}$ OPAL TPC	HFLAV Spring 2017 fit [54] [51]
$\Gamma_{93} = \pi^- K^- K^+ \nu_\tau$ $(0.1630 \pm 0.0270 \pm 0.0000) \cdot 10^{-2}$ $(0.1346 \pm 0.0010 \pm 0.0036) \cdot 10^{-2}$ $(0.1550 \pm 0.0010^{+0.0060}_{-0.0050}) \cdot 10^{-2}$ $(0.1550 \pm 0.0060 \pm 0.0090) \cdot 10^{-2}$	$(0.1434 \pm 0.0027) \cdot 10^{-2}$ ALEPH <i>BABAR</i> Belle CLEO3	HFLAV Spring 2017 fit [52] [44] [45] [46]
$\frac{\Gamma_{93}}{\Gamma_{60}} = \frac{\pi^- K^- K^+ \nu_\tau}{\pi^- \pi^+ \pi^- \nu_\tau}$ (ex. $K^0$ ) $(1.600 \pm 0.150 \pm 0.300) \cdot 10^{-2}$	$(1.593 \pm 0.030) \cdot 10^{-2}$ CLEO	HFLAV Spring 2017 fit [50]
$\Gamma_{94} = \pi^- K^- K^+ \pi^0 \nu_\tau$ $(7.500 \pm 3.265 \pm 0.000) \cdot 10^{-4}$ $(0.550 \pm 0.140 \pm 0.120) \cdot 10^{-4}$	$(0.611 \pm 0.183) \cdot 10^{-4}$ ALEPH CLEO3	HFLAV Spring 2017 fit [52] [53]
$\frac{\Gamma_{94}}{\Gamma_{69}} = \frac{\pi^- K^- K^+ \pi^0 \nu_\tau}{\pi^- \pi^+ \pi^- \pi^0 \nu_\tau}$ (ex. $K^0$ ) $(0.7900 \pm 0.4400 \pm 0.1600) \cdot 10^{-2}$	$(0.1353 \pm 0.0405) \cdot 10^{-2}$ CLEO	HFLAV Spring 2017 fit [50]
$\Gamma_{96} = K^- K^- K^+ \nu_\tau$ $(1.578 \pm 0.130 \pm 0.123) \cdot 10^{-5}$ $(3.290 \pm 0.170^{+0.190}_{-0.200}) \cdot 10^{-5}$	$(2.174 \pm 0.800) \cdot 10^{-5}$ <i>BABAR</i> Belle	HFLAV Spring 2017 fit [44] [45]
$\Gamma_{102} = 3h^- 2h^+ \geq 0$ neutrals $\nu_\tau$ (ex. $K^0$ ) $(0.0970 \pm 0.0050 \pm 0.0110) \cdot 10^{-2}$ $(0.1020 \pm 0.0290 \pm 0.0000) \cdot 10^{-2}$ $(0.1700 \pm 0.0220 \pm 0.0260) \cdot 10^{-2}$	$(0.0985 \pm 0.0037) \cdot 10^{-2}$ CLEO HRS L3	HFLAV Spring 2017 fit [55] [56] [41]
$\Gamma_{103} = 3h^- 2h^+ \nu_\tau$ (ex. $K^0$ ) $(7.200 \pm 1.500 \pm 0.000) \cdot 10^{-4}$ $(6.400 \pm 2.300 \pm 1.000) \cdot 10^{-4}$ $(7.700 \pm 0.500 \pm 0.900) \cdot 10^{-4}$ $(9.700 \pm 1.500 \pm 0.500) \cdot 10^{-4}$	$(8.216 \pm 0.316) \cdot 10^{-4}$ ALEPH ARGUS CLEO DELPHI	HFLAV Spring 2017 fit [10] [57] [55] [21]

**Table 1 – continued from previous page**

$\tau$ lepton branching fraction	Fit value / Exp.	HFLAV Fit / Ref.
$(5.100 \pm 2.000 \pm 0.000) \cdot 10^{-4}$	HRS	[56]
$(9.100 \pm 1.400 \pm 0.600) \cdot 10^{-4}$	OPAL	[58]
$\Gamma_{104} = 3h^- 2h^+ \pi^0 \nu_\tau$ (ex. $K^0$ )	$(1.634 \pm 0.114) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$(2.100 \pm 0.700 \pm 0.900) \cdot 10^{-4}$	ALEPH	[10]
$(1.700 \pm 0.200 \pm 0.200) \cdot 10^{-4}$	CLEO	[49]
$(1.600 \pm 1.200 \pm 0.600) \cdot 10^{-4}$	DELPHI	[21]
$(2.700 \pm 1.800 \pm 0.900) \cdot 10^{-4}$	OPAL	[58]
$\Gamma_{106} = (5\pi)^- \nu_\tau$	$(0.7748 \pm 0.0534) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{110} = X_s^- \nu_\tau$	$(2.909 \pm 0.048) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{126} = \pi^- \pi^0 \eta \nu_\tau$	$(0.1386 \pm 0.0072) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.1800 \pm 0.0447 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[59]
$(0.1350 \pm 0.0030 \pm 0.0070) \cdot 10^{-2}$	Belle	[60]
$(0.1700 \pm 0.0200 \pm 0.0200) \cdot 10^{-2}$	CLEO	[61]
$\Gamma_{128} = K^- \eta \nu_\tau$	$(1.547 \pm 0.080) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$(2.900^{+1.300}_{-1.200} \pm 0.700) \cdot 10^{-4}$	ALEPH	[59]
$(1.420 \pm 0.110 \pm 0.070) \cdot 10^{-4}$	BABAR	[62]
$(1.580 \pm 0.050 \pm 0.090) \cdot 10^{-4}$	Belle	[60]
$(2.600 \pm 0.500 \pm 0.500) \cdot 10^{-4}$	CLEO	[63]
$\Gamma_{130} = K^- \pi^0 \eta \nu_\tau$	$(0.483 \pm 0.116) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$(0.460 \pm 0.110 \pm 0.040) \cdot 10^{-4}$	Belle	[60]
$(1.770 \pm 0.560 \pm 0.710) \cdot 10^{-4}$	CLEO	[64]
$\Gamma_{132} = \pi^- \bar{K}^0 \eta \nu_\tau$	$(0.937 \pm 0.149) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$(0.880 \pm 0.140 \pm 0.060) \cdot 10^{-4}$	Belle	[60]
$(2.200 \pm 0.700 \pm 0.220) \cdot 10^{-4}$	CLEO	[64]
$\Gamma_{136} = \pi^- \pi^+ \pi^- \eta \nu_\tau$ (ex. $K^0$ )	$(2.184 \pm 0.130) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$\Gamma_{149} = h^- \omega \geq 0 \text{ neutrals} \nu_\tau$	$(2.401 \pm 0.075) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{150} = h^- \omega \nu_\tau$	$(1.995 \pm 0.064) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(1.910 \pm 0.092 \pm 0.000) \cdot 10^{-2}$	ALEPH	[59]
$(1.600 \pm 0.270 \pm 0.410) \cdot 10^{-2}$	CLEO	[65]
$\frac{\Gamma_{150}}{\Gamma_{66}} = \frac{h^- \omega \nu_\tau}{h^- h^- h^+ \pi^0 \nu_\tau \text{ (ex. } K^0)}$	$0.4332 \pm 0.0139$	HFLAV Spring 2017 fit
$0.4310 \pm 0.0330 \pm 0.0000$	ALEPH	[66]
$0.4640 \pm 0.0160 \pm 0.0170$	CLEO	[43]
$\Gamma_{151} = K^- \omega \nu_\tau$	$(4.100 \pm 0.922) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$(4.100 \pm 0.600 \pm 0.700) \cdot 10^{-4}$	CLEO3	[53]
$\Gamma_{152} = h^- \pi^0 \omega \nu_\tau$	$(0.4058 \pm 0.0419) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$(0.4300 \pm 0.0781 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[59]
$\frac{\Gamma_{152}}{\Gamma_{54}} = \frac{h^- \omega \pi^0 \nu_\tau}{h^- h^- h^+ \geq 0 \text{ neutrals} \geq 0 K_L^0 \nu_\tau}$	$(2.667 \pm 0.275) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\frac{\Gamma_{152}}{\Gamma_{76}} = \frac{h^- \omega \pi^0 \nu_\tau}{h^- h^- h^+ + 2\pi^0 \nu_\tau \text{ (ex. } K^0)}$	$0.8241 \pm 0.0757$	HFLAV Spring 2017 fit
$0.8100 \pm 0.0600 \pm 0.0600$	CLEO	[48]

**Table 1 – continued from previous page**

$\tau$ lepton branching fraction	Fit value / Exp.	HFLAV Fit / Ref.
$\Gamma_{167} = K^- \phi \nu_\tau$	$(4.445 \pm 1.636) \cdot 10^{-5}$	HFLAV Spring 2017 fit
$\Gamma_{168} = K^- \phi \nu_\tau (\phi \rightarrow K^+ K^-)$	$(2.174 \pm 0.800) \cdot 10^{-5}$	HFLAV Spring 2017 fit
$\Gamma_{169} = K^- \phi \nu_\tau (\phi \rightarrow K_S^0 K_L^0)$	$(1.520 \pm 0.560) \cdot 10^{-5}$	HFLAV Spring 2017 fit
$\Gamma_{800} = \pi^- \omega \nu_\tau$	$(1.954 \pm 0.065) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{802} = K^- \pi^- \pi^+ \nu_\tau$ (ex. $K^0, \omega$ )	$(0.2923 \pm 0.0067) \cdot 10^{-2}$	HFLAV Spring 2017 fit
$\Gamma_{803} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. $K^0, \omega, \eta$ )	$(4.103 \pm 1.429) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$\Gamma_{804} = \pi^- K_L^0 K_L^0 \nu_\tau$	$(2.332 \pm 0.065) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$\Gamma_{805} = a_1^- (\rightarrow \pi^- \gamma) \nu_\tau$ $(4.000 \pm 2.000 \pm 0.000) \cdot 10^{-4}$	$(4.000 \pm 2.000) \cdot 10^{-4}$ ALEPH	HFLAV Spring 2017 fit [10]
$\Gamma_{806} = \pi^- \pi^0 K_L^0 K_L^0 \nu_\tau$	$(1.815 \pm 0.207) \cdot 10^{-5}$	HFLAV Spring 2017 fit
$\Gamma_{810} = 2\pi^- \pi^+ 3\pi^0 \nu_\tau$ (ex. $K^0$ )	$(1.924 \pm 0.298) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$\Gamma_{811} = \pi^- 2\pi^0 \omega \nu_\tau$ (ex. $K^0$ ) $(7.300 \pm 1.200 \pm 1.200) \cdot 10^{-5}$	$(7.105 \pm 1.586) \cdot 10^{-5}$ BABAR	HFLAV Spring 2017 fit [67]
$\Gamma_{812} = 2\pi^- \pi^+ 3\pi^0 \nu_\tau$ (ex. $K^0, \eta, \omega, f_1$ ) $(1.000 \pm 0.800 \pm 3.000) \cdot 10^{-5}$	$(1.344 \pm 2.683) \cdot 10^{-5}$ BABAR	HFLAV Spring 2017 fit [67]
$\Gamma_{820} = 3\pi^- 2\pi^+ \nu_\tau$ (ex. $K^0, \omega$ )	$(8.197 \pm 0.315) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$\Gamma_{821} = 3\pi^- 2\pi^+ \nu_\tau$ (ex. $K^0, \omega, f_1$ ) $(7.680 \pm 0.040 \pm 0.400) \cdot 10^{-4}$	$(7.677 \pm 0.297) \cdot 10^{-4}$ BABAR	HFLAV Spring 2017 fit [67]
$\Gamma_{822} = K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. $K^0$ ) $(0.600 \pm 0.500 \pm 1.100) \cdot 10^{-6}$	$(0.596 \pm 1.208) \cdot 10^{-6}$ BABAR	HFLAV Spring 2017 fit [67]
$\Gamma_{830} = 3\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. $K^0$ )	$(1.623 \pm 0.114) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$\Gamma_{831} = 2\pi^- \pi^+ \omega \nu_\tau$ (ex. $K^0$ ) $(8.400 \pm 0.400 \pm 0.600) \cdot 10^{-5}$	$(8.359 \pm 0.626) \cdot 10^{-5}$ BABAR	HFLAV Spring 2017 fit [67]
$\Gamma_{832} = 3\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. $K^0, \eta, \omega, f_1$ ) $(3.600 \pm 0.300 \pm 0.900) \cdot 10^{-5}$	$(3.771 \pm 0.875) \cdot 10^{-5}$ BABAR	HFLAV Spring 2017 fit [67]
$\Gamma_{833} = K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. $K^0$ ) $(1.100 \pm 0.400 \pm 0.400) \cdot 10^{-6}$	$(1.108 \pm 0.566) \cdot 10^{-6}$ BABAR	HFLAV Spring 2017 fit [67]
$\Gamma_{910} = 2\pi^- \pi^+ \eta \nu_\tau$ ( $\eta \rightarrow 3\pi^0$ ) (ex. $K^0$ ) $(8.270 \pm 0.880 \pm 0.810) \cdot 10^{-5}$	$(7.136 \pm 0.424) \cdot 10^{-5}$ BABAR	HFLAV Spring 2017 fit [67]
$\Gamma_{911} = \pi^- 2\pi^0 \eta \nu_\tau$ ( $\eta \rightarrow \pi^+ \pi^- \pi^0$ ) (ex. $K^0$ ) $(4.570 \pm 0.770 \pm 0.500) \cdot 10^{-5}$	$(4.420 \pm 0.867) \cdot 10^{-5}$ BABAR	HFLAV Spring 2017 fit [67]
$\Gamma_{920} = \pi^- f_1 \nu_\tau$ ( $f_1 \rightarrow 2\pi^- 2\pi^+$ ) $(5.200 \pm 0.310 \pm 0.370) \cdot 10^{-5}$	$(5.197 \pm 0.444) \cdot 10^{-5}$ BABAR	HFLAV Spring 2017 fit [67]
$\Gamma_{930} = 2\pi^- \pi^+ \eta \nu_\tau$ ( $\eta \rightarrow \pi^+ \pi^- \pi^0$ ) (ex. $K^0$ ) $(5.390 \pm 0.270 \pm 0.410) \cdot 10^{-5}$	$(5.005 \pm 0.297) \cdot 10^{-5}$ BABAR	HFLAV Spring 2017 fit [67]
$\Gamma_{944} = 2\pi^- \pi^+ \eta \nu_\tau$ ( $\eta \rightarrow \gamma\gamma$ ) (ex. $K^0$ ) $(8.260 \pm 0.350 \pm 0.510) \cdot 10^{-5}$	$(8.606 \pm 0.511) \cdot 10^{-5}$ BABAR	HFLAV Spring 2017 fit [67]

**Table 1 – continued from previous page**

$\tau$ lepton branching fraction	Fit value / Exp.	HFLAV Fit / Ref.
$\Gamma_{945} = \pi^- 2\pi^0 \eta \nu_\tau$	$(1.929 \pm 0.378) \cdot 10^{-4}$	HFLAV Spring 2017 fit
$\Gamma_{998} = 1 - \Gamma_{\text{All}}$	$(0.0355 \pm 0.1031) \cdot 10^{-2}$	HFLAV Spring 2017 fit

## 2.6 Correlation terms between basis branching fractions uncertainties

The following tables report the correlation coefficients between basis quantities, in percent.

Table 2: Basis quantities correlation coefficients in percent, subtable 1.

$\Gamma_5$	23													
$\Gamma_9$	7	5												
$\Gamma_{10}$	3	5	1											
$\Gamma_{14}$	-13	-14	-12	-3										
$\Gamma_{16}$	0	-1	2	-1	-16									
$\Gamma_{20}$	-5	-5	-7	-1	-40	2								
$\Gamma_{23}$	0	0	0	-2	2	-13	-22							
$\Gamma_{27}$	-4	-3	-8	-1	0	3	-36	6						
$\Gamma_{28}$	0	0	0	-2	2	-13	5	-21	-29					
$\Gamma_{30}$	-5	-4	-11	-2	-9	0	6	0	-42	0				
$\Gamma_{35}$	0	0	0	0	0	0	0	1	0	1	0			
$\Gamma_{37}$	0	0	0	0	0	-2	1	-3	1	-3	0	-22		
$\Gamma_{40}$	0	0	0	0	0	1	0	1	-2	1	0	-12	4	
$\Gamma_3$	$\Gamma_5$	$\Gamma_9$	$\Gamma_{10}$	$\Gamma_{14}$	$\Gamma_{16}$	$\Gamma_{20}$	$\Gamma_{23}$	$\Gamma_{27}$	$\Gamma_{28}$	$\Gamma_{30}$	$\Gamma_{35}$	$\Gamma_{37}$	$\Gamma_{40}$	

Table 3: Basis quantities correlation coefficients in percent, subtable 2.

$\Gamma_{42}$	0	0	0	0	1	-3	1	-5	1	-5	0	2	-21	-20
$\Gamma_{44}$	0	0	0	0	0	0	0	0	0	0	0	-1	0	-4
$\Gamma_{47}$	0	0	0	0	0	0	0	0	0	0	0	-1	1	-4
$\Gamma_{48}$	0	0	0	0	0	0	0	0	0	0	0	-3	0	-2
$\Gamma_{50}$	0	0	0	0	0	0	0	-1	0	-1	0	0	7	0
$\Gamma_{51}$	0	0	0	0	0	0	0	0	0	0	0	-1	0	-1
$\Gamma_{53}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Gamma_{62}$	-3	-5	8	0	-4	5	-7	-1	-5	-1	-5	0	0	0
$\Gamma_{70}$	-6	-6	-7	-1	-8	-1	-1	0	-1	0	3	0	0	0
$\Gamma_{77}$	-1	0	-3	-1	-2	0	0	0	2	0	2	0	0	0
$\Gamma_{93}$	-1	-1	3	0	-1	2	-1	0	-1	0	-1	0	0	0
$\Gamma_{94}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Gamma_{126}$	0	0	0	0	0	-1	0	0	0	0	-2	0	0	0
$\Gamma_{128}$	0	0	1	0	0	1	0	-1	0	-1	0	0	0	0
$\Gamma_3$	$\Gamma_5$	$\Gamma_9$	$\Gamma_{10}$	$\Gamma_{14}$	$\Gamma_{16}$	$\Gamma_{20}$	$\Gamma_{23}$	$\Gamma_{27}$	$\Gamma_{28}$	$\Gamma_{30}$	$\Gamma_{35}$	$\Gamma_{37}$	$\Gamma_{40}$	

Table 4: Basis quantities correlation coefficients in percent, subtable 3.

	$\Gamma_{130}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\Gamma_{132}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\Gamma_{136}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\Gamma_{151}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\Gamma_{152}$	-1	0	-3	-1	-2	0	-1	0	2	0	2	0	0	0	0
	$\Gamma_{167}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\Gamma_{800}$	-2	-2	-2	0	-3	0	0	0	0	0	1	0	0	0	0
	$\Gamma_{802}$	-1	-1	0	0	-1	0	-2	0	-2	0	-1	0	0	0	0
	$\Gamma_{803}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\Gamma_{805}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\Gamma_{811}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\Gamma_{812}$	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\Gamma_{821}$	0	0	1	0	0	0	-1	0	0	0	-1	0	0	0	0
	$\Gamma_{822}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\Gamma_3$	$\Gamma_5$	$\Gamma_9$	$\Gamma_{10}$	$\Gamma_{14}$	$\Gamma_{16}$	$\Gamma_{20}$	$\Gamma_{23}$	$\Gamma_{27}$	$\Gamma_{28}$	$\Gamma_{30}$	$\Gamma_{35}$	$\Gamma_{37}$	$\Gamma_{40}$		

Table 5: Basis quantities correlation coefficients in percent, subtable 4.

	$\Gamma_{831}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\Gamma_{832}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\Gamma_{833}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\Gamma_{920}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\Gamma_{945}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\Gamma_3$	$\Gamma_5$	$\Gamma_9$	$\Gamma_{10}$	$\Gamma_{14}$	$\Gamma_{16}$	$\Gamma_{20}$	$\Gamma_{23}$	$\Gamma_{27}$	$\Gamma_{28}$	$\Gamma_{30}$	$\Gamma_{35}$	$\Gamma_{37}$	$\Gamma_{40}$		

Table 6: Basis quantities correlation coefficients in percent, subtable 5.

	$\Gamma_{44}$	0														
	$\Gamma_{47}$	1	0													
	$\Gamma_{48}$	-1	-6	0												
	$\Gamma_{50}$	5	0	-7	0											
	$\Gamma_{51}$	0	-3	0	-6	0										
	$\Gamma_{53}$	0	0	0	0	0	0									
	$\Gamma_{62}$	0	0	1	0	0	0	0	0							
	$\Gamma_{70}$	0	0	0	0	0	0	0	0	-20						
	$\Gamma_{77}$	0	0	0	0	0	0	0	-1	-7						
	$\Gamma_{93}$	0	0	0	0	0	0	0	14	-4	0					
	$\Gamma_{94}$	0	0	0	0	0	0	0	0	-2	0	0				
	$\Gamma_{126}$	0	0	1	0	0	0	0	1	0	-5	0	0			
	$\Gamma_{128}$	0	0	1	0	0	0	0	2	0	0	1	0	4		
	$\Gamma_{42}$	$\Gamma_{44}$	$\Gamma_{47}$	$\Gamma_{48}$	$\Gamma_{50}$	$\Gamma_{51}$	$\Gamma_{53}$	$\Gamma_{62}$	$\Gamma_{70}$	$\Gamma_{77}$	$\Gamma_{93}$	$\Gamma_{94}$	$\Gamma_{126}$	$\Gamma_{128}$		



## 2.7 Equality constraints

We list in the following the equality constraints that relate a branching fraction to a sum of branching fractions. The constraint equations include as coefficients the values of some non-tau branching fractions, denoted e.g., with the self-describing notation  $\Gamma_{K_S \rightarrow \pi^0 \pi^0}$ . Some coefficients are probabilities corresponding to modulus square amplitudes describing quantum mixtures of states such as  $K^0$ ,  $\bar{K}^0$ ,  $K_S$ ,  $K_L$ , denoted with e.g.,  $\Gamma_{<K^0|K_S>} = |<K^0|K_S>|^2$ . All non-tau quantities are taken from the PDG 2015 [9] fits (when available) or averages, and are used without accounting for their uncertainties, which are however in general small with respect to the uncertainties on the  $\tau$  branching fractions.

The following list does not include the constraints listed in Table 1, where some measured ratios of branching fractions are expressed as ratios of two branching fractions.

$$\begin{aligned}
\Gamma_1 &= \Gamma_3 + \Gamma_5 + \Gamma_9 + \Gamma_{10} + \Gamma_{14} + \Gamma_{16} \\
&\quad + \Gamma_{20} + \Gamma_{23} + \Gamma_{27} + \Gamma_{28} + \Gamma_{30} + \Gamma_{35} \\
&\quad + \Gamma_{40} + \Gamma_{44} + \Gamma_{37} + \Gamma_{42} + \Gamma_{47} + \Gamma_{48} \\
&\quad + \Gamma_{804} + \Gamma_{50} + \Gamma_{51} + \Gamma_{806} + \Gamma_{126} \cdot \Gamma_{\eta \rightarrow \text{neutral}} \\
&\quad + \Gamma_{128} \cdot \Gamma_{\eta \rightarrow \text{neutral}} + \Gamma_{130} \cdot \Gamma_{\eta \rightarrow \text{neutral}} + \Gamma_{132} \cdot \Gamma_{\eta \rightarrow \text{neutral}} \\
&\quad + \Gamma_{800} \cdot \Gamma_{\omega \rightarrow \pi^0 \gamma} + \Gamma_{151} \cdot \Gamma_{\omega \rightarrow \pi^0 \gamma} + \Gamma_{152} \cdot \Gamma_{\omega \rightarrow \pi^0 \gamma} \\
&\quad + \Gamma_{167} \cdot \Gamma_{\phi \rightarrow K_S K_L} \\
\Gamma_2 &= \Gamma_3 + \Gamma_5 + \Gamma_9 + \Gamma_{10} + \Gamma_{14} + \Gamma_{16} \\
&\quad + \Gamma_{20} + \Gamma_{23} + \Gamma_{27} + \Gamma_{28} + \Gamma_{30} + \Gamma_{35} \cdot (\Gamma_{<\bar{K}^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0} \\
&\quad + \Gamma_{<\bar{K}^0|K_L>} + \Gamma_{40} \cdot (\Gamma_{<\bar{K}^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0} + \Gamma_{<\bar{K}^0|K_L>})) + \Gamma_{44} \cdot (\Gamma_{<\bar{K}^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0} \\
&\quad + \Gamma_{<\bar{K}^0|K_L>} + \Gamma_{37} \cdot (\Gamma_{<\bar{K}^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0} + \Gamma_{<\bar{K}^0|K_L>})) + \Gamma_{42} \cdot (\Gamma_{<\bar{K}^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0} \\
&\quad + \Gamma_{<\bar{K}^0|K_L>} + \Gamma_{47} \cdot (\Gamma_{K_S \rightarrow \pi^0 \pi^0} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{48} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0} \\
&\quad + \Gamma_{804} + \Gamma_{50} \cdot (\Gamma_{K_S \rightarrow \pi^0 \pi^0} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{51} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0} \\
&\quad + \Gamma_{806} + \Gamma_{126} \cdot \Gamma_{\eta \rightarrow \text{neutral}} + \Gamma_{128} \cdot \Gamma_{\eta \rightarrow \text{neutral}} + \Gamma_{130} \cdot \Gamma_{\eta \rightarrow \text{neutral}} \\
&\quad + \Gamma_{132} \cdot (\Gamma_{\eta \rightarrow \text{neutral}} \cdot (\Gamma_{<\bar{K}^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0} + \Gamma_{<\bar{K}^0|K_L>})) + \Gamma_{800} \cdot \Gamma_{\omega \rightarrow \pi^0 \gamma} \\
&\quad + \Gamma_{151} \cdot \Gamma_{\omega \rightarrow \pi^0 \gamma} + \Gamma_{152} \cdot \Gamma_{\omega \rightarrow \pi^0 \gamma} + \Gamma_{167} \cdot (\Gamma_{\phi \rightarrow K_S K_L} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) \\
\Gamma_7 &= \Gamma_{35} \cdot \Gamma_{<\bar{K}^0|K_L>} + \Gamma_9 + \Gamma_{804} + \Gamma_{37} \cdot \Gamma_{<K^0|K_L>} \\
&\quad + \Gamma_{10} \\
\Gamma_8 &= \Gamma_9 + \Gamma_{10} \\
\Gamma_{11} &= \Gamma_{14} + \Gamma_{16} + \Gamma_{20} + \Gamma_{23} + \Gamma_{27} + \Gamma_{28} \\
&\quad + \Gamma_{30} + \Gamma_{35} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{37} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) \\
&\quad + \Gamma_{40} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{42} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) \\
&\quad + \Gamma_{47} \cdot (\Gamma_{K_S \rightarrow \pi^0 \pi^0} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{50} \cdot (\Gamma_{K_S \rightarrow \pi^0 \pi^0} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) \\
&\quad + \Gamma_{126} \cdot \Gamma_{\eta \rightarrow \text{neutral}} + \Gamma_{128} \cdot \Gamma_{\eta \rightarrow \text{neutral}} + \Gamma_{130} \cdot \Gamma_{\eta \rightarrow \text{neutral}} \\
&\quad + \Gamma_{132} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0} \cdot \Gamma_{\eta \rightarrow \text{neutral}}) + \Gamma_{151} \cdot \Gamma_{\omega \rightarrow \pi^0 \gamma} \\
&\quad + \Gamma_{152} \cdot \Gamma_{\omega \rightarrow \pi^0 \gamma} + \Gamma_{800} \cdot \Gamma_{\omega \rightarrow \pi^0 \gamma} \\
\Gamma_{12} &= \Gamma_{128} \cdot \Gamma_{\eta \rightarrow 3\pi^0} + \Gamma_{30} + \Gamma_{23} + \Gamma_{28} + \Gamma_{14} \\
&\quad + \Gamma_{16} + \Gamma_{20} + \Gamma_{27} + \Gamma_{126} \cdot \Gamma_{\eta \rightarrow 3\pi^0} + \Gamma_{130} \cdot \Gamma_{\eta \rightarrow 3\pi^0} \\
\Gamma_{13} &= \Gamma_{14} + \Gamma_{16} \\
\Gamma_{17} &= \Gamma_{128} \cdot \Gamma_{\eta \rightarrow 3\pi^0} + \Gamma_{30} + \Gamma_{23} + \Gamma_{28} + \Gamma_{35} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) \\
&\quad + \Gamma_{40} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{42} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) \\
&\quad + \Gamma_{20} + \Gamma_{27} + \Gamma_{47} \cdot (\Gamma_{K_S \rightarrow \pi^0 \pi^0} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{50} \cdot (\Gamma_{K_S \rightarrow \pi^0 \pi^0} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) \\
&\quad + \Gamma_{126} \cdot \Gamma_{\eta \rightarrow 3\pi^0} + \Gamma_{37} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{130} \cdot \Gamma_{\eta \rightarrow 3\pi^0} \\
\Gamma_{18} &= \Gamma_{23} + \Gamma_{35} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{20} + \Gamma_{37} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) \\
\Gamma_{19} &= \Gamma_{23} + \Gamma_{20}
\end{aligned}$$

$$\begin{aligned}
\Gamma_{24} &= \Gamma_{27} + \Gamma_{28} + \Gamma_{30} + \Gamma_{40} \cdot (\Gamma_{<\kappa^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) \\
&\quad + \Gamma_{42} \cdot (\Gamma_{<\kappa^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{47} \cdot (\Gamma_{K_S \rightarrow \pi^0 \pi^0} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) \\
&\quad + \Gamma_{50} \cdot (\Gamma_{K_S \rightarrow \pi^0 \pi^0} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{126} \cdot \Gamma_{\eta \rightarrow 3\pi^0} + \Gamma_{128} \cdot \Gamma_{\eta \rightarrow 3\pi^0} \\
&\quad + \Gamma_{130} \cdot \Gamma_{\eta \rightarrow 3\pi^0} + \Gamma_{132} \cdot (\Gamma_{<\kappa^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0} \cdot \Gamma_{\eta \rightarrow 3\pi^0}) \\
\Gamma_{25} &= \Gamma_{128} \cdot \Gamma_{\eta \rightarrow 3\pi^0} + \Gamma_{30} + \Gamma_{28} + \Gamma_{27} + \Gamma_{126} \cdot \Gamma_{\eta \rightarrow 3\pi^0} \\
&\quad + \Gamma_{130} \cdot \Gamma_{\eta \rightarrow 3\pi^0} \\
\Gamma_{26} &= \Gamma_{128} \cdot \Gamma_{\eta \rightarrow 3\pi^0} + \Gamma_{28} + \Gamma_{40} \cdot (\Gamma_{<\kappa^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) \\
&\quad + \Gamma_{42} \cdot (\Gamma_{<\kappa^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{27} \\
\Gamma_{29} &= \Gamma_{30} + \Gamma_{126} \cdot \Gamma_{\eta \rightarrow 3\pi^0} + \Gamma_{130} \cdot \Gamma_{\eta \rightarrow 3\pi^0} \\
\Gamma_{31} &= \Gamma_{128} \cdot \Gamma_{\eta \rightarrow \text{neutral}} + \Gamma_{23} + \Gamma_{28} + \Gamma_{42} + \Gamma_{16} \\
&\quad + \Gamma_{37} + \Gamma_{10} + \Gamma_{167} \cdot (\Gamma_{\phi \rightarrow K_S K_L} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) \\
\Gamma_{32} &= \Gamma_{16} + \Gamma_{23} + \Gamma_{28} + \Gamma_{37} + \Gamma_{42} + \Gamma_{128} \cdot \Gamma_{\eta \rightarrow \text{neutral}} \\
&\quad + \Gamma_{130} \cdot \Gamma_{\eta \rightarrow \text{neutral}} + \Gamma_{167} \cdot (\Gamma_{\phi \rightarrow K_S K_L} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) \\
\Gamma_{33} &= \Gamma_{35} \cdot \Gamma_{<\bar{\kappa}^0|K_S>} + \Gamma_{40} \cdot \Gamma_{<\bar{\kappa}^0|K_S>} + \Gamma_{42} \cdot \Gamma_{<\kappa^0|K_S>} \\
&\quad + \Gamma_{47} + \Gamma_{48} + \Gamma_{50} + \Gamma_{51} + \Gamma_{37} \cdot \Gamma_{<\kappa^0|K_S>} \\
&\quad + \Gamma_{132} \cdot (\Gamma_{<\bar{\kappa}^0|K_S>} \cdot \Gamma_{\eta \rightarrow \text{neutral}}) + \Gamma_{44} \cdot \Gamma_{<\bar{\kappa}^0|K_S>} + \Gamma_{167} \cdot \Gamma_{\phi \rightarrow K_S K_L} \\
\Gamma_{34} &= \Gamma_{35} + \Gamma_{37} \\
\Gamma_{38} &= \Gamma_{42} + \Gamma_{37} \\
\Gamma_{39} &= \Gamma_{40} + \Gamma_{42} \\
\Gamma_{43} &= \Gamma_{40} + \Gamma_{44} \\
\Gamma_{46} &= \Gamma_{48} + \Gamma_{47} + \Gamma_{804} \\
\Gamma_{49} &= \Gamma_{50} + \Gamma_{51} + \Gamma_{806} \\
\Gamma_{54} &= \Gamma_{35} \cdot (\Gamma_{<\kappa^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-}) + \Gamma_{37} \cdot (\Gamma_{<\kappa^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-}) \\
&\quad + \Gamma_{40} \cdot (\Gamma_{<\kappa^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-}) + \Gamma_{42} \cdot (\Gamma_{<\kappa^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-}) \\
&\quad + \Gamma_{47} \cdot (2 \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{48} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-} \\
&\quad + \Gamma_{50} \cdot (2 \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{51} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-} \\
&\quad + \Gamma_{53} \cdot (\Gamma_{<\bar{\kappa}^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0} + \Gamma_{<\bar{\kappa}^0|K_L>} + \Gamma_{62} + \Gamma_{70} \\
&\quad + \Gamma_{77} + \Gamma_{78} + \Gamma_{93} + \Gamma_{94} + \Gamma_{126} \cdot \Gamma_{\eta \rightarrow \text{charged}} \\
&\quad + \Gamma_{128} \cdot \Gamma_{\eta \rightarrow \text{charged}} + \Gamma_{130} \cdot \Gamma_{\eta \rightarrow \text{charged}} + \Gamma_{132} \cdot (\Gamma_{<\bar{\kappa}^0|K_L>} \cdot \Gamma_{\eta \rightarrow \pi^+ \pi^- \pi^0} \\
&\quad + \Gamma_{<\bar{\kappa}^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0} \cdot \Gamma_{\eta \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{<\bar{\kappa}^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-} \cdot \Gamma_{\eta \rightarrow 3\pi^0}) \\
&\quad + \Gamma_{151} \cdot (\Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{\omega \rightarrow \pi^+ \pi^-}) + \Gamma_{152} \cdot (\Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{\omega \rightarrow \pi^+ \pi^-}) \\
&\quad + \Gamma_{167} \cdot (\Gamma_{\phi \rightarrow K^+ K^-} + \Gamma_{\phi \rightarrow K_S K_L} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-}) + \Gamma_{802} + \Gamma_{803} \\
&\quad + \Gamma_{800} \cdot (\Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{\omega \rightarrow \pi^+ \pi^-}) \\
\Gamma_{55} &= \Gamma_{128} \cdot \Gamma_{\eta \rightarrow \text{charged}} + \Gamma_{152} \cdot (\Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{\omega \rightarrow \pi^+ \pi^-}) + \Gamma_{78} \\
&\quad + \Gamma_{77} + \Gamma_{94} + \Gamma_{62} + \Gamma_{70} + \Gamma_{93} + \Gamma_{126} \cdot \Gamma_{\eta \rightarrow \text{charged}} \\
&\quad + \Gamma_{802} + \Gamma_{803} + \Gamma_{800} \cdot (\Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{\omega \rightarrow \pi^+ \pi^-}) + \Gamma_{151} \cdot (\Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} \\
&\quad + \Gamma_{\omega \rightarrow \pi^+ \pi^-}) + \Gamma_{130} \cdot \Gamma_{\eta \rightarrow \text{charged}} + \Gamma_{168} \\
\Gamma_{56} &= \Gamma_{35} \cdot (\Gamma_{<\kappa^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-}) + \Gamma_{62} + \Gamma_{93} + \Gamma_{37} \cdot (\Gamma_{<\kappa^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-}) \\
&\quad + \Gamma_{802} + \Gamma_{800} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^-} + \Gamma_{151} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^-} + \Gamma_{168} \\
\Gamma_{57} &= \Gamma_{62} + \Gamma_{93} + \Gamma_{802} + \Gamma_{800} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^-} + \Gamma_{151} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^-} \\
&\quad + \Gamma_{167} \cdot \Gamma_{\phi \rightarrow K^+ K^-} \\
\Gamma_{58} &= \Gamma_{62} + \Gamma_{93} + \Gamma_{802} + \Gamma_{167} \cdot \Gamma_{\phi \rightarrow K^+ K^-}
\end{aligned}$$



$$\begin{aligned}
\Gamma_{92} &= \Gamma_{94} + \Gamma_{93} \\
\Gamma_{96} &= \Gamma_{167} \cdot \Gamma_{\phi \rightarrow K+K-} \\
\Gamma_{102} &= \Gamma_{103} + \Gamma_{104} \\
\Gamma_{103} &= \Gamma_{820} + \Gamma_{822} + \Gamma_{831} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^-} \\
\Gamma_{104} &= \Gamma_{830} + \Gamma_{833} \\
\Gamma_{106} &= \Gamma_{30} + \Gamma_{44} \cdot \Gamma_{<\bar{K}^0|K_S>} + \Gamma_{47} + \Gamma_{53} \cdot \Gamma_{<K^0|K_S>} \\
&\quad + \Gamma_{77} + \Gamma_{103} + \Gamma_{126} \cdot (\Gamma_{\eta \rightarrow 3\pi^0} + \Gamma_{\eta \rightarrow \pi^+ \pi^- \pi^0}) + \Gamma_{152} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} \\
\Gamma_{110} &= \Gamma_{10} + \Gamma_{16} + \Gamma_{23} + \Gamma_{28} + \Gamma_{35} + \Gamma_{40} \\
&\quad + \Gamma_{128} + \Gamma_{802} + \Gamma_{803} + \Gamma_{151} + \Gamma_{130} + \Gamma_{132} \\
&\quad + \Gamma_{44} + \Gamma_{53} + \Gamma_{168} + \Gamma_{169} + \Gamma_{822} + \Gamma_{833} \\
\Gamma_{149} &= \Gamma_{152} + \Gamma_{800} + \Gamma_{151} \\
\Gamma_{150} &= \Gamma_{800} + \Gamma_{151} \\
\Gamma_{168} &= \Gamma_{167} \cdot \Gamma_{\phi \rightarrow K+K-} \\
\Gamma_{169} &= \Gamma_{167} \cdot \Gamma_{\phi \rightarrow K_SK_L} \\
\Gamma_{804} &= \Gamma_{47} \cdot ((\Gamma_{<K^0|K_L>} \cdot \Gamma_{<\bar{K}^0|K_L>}) / (\Gamma_{<K^0|K_S>} \cdot \Gamma_{<\bar{K}^0|K_S>})) \\
\Gamma_{806} &= \Gamma_{50} \cdot ((\Gamma_{<K^0|K_L>} \cdot \Gamma_{<\bar{K}^0|K_L>}) / (\Gamma_{<K^0|K_S>} \cdot \Gamma_{<\bar{K}^0|K_S>})) \\
\Gamma_{810} &= \Gamma_{910} + \Gamma_{911} + \Gamma_{811} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{812} \\
\Gamma_{820} &= \Gamma_{920} + \Gamma_{821} \\
\Gamma_{830} &= \Gamma_{930} + \Gamma_{831} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{832} \\
\Gamma_{910} &= \Gamma_{136} \cdot \Gamma_{\eta \rightarrow 3\pi^0} \\
\Gamma_{911} &= \Gamma_{945} \cdot \Gamma_{\eta \rightarrow \pi^+ \pi^- \pi^0} \\
\Gamma_{930} &= \Gamma_{136} \cdot \Gamma_{\eta \rightarrow \pi^+ \pi^- \pi^0} \\
\Gamma_{944} &= \Gamma_{136} \cdot \Gamma_{\eta \rightarrow \gamma\gamma} \\
\Gamma_{\text{All}} &= \Gamma_3 + \Gamma_5 + \Gamma_9 + \Gamma_{10} + \Gamma_{14} + \Gamma_{16} \\
&\quad + \Gamma_{20} + \Gamma_{23} + \Gamma_{27} + \Gamma_{28} + \Gamma_{30} + \Gamma_{35} \\
&\quad + \Gamma_{37} + \Gamma_{40} + \Gamma_{42} + \Gamma_{47} \cdot (1 + ((\Gamma_{<K^0|K_L>} \cdot \Gamma_{<\bar{K}^0|K_L>}) / (\Gamma_{<K^0|K_S>} \cdot \Gamma_{<\bar{K}^0|K_S>}))) \\
&\quad + \Gamma_{48} + \Gamma_{62} + \Gamma_{70} + \Gamma_{77} + \Gamma_{811} + \Gamma_{812} \\
&\quad + \Gamma_{93} + \Gamma_{94} + \Gamma_{832} + \Gamma_{833} + \Gamma_{126} + \Gamma_{128} \\
&\quad + \Gamma_{802} + \Gamma_{803} + \Gamma_{800} + \Gamma_{151} + \Gamma_{130} + \Gamma_{132} \\
&\quad + \Gamma_{44} + \Gamma_{53} + \Gamma_{50} \cdot (1 + ((\Gamma_{<K^0|K_L>} \cdot \Gamma_{<\bar{K}^0|K_L>}) / (\Gamma_{<K^0|K_S>} \cdot \Gamma_{<\bar{K}^0|K_S>}))) \\
&\quad + \Gamma_{51} + \Gamma_{167} \cdot (\Gamma_{\phi \rightarrow K+K-} + \Gamma_{\phi \rightarrow K_SK_L}) + \Gamma_{152} + \Gamma_{920} \\
&\quad + \Gamma_{821} + \Gamma_{822} + \Gamma_{831} + \Gamma_{136} + \Gamma_{945} + \Gamma_{805}
\end{aligned}$$

### 3 Tests of lepton universality

Lepton universality tests probe the Standard Model prediction that the charged weak current interaction has the same coupling for all lepton generations. The precision of such tests has been significantly improved since the 2014 edition by the addition of the Belle  $\tau$  lifetime measurement [68], while improvements from the  $\tau$  branching fraction

fit are negligible. We compute the universality tests as in the previous report by using ratios of the partial widths of a heavier lepton  $\lambda$  decaying to a lighter lepton  $\rho$  [69],

$$\Gamma(\lambda \rightarrow \nu_\lambda \rho \bar{\nu}_\rho(\gamma)) = \frac{B(\lambda \rightarrow \nu_\lambda \rho \bar{\nu}_\rho)}{\tau_\lambda} = \frac{G_\lambda G_\rho m_\lambda^5}{192\pi^3} f\left(\frac{m_\rho^2}{m_\lambda^2}\right) R_W^\lambda R_\gamma^\lambda ,$$

where

$$G_\rho = \frac{g_\rho^2}{4\sqrt{2}M_W^2} , \quad f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x ,$$

$$R_W^\lambda = 1 + \frac{3}{5} \frac{m_\lambda^2}{M_W^2} + \frac{9}{5} \frac{m_\rho^2}{M_W^2} \text{ [70, 71, 72]}, \quad R_\gamma^\lambda = 1 + \frac{\alpha(m_\lambda)}{2\pi} \left( \frac{25}{4} - \pi^2 \right) .$$

We use  $R_\gamma^\tau = 1 - 43.2 \cdot 10^{-4}$  and  $R_\gamma^\mu = 1 - 42.4 \cdot 10^{-4}$  [69] and  $M_W$  from PDG 2015 [9]. We use HFLAV Spring 2017 averages and PDG 2015 for the other quantities. Using pure leptonic processes we obtain

$$\left(\frac{g_\tau}{g_\mu}\right) = 1.0010 \pm 0.0015 , \quad \left(\frac{g_\tau}{g_e}\right) = 1.0029 \pm 0.0015 , \quad \left(\frac{g_\mu}{g_e}\right) = 1.0019 \pm 0.0014 .$$

Using the expressions for the  $\tau$  semi-hadronic partial widths, we obtain

$$\left(\frac{g_\tau}{g_\mu}\right)^2 = \frac{B(\tau \rightarrow h \nu_\tau)}{B(h \rightarrow \mu \bar{\nu}_\mu)} \frac{2m_h m_\mu^2 \tau_h}{(1 + \delta R_{\tau/h}) m_\tau^3 \tau_\tau} \left(\frac{1 - m_\mu^2/m_h^2}{1 - m_h^2/m_\tau^2}\right)^2 ,$$

where  $h = \pi$  or  $K$  and the radiative corrections are  $\delta R_{\tau/\pi} = (0.16 \pm 0.14)\%$  and  $\delta R_{\tau/K} = (0.90 \pm 0.22)\%$  [73, 74, 75, 76]. We measure:

$$\left(\frac{g_\tau}{g_\mu}\right)_\pi = 0.9961 \pm 0.0027 , \quad \left(\frac{g_\tau}{g_\mu}\right)_K = 0.9860 \pm 0.0070 .$$

Similar tests could be performed with decays to electrons, however they are less precise because the hadron two body decays to electrons are helicity-suppressed. Averaging the three  $g_\tau/g_\mu$  ratios we obtain

$$\left(\frac{g_\tau}{g_\mu}\right)_{\tau+\pi+K} = 1.0000 \pm 0.0014 ,$$

accounting for statistical correlations. Table 12 reports the statistical correlation coefficients for the fitted coupling ratios.

Table 12: Universality coupling ratios correlation coefficients (%).

$\left(\frac{g_\tau}{g_e}\right)$	53			
$\left(\frac{g_\mu}{g_e}\right)$	-49	48		
$\left(\frac{g_\tau}{g_\mu}\right)_\pi$	24	26	2	
$\left(\frac{g_\tau}{g_\mu}\right)_K$	11	10	-1	6
	$\left(\frac{g_\tau}{g_\mu}\right)$	$\left(\frac{g_\tau}{g_e}\right)$	$\left(\frac{g_\mu}{g_e}\right)$	$\left(\frac{g_\tau}{g_\mu}\right)_\pi$

Since there is 100% correlation between  $g_\tau/g_\mu$ ,  $g_\tau/g_e$  and  $g_\mu/g_e$ , the correlation matrix is expected to be positive semi-definite, with one eigenvalue equal to zero. Due to numerical inaccuracies, one eigenvalue is expected to be close to zero rather than exactly zero.

## 4 Universality improved $B(\tau \rightarrow e \nu \bar{\nu})$ and $R_{\text{had}}$

We compute two quantities that are used in this report and that have been traditionally used for further elaborations and tests involving the  $\tau$  branching fractions:

- the “universality improved” experimental determination of  $B_e = B(\tau \rightarrow e\nu\bar{\nu})$ , which relies on the assumption that the Standard Model and lepton universality hold;
- the ratio  $R_{\text{had}}$  between the total branching fraction of the  $\tau$  to hadrons and the universality improved  $B_e$ , which is the same as the ratio of the two respective partial widths.

Following Ref. [77], we obtain a more precise experimental determination of  $B_e$  using the  $\tau$  branching fraction to  $\mu\nu\bar{\nu}$ ,  $B_\mu$ , and the  $\tau$  lifetime. We average:

- the  $B_e$  fit value  $\Gamma_5$ ,
- the  $B_e$  determination from the  $B_\mu = B(\tau \rightarrow \mu\nu\bar{\nu})$  fit value  $\Gamma_3$  assuming that  $g_\mu/g_e = 1$ , hence (see also Section 3)  $B_e = B_\mu \cdot f(m_e^2/m_\tau^2)/f(m_\mu^2/m_\tau^2)$ ,
- the  $B_e$  determination from the  $\tau$  lifetime assuming that  $g_\tau/g_\mu = 1$ , hence  $B_e = B(\mu \rightarrow e\bar{\nu}_e\nu_\mu) \cdot (\tau_\tau/\tau_\mu) \cdot (m_\tau/m_\mu)^5 \cdot f(m_e^2/m_\tau^2)/f(m_\mu^2/m_\tau^2) \cdot (\delta_\gamma^\tau \delta_W^\tau)/(\delta_\gamma^\mu \delta_W^\mu)$  where  $B(\mu \rightarrow e\bar{\nu}_e\nu_\mu) = 1$ .

Accounting for statistical correlations, we obtain

$$B_e^{\text{uni}} = (17.815 \pm 0.023)\%.$$

We use  $B_e^{\text{uni}}$  to obtain the ratio

$$R_{\text{had}} = \frac{\Gamma(\tau \rightarrow \text{hadrons})}{\Gamma(\tau \rightarrow e\nu\bar{\nu})} = \frac{\Gamma_{\text{hadrons}}}{B_e^{\text{uni}}} = 3.6349 \pm 0.0082,$$

where  $\Gamma(\tau \rightarrow \text{hadrons})$  and  $\Gamma(\tau \rightarrow e\nu\bar{\nu})$  indicate the partial widths and  $\Gamma_{\text{hadrons}}$  is the total branching fraction of the  $\tau$  to hadrons, or the total branching fraction in any measured final state minus the leptonic branching fractions, *i.e.*, with our notation  $\Gamma_{\text{hadrons}} = \Gamma_{\text{All}} - \Gamma_3 - \Gamma_5 = (64.76 \pm 0.10)\%$  (see Section 2 and Table 1 for the definitions of  $\Gamma_{\text{All}}$ ,  $\Gamma_3$ ,  $\Gamma_5$ ). We underline that this report’s definition of  $\Gamma_{\text{hadrons}}$  corresponds to summing all  $\tau$  hadronic decay modes, like in the previous report, rather than – as done elsewhere – subtracting the leptonic branching fractions from unity, *i.e.*,  $\Gamma_{\text{hadrons}} = 1 - \Gamma_3 - \Gamma_5$ .

## 5 $|V_{us}|$ measurement

The CKM matrix element magnitude  $|V_{us}|$  is most precisely determined from kaon decays [78] (see Figure 1), and its precision is limited by the uncertainties of the lattice QCD estimates of the meson decay constants  $f_+^{K\pi}(0)$  and  $f_K/f_\pi$ . Using the  $\tau$  branching fractions, it is possible to determine  $|V_{us}|$  in an alternative way [79, 80] that does not depend on lattice QCD and has small theory uncertainties (as discussed in Section 5.1). Moreover,  $|V_{us}|$  can be determined using the  $\tau$  branching fractions similarly to the kaon case, using the same meson decay constants from Lattice QCD.

### 5.1 $|V_{us}|$ from $B(\tau \rightarrow X_s \nu)$

The  $\tau$  hadronic partial width is the sum of the  $\tau$  partial widths to strange and to non-strange hadronic final states,  $\Gamma_{\text{had}} = \Gamma_s + \Gamma_{\text{VA}}$ . The suffix “VA” traditionally denotes the sum of the  $\tau$  partial widths to non-strange final states, which proceed through either vector or axial-vector currents.

Dividing any partial width  $\Gamma_x$  by the electronic partial width,  $\Gamma_e$ , we obtain partial width ratios  $R_x$  (which are equal to the respective branching fraction ratios  $B_x/B_e$ ) for which  $R_{\text{had}} = R_s + R_{\text{VA}}$ . In terms of such ratios,  $|V_{us}|$  can be measured as [79, 80]

$$|V_{us}|_{\tau s} = \sqrt{R_s / \left[ \frac{R_{\text{VA}}}{|V_{ud}|^2} - \delta R_{\text{theory}} \right]},$$

where  $\delta R_{\text{theory}}$  can be determined in the context of low energy QCD theory, partly relying on experimental low energy scattering data. The literature reports several calculations [81, 82, 83]. In this report we use Ref. [81], whose estimated uncertainty size is intermediate between the two other ones. We use the information in that paper and the PDG 2015 value for the  $s$ -quark mass  $m_s = 95.00 \pm 5.00$  MeV [9] to calculate  $\delta R_{\text{theory}} = 0.242 \pm 0.032$ .

Table 13: HFLAV Spring 2017  $\tau$  branching fractions to strange final states.

Branching fraction	HFLAV Spring 2017 fit (%)
$K^-\nu_\tau$	$0.6960 \pm 0.0096$
$K^-\pi^0\nu_\tau$	$0.4327 \pm 0.0149$
$K^-2\pi^0\nu_\tau$ (ex. $K^0$ )	$0.0640 \pm 0.0220$
$K^-3\pi^0\nu_\tau$ (ex. $K^0, \eta$ )	$0.0428 \pm 0.0216$
$\pi^-\bar{K}^0\nu_\tau$	$0.8386 \pm 0.0141$
$\pi^-\bar{K}^0\pi^0\nu_\tau$	$0.3812 \pm 0.0129$
$\pi^-\bar{K}^0\pi^0\pi^0\nu_\tau$ (ex. $K^0$ )	$0.0234 \pm 0.0231$
$\bar{K}^0 h^- h^+ \nu_\tau$	$0.0222 \pm 0.0202$
$K^-\eta\nu_\tau$	$0.0155 \pm 0.0008$
$K^-\pi^0\eta\nu_\tau$	$0.0048 \pm 0.0012$
$\pi^-\bar{K}^0\eta\nu_\tau$	$0.0094 \pm 0.0015$
$K^-\omega\nu_\tau$	$0.0410 \pm 0.0092$
$K^-\phi\nu_\tau$ ( $\phi \rightarrow K^+K^-$ )	$0.0022 \pm 0.0008$
$K^-\phi\nu_\tau$ ( $\phi \rightarrow K_S^0 K_L^0$ )	$0.0015 \pm 0.0006$
$K^-\pi^-\pi^+\nu_\tau$ (ex. $K^0, \omega$ )	$0.2923 \pm 0.0067$
$K^-\pi^-\pi^+\pi^0\nu_\tau$ (ex. $K^0, \omega, \eta$ )	$0.0410 \pm 0.0143$
$K^-2\pi^-2\pi^+\nu_\tau$ (ex. $K^0$ )	$0.0001 \pm 0.0001$
$K^-2\pi^-2\pi^+\pi^0\nu_\tau$ (ex. $K^0$ )	$0.0001 \pm 0.0001$
$X_s^-\nu_\tau$	$2.9087 \pm 0.0482$

We proceed following the same procedure of the 2012 HFLAV report [3]. We sum the relevant  $\tau$  branching fractions to compute  $B_{VA}$  and  $B_s$  and we use the universality improved  $B_e^{\text{uni}}$  (see Section 4) to compute the  $R_{VA}$  and  $R_s$  ratios. In past determinations of  $|V_{us}|$ , for example in the 2009 HFLAV report [1], the total hadronic branching fraction has been computed using unitarity as  $B_{\text{had}}^{\text{uni}} = 1 - B_e - B_\mu$ , obtaining then  $B_s$  from the sum of the strange branching fractions and  $B_{VA}$  from  $B_{\text{had}}^{\text{uni}} - B_s$ . We prefer to use the more direct experimental determination of  $B_{VA}$  for two reasons. First, both methods result in comparable uncertainties on  $|V_{us}|$ , since the better precision on  $B_{\text{had}}^{\text{uni}} = 1 - B_e - B_\mu$  is vanquished by increased statistical correlations in the expressions  $(1 - B_e - B_\mu)/B_e^{\text{univ}}$  and  $B_s/(B_{\text{had}} - B_s)$  in the  $|V_{us}|$  calculation. Second, if there are unobserved  $\tau$  hadronic decay modes, they would affect  $B_{VA}$  and  $B_s$  in a more asymmetric way when using unitarity.

Using the  $\tau$  branching fraction fit results with their uncertainties and correlations (Section 2), we compute  $B_s = (2.909 \pm 0.048)\%$  (see also Table 13) and  $B_{VA} = B_{\text{hadrons}} - B_s = (61.85 \pm 0.10)\%$ , where  $B_{\text{hadrons}}$  is equal to  $\Gamma_{\text{hadrons}}$  defined in section 4. PDG 2015 averages are used for non- $\tau$  quantities, and  $|V_{ud}| = 0.97417 \pm 0.00021$  [84].

We obtain  $|V_{us}|_{\tau s} = 0.2186 \pm 0.0021$ , which is  $3.1\sigma$  lower than the unitarity CKM prediction  $|V_{us}|_{\text{uni}} = 0.22582 \pm 0.00089$ , from  $(|V_{us}|_{\text{uni}})^2 = 1 - |V_{ud}|^2$ . The  $|V_{us}|_{\tau s}$  uncertainty includes a systematic error contribution of 0.47% from the theory uncertainty on  $\delta R_{\text{theory}}$ . There is no significant change with respect to the previous HFLAV report.

## 5.2 $|V_{us}|$ from $B(\tau \rightarrow K\nu)/B(\tau \rightarrow \pi\nu)$

We compute  $|V_{us}|$  from the ratio of branching fractions  $B(\tau \rightarrow K^-\nu_\tau)/B(\tau \rightarrow \pi^-\nu_\tau) = (6.438 \pm 0.094) \cdot 10^{-2}$  from the equation [70]:

$$\frac{B(\tau \rightarrow K^-\nu_\tau)}{B(\tau \rightarrow \pi^-\nu_\tau)} = \frac{f_K^2 |V_{us}|^2}{f_\pi^2 |V_{ud}|^2} \frac{(m_\tau^2 - m_K^2)^2}{(m_\tau^2 - m_\pi^2)^2} \frac{1 + \delta R_{\tau/K}}{1 + \delta R_{\tau/\pi}} (1 + \delta R_{K/\pi})$$

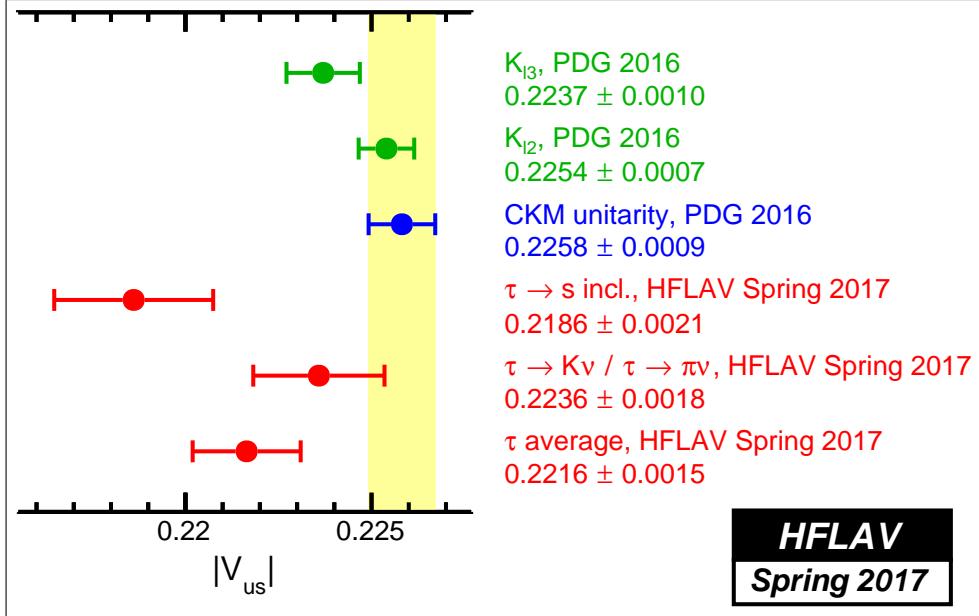


Figure 1:  $|V_{us}|$  averages.

We use  $f_K/f_\pi = 1.1930 \pm 0.0030$  from the FLAG 2016 Lattice averages with  $N_f = 2 + 1 + 1$  [85],

$$\frac{1 + \delta R_{\tau/K}}{1 + \delta R_{\tau/\pi}} = \frac{1 + (0.90 \pm 0.22)\%}{1 + (0.16 \pm 0.14)\%} [73, 74, 75, 76] ,$$

$$1 + \delta R_{K/\pi} = 1 + (-1.13 \pm 0.23)\% [70, 86, 87] .$$

We compute  $|V_{us}|_{\tau K/\pi} = 0.2236 \pm 0.0018$ ,  $1.1\sigma$  below the CKM unitarity prediction.

### 5.3 $|V_{us}|$ from $\tau$ summary

We summarize the  $|V_{us}|$  results reporting the values, the discrepancy with respect to the  $|V_{us}|$  determination from CKM unitarity, and an illustration of the measurement method:

$$|V_{us}|_{\text{uni}} = 0.22582 \pm 0.00089 \quad [\text{from } \sqrt{1 - |V_{ud}|^2} \text{ (CKM unitarity)}] ,$$

$$|V_{us}|_{\tau s} = 0.2186 \pm 0.0021 \quad -3.1\sigma \quad [\text{from } \Gamma(\tau^- \rightarrow X_s^- \nu_\tau)] ,$$

$$|V_{us}|_{\tau K/\pi} = 0.2236 \pm 0.0018 \quad -1.1\sigma \quad [\text{from } \Gamma(\tau^- \rightarrow K^- \nu_\tau)/\Gamma(\tau^- \rightarrow \pi^- \nu_\tau)] .$$

Averaging the two above  $|V_{us}|$  determinations that rely on the  $\tau$  branching fractions (taking into account all correlations due to the  $\tau$  HFLAV and other mentioned inputs) we obtain, for  $|V_{us}|$  and its discrepancy:

$$|V_{us}|_\tau = 0.2216 \pm 0.0015 \quad -2.4\sigma \quad [\text{average of 2 } |V_{us}| \text{ } \tau \text{ measurements}] .$$

All  $|V_{us}|$  determinations based on measured  $\tau$  branching fractions are lower than both the kaon and the CKM-unitarity determinations. This is correlated with the fact that the direct measurements of the three major  $\tau$  branching fractions to kaons [ $B(\tau \rightarrow K^- \nu_\tau)$ ,  $B(\tau \rightarrow K^- \pi^0 \nu_\tau)$  and  $B(\tau \rightarrow \pi^- \bar{K}^0 \nu_\tau)$ ] are lower than their determinations from the kaon branching fractions into final states with leptons within the SM [70, 88, 89].

A recent determination of  $|V_{us}|$  [90, 91] that relies on the  $\tau$  spectral functions in addition to the inclusive  $\tau \rightarrow X_s \nu$  branching fraction reports a  $|V_{us}|$  value about  $1\sigma$  lower than the CKM-unitarity determination. This determination uses inputs that are partially different from the ones used in this report. Specifically, the HFLAV average of  $B(\tau \rightarrow K^- \nu_\tau)$  has been replaced with the SM prediction based on the measured  $B(K^- \rightarrow \mu^- \bar{\nu}_\mu)$  and the HFLAV average of  $B(\tau \rightarrow K^- \pi^0 \nu_\tau)$  has been replaced with an in-progress *BABAR* measurement that is published in a PhD thesis. Both changes increase the resulting  $\tau \rightarrow X_s \nu$  inclusive branching fraction. This study claims that the newly proposed  $|V_{us}|$  calculation has a more stable and reliable theory uncertainty, which could possibly have been underestimated in former studies, which are used for the HFLAV  $|V_{us}|$  average.

In previous editions of the HFLAV report, we also computed  $|V_{us}|$  using the branching fraction  $B(\tau \rightarrow K\nu)$  and without taking the ratio with  $B(\tau \rightarrow \pi\nu)$ . We do not report this additional determination because it did not include the long-distance radiative corrections in addition to the short-distance contribution, and because it had a negligible effect on the overall precision of the  $|V_{us}|$  calculation with  $\tau$  data.

Figure 1 reports the HFLAV  $|V_{us}|$  determinations that use the  $\tau$  branching fractions, compared to two  $|V_{us}|$  determinations based on kaon data [2] and to  $|V_{us}|$  obtained from  $|V_{ud}|$  and the CKM matrix unitarity [2].

## 6 Upper limits on $\tau$ lepton-flavour-violating branching fractions

The Standard Model predicts that the  $\tau$  lepton-flavour-violating (LFV) branching fractions are too small to be measured with the available experimental precision. We report in Table 14 and Figure 2 the experimental upper limits on these branching fractions that have been published by the  $B$ -factories *BABAR* and *Belle* and later experiments. We omit previous weaker upper limits (mainly from CLEO) and all preliminary results presented several years ago. The previous HFLAV report [4] still included a few preliminary results, which have all been removed now.

Table 14: Experimental upper limits on lepton flavour violating  $\tau$  decays. The modes are grouped according to the properties of their final states. Modes with baryon number violation are labelled with “BNV”.

Decay mode	Category	90% CL Limit	Exp.	Ref.
$\Gamma_{156} = e^-\gamma$	$\ell\gamma$	$3.3 \cdot 10^{-8}$	<i>BABAR</i>	[92]
$\Gamma_{156} = e^-\gamma$		$1.2 \cdot 10^{-7}$	<i>Belle</i>	[93]
$\Gamma_{157} = \mu^-\gamma$		$4.4 \cdot 10^{-8}$	<i>BABAR</i>	[92]
$\Gamma_{157} = \mu^-\gamma$		$4.5 \cdot 10^{-8}$	<i>Belle</i>	[93]
$\Gamma_{158} = e^-\pi^0$	$\ell P^0$	$1.3 \cdot 10^{-7}$	<i>BABAR</i>	[94]
$\Gamma_{158} = e^-\pi^0$		$8.0 \cdot 10^{-8}$	<i>Belle</i>	[95]
$\Gamma_{159} = \mu^-\pi^0$		$1.1 \cdot 10^{-7}$	<i>BABAR</i>	[94]
$\Gamma_{159} = \mu^-\pi^0$		$1.2 \cdot 10^{-7}$	<i>Belle</i>	[95]
$\Gamma_{160} = e^-K_S^0$		$3.3 \cdot 10^{-8}$	<i>BABAR</i>	[96]
$\Gamma_{160} = e^-K_S^0$		$2.6 \cdot 10^{-8}$	<i>Belle</i>	[97]
$\Gamma_{161} = \mu^-K_S^0$		$4.0 \cdot 10^{-8}$	<i>BABAR</i>	[96]
$\Gamma_{161} = \mu^-K_S^0$		$2.3 \cdot 10^{-8}$	<i>Belle</i>	[97]
$\Gamma_{162} = e^-\eta$		$1.6 \cdot 10^{-7}$	<i>BABAR</i>	[94]
$\Gamma_{162} = e^-\eta$		$9.2 \cdot 10^{-8}$	<i>Belle</i>	[95]
$\Gamma_{163} = \mu^-\eta$		$1.5 \cdot 10^{-7}$	<i>BABAR</i>	[94]
$\Gamma_{163} = \mu^-\eta$		$6.5 \cdot 10^{-8}$	<i>Belle</i>	[95]
$\Gamma_{172} = e^-\eta'(958)$		$2.4 \cdot 10^{-7}$	<i>BABAR</i>	[94]
$\Gamma_{172} = e^-\eta'(958)$		$1.6 \cdot 10^{-7}$	<i>Belle</i>	[95]
$\Gamma_{173} = \mu^-\eta'(958)$		$1.4 \cdot 10^{-7}$	<i>BABAR</i>	[94]
$\Gamma_{173} = \mu^-\eta'(958)$		$1.3 \cdot 10^{-7}$	<i>Belle</i>	[95]
$\Gamma_{164} = e^-\rho^0$	$\ell V^0$	$4.6 \cdot 10^{-8}$	<i>BABAR</i>	[98]
$\Gamma_{164} = e^-\rho^0$		$1.8 \cdot 10^{-8}$	<i>Belle</i>	[99]
$\Gamma_{165} = \mu^-\rho^0$		$2.6 \cdot 10^{-8}$	<i>BABAR</i>	[98]
$\Gamma_{165} = \mu^-\rho^0$		$1.2 \cdot 10^{-8}$	<i>Belle</i>	[99]
$\Gamma_{166} = e^-\omega$		$1.1 \cdot 10^{-7}$	<i>BABAR</i>	[100]
$\Gamma_{166} = e^-\omega$		$4.8 \cdot 10^{-8}$	<i>Belle</i>	[99]
$\Gamma_{167} = \mu^-\omega$		$1.0 \cdot 10^{-7}$	<i>BABAR</i>	[100]
$\Gamma_{167} = \mu^-\omega$		$4.7 \cdot 10^{-8}$	<i>Belle</i>	[99]
$\Gamma_{168} = e^-K^*(892)^0$		$5.9 \cdot 10^{-8}$	<i>BABAR</i>	[98]
$\Gamma_{168} = e^-K^*(892)^0$		$3.2 \cdot 10^{-8}$	<i>Belle</i>	[99]
$\Gamma_{169} = \mu^-K^*(892)^0$		$1.7 \cdot 10^{-7}$	<i>BABAR</i>	[98]
$\Gamma_{169} = \mu^-K^*(892)^0$		$7.2 \cdot 10^{-8}$	<i>Belle</i>	[99]
$\Gamma_{170} = e^-\overline{K}^*(892)^0$		$4.6 \cdot 10^{-8}$	<i>BABAR</i>	[98]

**Table 14 – continued from previous page**

Decay mode	Category	90% CL Limit	Exp.	Ref.
$\Gamma_{170} = e^- \bar{K}^*(892)^0$		$3.4 \cdot 10^{-8}$	Belle	[99]
$\Gamma_{171} = \mu^- \bar{K}^*(892)^0$		$7.3 \cdot 10^{-8}$	<i>BABAR</i>	[98]
$\Gamma_{171} = \mu^- \bar{K}^*(892)^0$		$7.0 \cdot 10^{-8}$	Belle	[99]
$\Gamma_{176} = e^- \phi$		$3.1 \cdot 10^{-8}$	<i>BABAR</i>	[98]
$\Gamma_{176} = e^- \phi$		$3.1 \cdot 10^{-8}$	Belle	[99]
$\Gamma_{177} = \mu^- \phi$		$1.9 \cdot 10^{-7}$	<i>BABAR</i>	[98]
$\Gamma_{177} = \mu^- \phi$		$8.4 \cdot 10^{-8}$	Belle	[99]
$\Gamma_{174} = e^- f_0(980)$	$\ell S^0$	$3.2 \cdot 10^{-8}$	Belle	[101]
$\Gamma_{175} = \mu^- f_0(980)$		$3.4 \cdot 10^{-8}$	Belle	[101]
$\Gamma_{178} = e^- e^+ e^-$	$\ell \ell \ell$	$2.9 \cdot 10^{-8}$	<i>BABAR</i>	[102]
$\Gamma_{178} = e^- e^+ e^-$		$2.7 \cdot 10^{-8}$	Belle	[103]
$\Gamma_{179} = e^- \mu^+ \mu^-$		$3.2 \cdot 10^{-8}$	<i>BABAR</i>	[102]
$\Gamma_{179} = e^- \mu^+ \mu^-$		$2.7 \cdot 10^{-8}$	Belle	[103]
$\Gamma_{180} = \mu^- e^+ \mu^-$		$2.6 \cdot 10^{-8}$	<i>BABAR</i>	[102]
$\Gamma_{180} = \mu^- e^+ \mu^-$		$1.7 \cdot 10^{-8}$	Belle	[103]
$\Gamma_{181} = \mu^- e^+ e^-$		$2.2 \cdot 10^{-8}$	<i>BABAR</i>	[102]
$\Gamma_{181} = \mu^- e^+ e^-$		$1.8 \cdot 10^{-8}$	Belle	[103]
$\Gamma_{182} = e^- \mu^+ e^-$		$1.8 \cdot 10^{-8}$	<i>BABAR</i>	[102]
$\Gamma_{182} = e^- \mu^+ e^-$		$1.5 \cdot 10^{-8}$	Belle	[103]
$\Gamma_{183} = \mu^- \mu^+ \mu^-$		$3.8 \cdot 10^{-7}$	ATLAS	[104]
$\Gamma_{183} = \mu^- \mu^+ \mu^-$		$3.3 \cdot 10^{-8}$	<i>BABAR</i>	[102]
$\Gamma_{183} = \mu^- \mu^+ \mu^-$		$2.1 \cdot 10^{-8}$	Belle	[103]
$\Gamma_{183} = \mu^- \mu^+ \mu^-$		$4.6 \cdot 10^{-8}$	LHCb	[105]
$\Gamma_{184} = e^- \pi^+ \pi^-$	$\ell hh$	$1.2 \cdot 10^{-7}$	<i>BABAR</i>	[106]
$\Gamma_{184} = e^- \pi^+ \pi^-$		$2.3 \cdot 10^{-8}$	Belle	[107]
$\Gamma_{185} = e^+ \pi^- \pi^-$		$2.7 \cdot 10^{-7}$	<i>BABAR</i>	[106]
$\Gamma_{185} = e^+ \pi^- \pi^-$		$2.0 \cdot 10^{-8}$	Belle	[107]
$\Gamma_{186} = \mu^- \pi^+ \pi^-$		$2.9 \cdot 10^{-7}$	<i>BABAR</i>	[106]
$\Gamma_{186} = \mu^- \pi^+ \pi^-$		$2.1 \cdot 10^{-8}$	Belle	[107]
$\Gamma_{187} = \mu^+ \pi^- \pi^-$		$7.0 \cdot 10^{-8}$	<i>BABAR</i>	[106]
$\Gamma_{187} = \mu^+ \pi^- \pi^-$		$3.9 \cdot 10^{-8}$	Belle	[107]
$\Gamma_{188} = e^- \pi^+ K^-$		$3.2 \cdot 10^{-7}$	<i>BABAR</i>	[106]
$\Gamma_{188} = e^- \pi^+ K^-$		$3.7 \cdot 10^{-8}$	Belle	[107]
$\Gamma_{189} = e^- K^+ \pi^-$		$1.7 \cdot 10^{-7}$	<i>BABAR</i>	[106]
$\Gamma_{189} = e^- K^+ \pi^-$		$3.1 \cdot 10^{-8}$	Belle	[107]
$\Gamma_{190} = e^+ \pi^- K^-$		$1.8 \cdot 10^{-7}$	<i>BABAR</i>	[106]
$\Gamma_{190} = e^+ \pi^- K^-$		$3.2 \cdot 10^{-8}$	Belle	[107]
$\Gamma_{191} = e^- K_S^0 K_S^0$		$7.1 \cdot 10^{-8}$	Belle	[97]
$\Gamma_{192} = e^- K^+ K^-$		$1.4 \cdot 10^{-7}$	<i>BABAR</i>	[106]
$\Gamma_{192} = e^- K^+ K^-$		$3.4 \cdot 10^{-8}$	Belle	[107]
$\Gamma_{193} = e^+ K^- K^-$		$1.5 \cdot 10^{-7}$	<i>BABAR</i>	[106]
$\Gamma_{193} = e^+ K^- K^-$		$3.3 \cdot 10^{-8}$	Belle	[107]
$\Gamma_{194} = \mu^- \pi^+ K^-$		$2.6 \cdot 10^{-7}$	<i>BABAR</i>	[106]
$\Gamma_{194} = \mu^- \pi^+ K^-$		$8.6 \cdot 10^{-8}$	Belle	[107]
$\Gamma_{195} = \mu^- K^+ \pi^-$		$3.2 \cdot 10^{-7}$	<i>BABAR</i>	[106]
$\Gamma_{195} = \mu^- K^+ \pi^-$		$4.5 \cdot 10^{-8}$	Belle	[107]
$\Gamma_{196} = \mu^+ \pi^- K^-$		$2.2 \cdot 10^{-7}$	<i>BABAR</i>	[106]
$\Gamma_{196} = \mu^+ \pi^- K^-$		$4.8 \cdot 10^{-8}$	Belle	[107]
$\Gamma_{197} = \mu^- K_S^0 K_S^0$		$8.0 \cdot 10^{-8}$	Belle	[97]
$\Gamma_{198} = \mu^- K^+ K^-$		$2.5 \cdot 10^{-7}$	<i>BABAR</i>	[106]
$\Gamma_{198} = \mu^- K^+ K^-$		$4.4 \cdot 10^{-8}$	Belle	[107]
$\Gamma_{199} = \mu^+ K^- K^-$		$4.8 \cdot 10^{-7}$	<i>BABAR</i>	[106]

**Table 14 – continued from previous page**

Decay mode	Category	90% CL Limit	Exp.	Ref.
$\Gamma_{199} = \mu^+ K^- K^-$		$4.7 \cdot 10^{-8}$	Belle	[107]
$\Gamma_{211} = \pi^- \Lambda$	BNV	$7.2 \cdot 10^{-8}$	Belle	[108]
$\Gamma_{212} = \pi^- \bar{\Lambda}$		$1.4 \cdot 10^{-7}$	Belle	[108]
$\Gamma_{215} = p\mu^-\mu^-$		$4.4 \cdot 10^{-7}$	LHCb	[109]
$\Gamma_{216} = \bar{p}\mu^+\mu^-$		$3.3 \cdot 10^{-7}$	LHCb	[109]

## 7 Combination of upper limits on $\tau$ lepton-flavour-violating branching fractions

Combining upper limits is a delicate issue, since there is no standard and generally agreed procedure. Furthermore, the  $\tau$  LFV searches published limits are extracted from the data with a variety of methods, and cannot be directly combined with a uniform procedure. It is however possible to use a single and effective upper limit combination procedure for all modes by re-computing the published upper limits with just one extraction method, using the published information that documents the upper limit determination: number of observed candidates, expected background, signal efficiency and number of analyzed  $\tau$  decays.

We chose to use the  $CL_s$  method [110] to re-compute the  $\tau$  LFV upper limits, since it is well known and widely used (see the Statistics review of PDG 2013 [2]), and since the limits computed with the  $CL_s$  method can be combined in a straightforward way (see below). The  $CL_s$  method is based on two hypotheses: signal plus background and background only. We calculate the observed confidence levels for the two hypotheses:

$$CL_{s+b} = P_{s+b}(Q \leq Q_{obs}) = \int_{-\infty}^{Q_{obs}} \frac{dP_{s+b}}{dQ} dQ, \quad (10)$$

$$CL_b = P_b(Q \leq Q_{obs}) = \int_{-\infty}^{Q_{obs}} \frac{dP_b}{dQ} dQ, \quad (11)$$

where  $CL_{s+b}$  is the confidence level observed for the signal plus background hypotheses,  $CL_b$  is the confidence level observed for the background only hypothesis,  $\frac{dP_{s+b}}{dQ}$  and  $\frac{dP_b}{dQ}$  are the probability distribution functions (PDFs) for the two corresponding hypothesis and  $Q$  is called the test statistic. The  $CL_s$  value is defined as the ratio between the confidence level for the signal plus background hypothesis and the confidence level for the background hypothesis:

$$CL_s = \frac{CL_{s+b}}{CL_b}. \quad (12)$$

When multiple results are combined, the PDFs in Eqs. (10) and (11) are the product of the individual PDFs,

$$CL_s = \frac{\prod_{i=1}^N \sum_{n=0}^{n_i} \frac{e^{-(s_i+b_i)}(s_i + b_i)^n}{n!} \prod_{j=1}^N [s_i S_i(x_{ij}) + b_i B_i(x_{ij})]}{\prod_{i=1}^N \sum_{n=0}^{n_i} \frac{e^{-b_i} b_i^n}{n!}}, \quad (13)$$

where  $N$  is the number of results (or channels), and, for each channel  $i$ ,  $n_i$  is the number of observed candidates,  $x_{ij}$  are the values of the discriminating variables (with index  $j$ ),  $s_i$  and  $b_i$  are the number of signal and background events and  $S_i$ ,  $B_i$  are the probability distribution functions of the discriminating variables. The discriminating variables  $x_{ij}$  are assumed to be uncorrelated. The expected signal  $s_i$  is related to the  $\tau$  lepton branching fraction  $B(\tau \rightarrow f_i)$  into the searched final state  $f_i$  by  $s_i = N_i \epsilon_i B(\tau \rightarrow f_i)$ , where  $N_i$  is the number of produced  $\tau$  leptons and  $\epsilon_i$  is the detection efficiency for observing the decay  $\tau \rightarrow f_i$ . For  $e^+e^-$  experiments,  $N_i = 2\mathcal{L}_i \sigma_{\tau\tau}$ , where  $\mathcal{L}_i$  is the integrated luminosity and  $\sigma_{\tau\tau}$  is the  $\tau$  pair production cross section  $\sigma(e^+e^- \rightarrow \tau^+\tau^-)$  [111]. In experiments where  $\tau$  leptons are produced in more complex multiple reactions, the effective  $N_i$  is typically estimated with Monte Carlo simulations calibrated with related data yields.

The extraction of the upper limits is performed using the code provided by Tom Junk [112]. The systematic uncertainties are modeled in the Monte Carlo toy experiments by convolving the  $S_i$  and  $B_i$  PDFs with Gaussian distributions corresponding to the nuisance parameters.

Table 15 reports the HFLAV combinations of the  $\tau$  LFV limits. Since there is negligible gain in combining limits of very different strength, the combinations do not include the CLEO searches and do not include results where the single event sensitivity is more than a factor of 5 lower than the value for the search with the best limit.

Figure 3 reports a graphical representation of the limits in Table 15. The published information that has been used to obtain these limits is reported in Table 16.

Table 15: Combinations of upper limits on lepton flavour violating  $\tau$  decay modes. The modes are grouped according to the properties of their final states. Modes with baryon number violation are labelled with “BNV”.

Decay mode	Category	90% CL Limit	Refs.
$\Gamma_{156} = e^-\gamma$	$\ell\gamma$	$5.4 \cdot 10^{-8}$	[93, 92]
$\Gamma_{157} = \mu^-\gamma$	$\ell\gamma$	$5.0 \cdot 10^{-8}$	[93, 92]
$\Gamma_{158} = e^-\pi^0$	$\ell P^0$	$4.9 \cdot 10^{-8}$	[95, 94]
$\Gamma_{159} = \mu^-\pi^0$	$\ell P^0$	$3.6 \cdot 10^{-8}$	[95, 94]
$\Gamma_{160} = e^-K_S^0$	$\ell P^0$	$1.4 \cdot 10^{-8}$	[97, 96]
$\Gamma_{161} = \mu^-K_S^0$	$\ell P^0$	$1.5 \cdot 10^{-8}$	[97, 96]
$\Gamma_{162} = e^-\eta$	$\ell P^0$	$5.5 \cdot 10^{-8}$	[95, 94]
$\Gamma_{163} = \mu^-\eta$	$\ell P^0$	$3.8 \cdot 10^{-8}$	[95, 94]
$\Gamma_{172} = e^-\eta'(958)$	$\ell P^0$	$9.9 \cdot 10^{-8}$	[95, 94]
$\Gamma_{173} = \mu^-\eta'(958)$	$\ell P^0$	$6.3 \cdot 10^{-8}$	[95, 94]
$\Gamma_{164} = e^-\rho^0$	$\ell V^0$	$1.5 \cdot 10^{-8}$	[99, 98]
$\Gamma_{165} = \mu^-\rho^0$	$\ell V^0$	$1.5 \cdot 10^{-8}$	[99, 98]
$\Gamma_{166} = e^-\omega$	$\ell V^0$	$3.3 \cdot 10^{-8}$	[99, 100]
$\Gamma_{167} = \mu^-\omega$	$\ell V^0$	$4.0 \cdot 10^{-8}$	[99, 100]
$\Gamma_{168} = e^-K^*(892)^0$	$\ell V^0$	$2.3 \cdot 10^{-8}$	[99, 98]
$\Gamma_{169} = \mu^-K^*(892)^0$	$\ell V^0$	$6.0 \cdot 10^{-8}$	[99, 98]
$\Gamma_{170} = e^-\bar{K}^*(892)^0$	$\ell V^0$	$2.2 \cdot 10^{-8}$	[99, 98]
$\Gamma_{171} = \mu^-\bar{K}^*(892)^0$	$\ell V^0$	$4.2 \cdot 10^{-8}$	[99, 98]
$\Gamma_{176} = e^-\phi$	$\ell V^0$	$2.0 \cdot 10^{-8}$	[99, 98]
$\Gamma_{177} = \mu^-\phi$	$\ell V^0$	$6.8 \cdot 10^{-8}$	[99, 98]
$\Gamma_{178} = e^-e^+e^-$	$\ell\ell\ell$	$1.4 \cdot 10^{-8}$	[103, 102]
$\Gamma_{179} = e^-\mu^+\mu^-$	$\ell\ell\ell$	$1.6 \cdot 10^{-8}$	[103, 102]
$\Gamma_{180} = \mu^-\epsilon^+\mu^-$	$\ell\ell\ell$	$9.8 \cdot 10^{-9}$	[103, 102]
$\Gamma_{181} = \mu^-\epsilon^+e^-$	$\ell\ell\ell$	$1.1 \cdot 10^{-8}$	[103, 102]
$\Gamma_{182} = e^-\mu^+e^-$	$\ell\ell\ell$	$8.4 \cdot 10^{-9}$	[103, 102]
$\Gamma_{183} = \mu^-\mu^+\mu^-$	$\ell\ell\ell$	$1.2 \cdot 10^{-8}$	[103, 102, 105]

Table 16: Published information that has been used to re-compute upper limits with the  $\text{CL}_s$  method, *i.e.* the number of  $\tau$  leptons produced, the signal detection efficiency and its uncertainty, the number of expected background events and its uncertainty, and the number of observed events. The uncertainty on the efficiency includes the minor uncertainty contribution on the number of  $\tau$  leptons (typically originating on the uncertainties on the integrated luminosity and on the production cross-section). The additional limit used in the combinations (from LHCb) has been originally determined with the  $\text{CL}_s$  method.

Decay mode	Exp.	Ref.	$N_\tau$ (millions)	efficiency (%)	$N_{\text{bkg}}$	$N_{\text{obs}}$
$\Gamma_{156} = e^- \gamma$	BABAR	[92]	963	$3.90 \pm 0.30$	$1.60 \pm 0.40$	0
$\Gamma_{156} = e^- \gamma$	Belle	[93]	983	$3.00 \pm 0.10$	$5.14 \pm 3.30$	5
$\Gamma_{157} = \mu^- \gamma$	BABAR	[92]	963	$6.10 \pm 0.50$	$3.60 \pm 0.70$	2
$\Gamma_{157} = \mu^- \gamma$	Belle	[93]	983	$5.07 \pm 0.20$	$13.90 \pm 5.00$	10
$\Gamma_{158} = e^- \pi^0$	BABAR	[94]	339	$2.83 \pm 0.25$	$0.17 \pm 0.04$	0
$\Gamma_{158} = e^- \pi^0$	Belle	[95]	401	$3.93 \pm 0.18$	$0.20 \pm 0.20$	0
$\Gamma_{159} = \mu^- \pi^0$	BABAR	[94]	339	$4.75 \pm 0.37$	$1.33 \pm 0.15$	1
$\Gamma_{159} = \mu^- \pi^0$	Belle	[95]	401	$4.53 \pm 0.20$	$0.58 \pm 0.34$	1
$\Gamma_{160} = e^- K^0_S$	BABAR	[96]	862	$9.10 \pm 1.73$	$0.59 \pm 0.25$	1
$\Gamma_{160} = e^- K^0_S$	Belle	[97]	1274	$10.20 \pm 0.67$	$0.18 \pm 0.18$	0
$\Gamma_{161} = \mu^- K^0_S$	BABAR	[96]	862	$6.14 \pm 0.20$	$0.30 \pm 0.18$	1
$\Gamma_{161} = \mu^- K^0_S$	Belle	[97]	1274	$10.70 \pm 0.73$	$0.35 \pm 0.21$	0
$\Gamma_{162} = e^- \eta$	BABAR	[94]	339	$2.12 \pm 0.20$	$0.22 \pm 0.05$	0
$\Gamma_{162} = e^- \eta$	Belle	[95]	401	$2.87 \pm 0.20$	$0.78 \pm 0.78$	0
$\Gamma_{163} = \mu^- \eta$	BABAR	[94]	339	$3.59 \pm 0.41$	$0.75 \pm 0.08$	1
$\Gamma_{163} = \mu^- \eta$	Belle	[95]	401	$4.08 \pm 0.28$	$0.64 \pm 0.04$	0
$\Gamma_{172} = e^- \eta'(958)$	BABAR	[94]	339	$1.53 \pm 0.16$	$0.12 \pm 0.03$	0
$\Gamma_{172} = e^- \eta'(958)$	Belle	[95]	401	$1.59 \pm 0.13$	$0.01 \pm 0.41$	0
$\Gamma_{173} = \mu^- \eta'(958)$	BABAR	[94]	339	$2.18 \pm 0.26$	$0.49 \pm 0.26$	0
$\Gamma_{173} = \mu^- \eta'(958)$	Belle	[95]	401	$2.47 \pm 0.20$	$0.23 \pm 0.46$	0
$\Gamma_{164} = e^- \rho^0$	BABAR	[98]	829	$7.31 \pm 0.20$	$1.32 \pm 0.17$	1
$\Gamma_{164} = e^- \rho^0$	Belle	[99]	1554	$7.58 \pm 0.41$	$0.29 \pm 0.15$	0
$\Gamma_{165} = \mu^- \rho^0$	BABAR	[98]	829	$4.52 \pm 0.40$	$2.04 \pm 0.19$	0
$\Gamma_{165} = \mu^- \rho^0$	Belle	[99]	1554	$7.09 \pm 0.37$	$1.48 \pm 0.35$	0
$\Gamma_{166} = e^- \omega$	BABAR	[100]	829	$2.96 \pm 0.13$	$0.35 \pm 0.06$	0
$\Gamma_{166} = e^- \omega$	Belle	[99]	1554	$2.92 \pm 0.18$	$0.30 \pm 0.14$	0
$\Gamma_{167} = \mu^- \omega$	BABAR	[100]	829	$2.56 \pm 0.16$	$0.73 \pm 0.03$	0
$\Gamma_{167} = \mu^- \omega$	Belle	[99]	1554	$2.38 \pm 0.14$	$0.72 \pm 0.18$	0
$\Gamma_{168} = e^- K^*(892)^0$	BABAR	[98]	829	$8.00 \pm 0.20$	$1.65 \pm 0.23$	2
$\Gamma_{168} = e^- K^*(892)^0$	Belle	[99]	1554	$4.37 \pm 0.24$	$0.29 \pm 0.14$	0
$\Gamma_{169} = \mu^- K^*(892)^0$	BABAR	[98]	829	$4.60 \pm 0.40$	$1.79 \pm 0.21$	4
$\Gamma_{169} = \mu^- K^*(892)^0$	Belle	[99]	1554	$3.39 \pm 0.19$	$0.53 \pm 0.20$	1
$\Gamma_{170} = e^- \bar{K}^*(892)^0$	BABAR	[98]	829	$7.80 \pm 0.20$	$2.76 \pm 0.28$	2
$\Gamma_{170} = e^- \bar{K}^*(892)^0$	Belle	[99]	1554	$4.41 \pm 0.25$	$0.08 \pm 0.08$	0
$\Gamma_{171} = \mu^- \bar{K}^*(892)^0$	BABAR	[98]	829	$4.10 \pm 0.30$	$1.72 \pm 0.17$	1
$\Gamma_{171} = \mu^- \bar{K}^*(892)^0$	Belle	[99]	1554	$3.60 \pm 0.20$	$0.45 \pm 0.17$	1
$\Gamma_{176} = e^- \phi$	BABAR	[98]	829	$6.40 \pm 0.20$	$0.68 \pm 0.12$	0
$\Gamma_{176} = e^- \phi$	Belle	[99]	1554	$4.18 \pm 0.25$	$0.47 \pm 0.19$	0
$\Gamma_{177} = \mu^- \phi$	BABAR	[98]	829	$5.20 \pm 0.30$	$2.76 \pm 0.16$	6
$\Gamma_{177} = \mu^- \phi$	Belle	[99]	1554	$3.21 \pm 0.19$	$0.06 \pm 0.06$	1
$\Gamma_{178} = e^- e^+ e^-$	BABAR	[102]	868	$8.60 \pm 0.20$	$0.12 \pm 0.02$	0
$\Gamma_{178} = e^- e^+ e^-$	Belle	[103]	1437	$6.00 \pm 0.59$	$0.21 \pm 0.15$	0
$\Gamma_{179} = e^- \mu^+ \mu^-$	BABAR	[102]	868	$6.40 \pm 0.40$	$0.54 \pm 0.14$	0
$\Gamma_{179} = e^- \mu^+ \mu^-$	Belle	[103]	1437	$6.10 \pm 0.58$	$0.10 \pm 0.04$	0
$\Gamma_{180} = \mu^- e^+ \mu^-$	BABAR	[102]	868	$10.20 \pm 0.60$	$0.03 \pm 0.02$	0
$\Gamma_{180} = \mu^- e^+ \mu^-$	Belle	[103]	1437	$10.10 \pm 0.77$	$0.02 \pm 0.02$	0
$\Gamma_{181} = \mu^- e^+ e^-$	BABAR	[102]	868	$8.80 \pm 0.50$	$0.64 \pm 0.19$	0
$\Gamma_{181} = \mu^- e^+ e^-$	Belle	[103]	1437	$9.30 \pm 0.73$	$0.04 \pm 0.04$	0
$\Gamma_{182} = e^- \mu^+ e^-$	BABAR	[102]	868	$12.70 \pm 0.70$	$0.34 \pm 0.12$	0
$\Gamma_{182} = e^- \mu^+ e^-$	Belle	[103]	1437	$11.50 \pm 0.89$	$0.01 \pm 0.01$	0
$\Gamma_{183} = \mu^- \mu^+ \mu^-$	BABAR	[102]	868	$6.60 \pm 0.60$	$0.44 \pm 0.17$	0
$\Gamma_{183} = \mu^- \mu^+ \mu^-$	Belle	[103]	1437	$7.60 \pm 0.56$	$0.13 \pm 0.20$	0

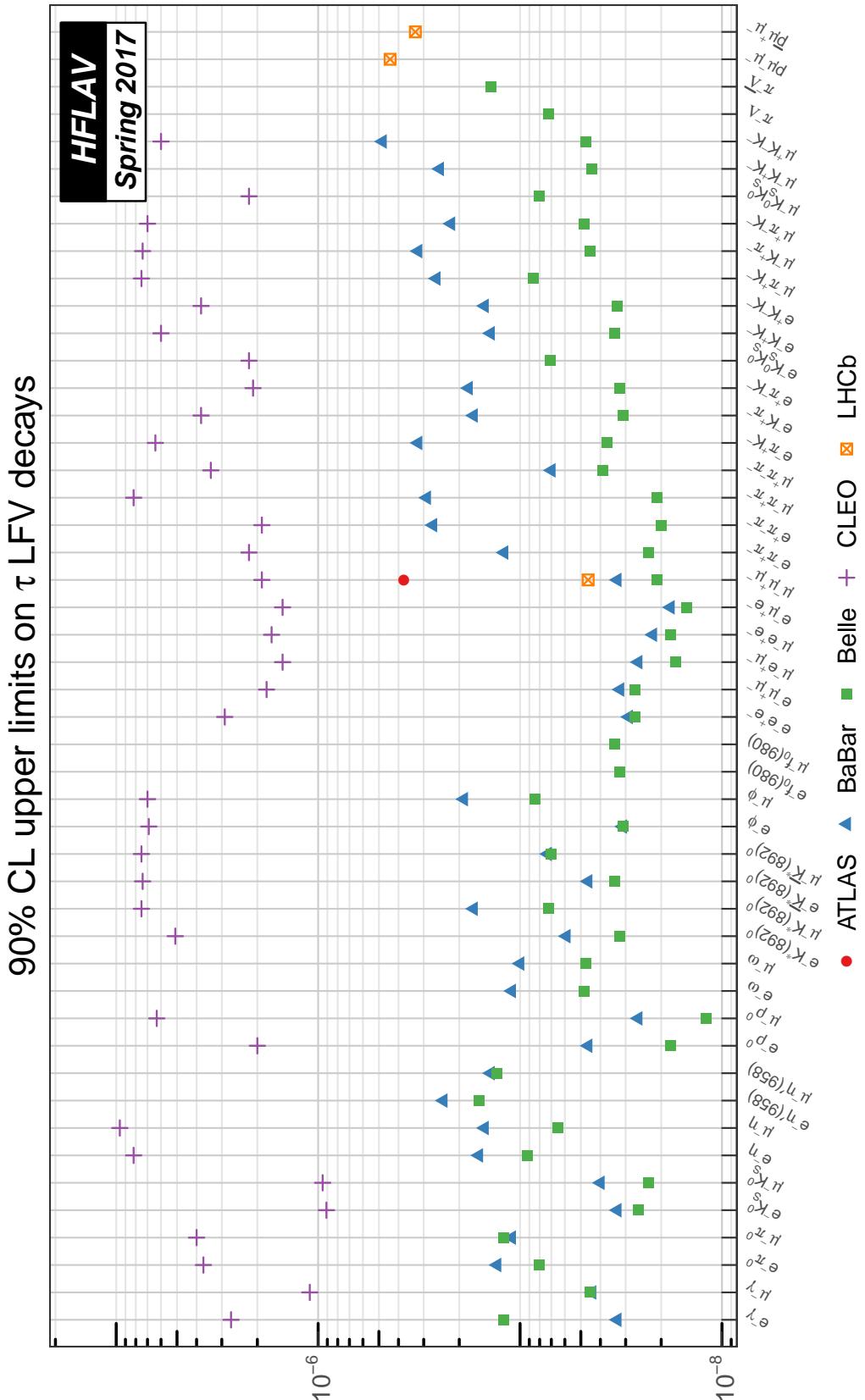


Figure 2: Tau lepton-flavor-violating branching fraction upper limits summary plot. In order to appreciate the physics reach improvement over time, the plot includes also the CLEO upper limits reported by PDG 2016 [2].

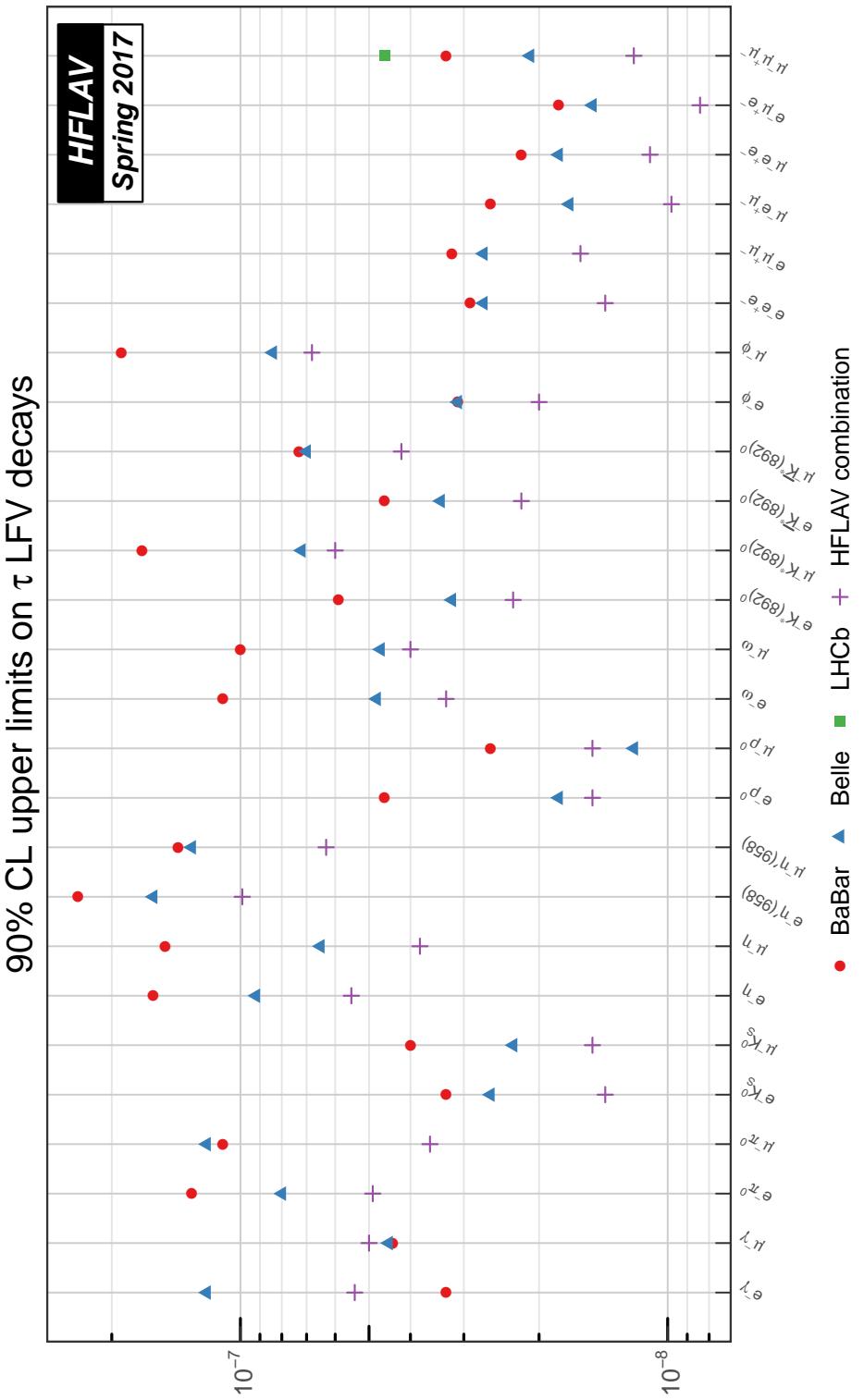


Figure 3: Tau lepton-flavour-violating branching fraction upper limits combinations summary plot. For each channel we report the HFLAV combined limit, and the experimental published limits. In some cases, the combined limit is weaker than the limit published by a single experiment. This arises since the  $CL_s$  method used in the combination can be more conservative compared to other legitimate methods, especially when the number of observed events fluctuates below the expected background.

## A Branching fractions fit measurement list by reference

Table 17 reports the measurements used for the HFLAV-Tau branching fraction fit grouped by their bibliographic reference.

Table 17: By-reference measurements list.

Reference / Branching Fraction	Value
BARATE 98 (ALEPH) [52]	
$\Gamma_{85} = K^- \pi^+ \pi^- \nu_\tau$ (ex. $K^0$ )	$0.00214 \pm 0.0004701$
$\Gamma_{88} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. $K^0$ )	$0.00061 \pm 0.0004295$
$\Gamma_{93} = \pi^- K^- K^+ \nu_\tau$	$0.00163 \pm 0.0002702$
$\Gamma_{94} = \pi^- K^- K^+ \pi^0 \nu_\tau$	$0.00075 \pm 0.0003265$
BARATE 98E (ALEPH) [8]	
$\Gamma_{33} = K_S^0 (\text{particles})^- \nu_\tau$	$0.0097 \pm 0.000849$
$\Gamma_{37} = K^- K^0 \nu_\tau$	$0.00158 \pm 0.0004531$
$\Gamma_{40} = \pi^- \bar{K}^0 \pi^0 \nu_\tau$	$0.00294 \pm 0.0008184$
$\Gamma_{42} = K^- \pi^0 K^0 \nu_\tau$	$0.00152 \pm 0.0007885$
$\Gamma_{47} = \pi^- K_S^0 K_S^0 \nu_\tau$	$0.00026 \pm 0.0001118$
$\Gamma_{48} = \pi^- K_S^0 K_L^0 \nu_\tau$	$0.00101 \pm 0.0002642$
$\Gamma_{51} = \pi^- \pi^0 K_S^0 K_L^0 \nu_\tau$	$(3.1 \pm 1.1 \pm 0.5) \cdot 10^{-4}$
$\Gamma_{53} = \bar{K}^0 h^- h^- h^+ \nu_\tau$	$0.00023 \pm 0.000202485$
BARATE 99K (ALEPH) [23]	
$\Gamma_{10} = K^- \nu_\tau$	$0.00696 \pm 0.0002865$
$\Gamma_{16} = K^- \pi^0 \nu_\tau$	$0.00444 \pm 0.0003538$
$\Gamma_{23} = K^- \pi^0 \nu_\tau$ (ex. $K^0$ )	$0.00056 \pm 0.00025$
$\Gamma_{28} = K^- 3\pi^0 \nu_\tau$ (ex. $K^0, \eta$ )	$0.00037 \pm 0.0002371$
$\Gamma_{35} = \pi^- \bar{K}^0 \nu_\tau$	$0.00928 \pm 0.000564$
$\Gamma_{37} = K^- K^0 \nu_\tau$	$0.00162 \pm 0.0002371$
$\Gamma_{40} = \pi^- \bar{K}^0 \pi^0 \nu_\tau$	$0.00347 \pm 0.0006464$
$\Gamma_{42} = K^- \pi^0 K^0 \nu_\tau$	$0.00143 \pm 0.0002915$
BARATE 99R (ALEPH) [36]	
$\Gamma_{44} = \pi^- \bar{K}^0 \pi^0 \pi^0 \nu_\tau$ (ex. $K^0$ )	$0.00026 \pm 0.00024$
BUSKULIC 96 (ALEPH) [66]	
$\frac{\Gamma_{150}}{\Gamma_{66}} = \frac{h^- \omega \nu_\tau}{h^- h^- h^+ \pi^0 \nu_\tau}$ (ex. $K^0$ )	$0.431 \pm 0.033$
BUSKULIC 97C (ALEPH) [59]	
$\Gamma_{126} = \pi^- \pi^0 \eta \nu_\tau$	$0.0018 \pm 0.0004472$
$\Gamma_{128} = K^- \eta \nu_\tau$	$(2.9_{-1.2}^{+1.3 \cdot 10^{-4}} \pm 0.7) \cdot 10^{-4}$
$\Gamma_{150} = h^- \omega \nu_\tau$	$0.0191 \pm 0.000922$
$\Gamma_{152} = h^- \pi^0 \omega \nu_\tau$	$0.0043 \pm 0.000781$
SCHAEL 05C (ALEPH) [10]	
$\Gamma_3 = \mu^- \bar{\nu}_\mu \nu_\tau$	$0.17319 \pm 0.000769675$
$\Gamma_5 = e^- \bar{\nu}_e \nu_\tau$	$0.17837 \pm 0.000804984$
$\Gamma_8 = h^- \nu_\tau$	$0.11524 \pm 0.00104805$
$\Gamma_{13} = h^- \pi^0 \nu_\tau$	$0.25924 \pm 0.00128973$
$\Gamma_{19} = h^- 2\pi^0 \nu_\tau$ (ex. $K^0$ )	$0.09295 \pm 0.00121655$
$\Gamma_{26} = h^- 3\pi^0 \nu_\tau$	$0.01082 \pm 0.000925581$
$\Gamma_{30} = h^- 4\pi^0 \nu_\tau$ (ex. $K^0, \eta$ )	$0.00112 \pm 0.000509313$
$\Gamma_{58} = h^- h^- h^+ \nu_\tau$ (ex. $K^0, \omega$ )	$0.09469 \pm 0.000957758$
$\Gamma_{66} = h^- h^- h^+ \pi^0 \nu_\tau$ (ex. $K^0$ )	$0.04734 \pm 0.000766942$

Table 17 – continued from previous page

Reference / Branching Fraction	Value
$\Gamma_{76} = h^- h^+ 2\pi^0 \nu_\tau$ (ex. $K^0$ )	$0.00435 \pm 0.000460977$
$\Gamma_{103} = 3h^- 2h^+ \nu_\tau$ (ex. $K^0$ )	$0.00072 \pm 0.00015$
$\Gamma_{104} = 3h^- 2h^+ \pi^0 \nu_\tau$ (ex. $K^0$ )	$(0.021 \pm 0.007 \pm 0.009) \cdot 10^{-2}$
$\Gamma_{805} = a_1^- (\rightarrow \pi^- \gamma) \nu_\tau$	$(4 \pm 2) \cdot 10^{-4}$
ALBRECHT 88B (ARGUS) [57]	
$\Gamma_{103} = 3h^- 2h^+ \nu_\tau$ (ex. $K^0$ )	$0.00064 \pm 0.00023 \pm 0.0001$
ALBRECHT 92D (ARGUS) [14]	
$\frac{\Gamma_3}{\Gamma_5} = \frac{\mu^- \bar{\nu}_\mu \nu_\tau}{e^- \bar{\nu}_e \nu_\tau}$	$0.997 \pm 0.035 \pm 0.04$
AUBERT 07AP ( <i>BABAR</i> ) [29]	
$\Gamma_{16} = K^- \pi^0 \nu_\tau$	$0.00416 \pm 3 \cdot 10^{-5} \pm 0.00018$
AUBERT 08 ( <i>BABAR</i> ) [44]	
$\Gamma_{60} = \pi^- \pi^+ \pi^- \nu_\tau$ (ex. $K^0$ )	$0.0883 \pm 0.0001 \pm 0.0013$
$\Gamma_{85} = K^- \pi^+ \pi^- \nu_\tau$ (ex. $K^0$ )	$0.00273 \pm 2 \cdot 10^{-5} \pm 9 \cdot 10^{-5}$
$\Gamma_{93} = \pi^- K^- K^+ \nu_\tau$	$0.001346 \pm 1 \cdot 10^{-5} \pm 3.6 \cdot 10^{-5}$
$\Gamma_{96} = K^- K^- K^+ \nu_\tau$	$1.5777 \cdot 10^{-5} \pm 1.3 \cdot 10^{-6} \pm 1.2308 \cdot 10^{-6}$
AUBERT 10F ( <i>BABAR</i> ) [15]	
$\frac{\Gamma_3}{\Gamma_5} = \frac{\mu^- \bar{\nu}_\mu \nu_\tau}{e^- \bar{\nu}_e \nu_\tau}$	$0.9796 \pm 0.0016 \pm 0.0036$
$\frac{\Gamma_9}{\Gamma_5} = \frac{\pi^- \nu_\tau}{e^- \bar{\nu}_e \nu_\tau}$	$0.5945 \pm 0.0014 \pm 0.0061$
$\frac{\Gamma_{10}}{\Gamma_5} = \frac{K^- \nu_\tau}{e^- \bar{\nu}_e \nu_\tau}$	$0.03882 \pm 0.00032 \pm 0.00057$
DEL-AMO-SANCHEZ 11E ( <i>BABAR</i> ) [62]	
$\Gamma_{128} = K^- \eta \nu_\tau$	$0.000142 \pm 1.1 \cdot 10^{-5} \pm 7 \cdot 10^{-6}$
LEES 12X ( <i>BABAR</i> ) [67]	
$\Gamma_{811} = \pi^- 2\pi^0 \omega \nu_\tau$ (ex. $K^0$ )	$(7.3 \pm 1.2 \pm 1.2) \cdot 10^{-5}$
$\Gamma_{812} = 2\pi^- \pi^+ 3\pi^0 \nu_\tau$ (ex. $K^0, \eta, \omega, f_1$ )	$(0.1 \pm 0.08 \pm 0.30) \cdot 10^{-4}$
$\Gamma_{821} = 3\pi^- 2\pi^+ \nu_\tau$ (ex. $K^0, \omega, f_1$ )	$(7.68 \pm 0.04 \pm 0.40) \cdot 10^{-4}$
$\Gamma_{822} = K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. $K^0$ )	$(0.6 \pm 0.5 \pm 1.1) \cdot 10^{-6}$
$\Gamma_{831} = 2\pi^- \pi^+ \omega \nu_\tau$ (ex. $K^0$ )	$(8.4 \pm 0.4 \pm 0.6) \cdot 10^{-5}$
$\Gamma_{832} = 3\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. $K^0, \eta, \omega, f_1$ )	$(0.36 \pm 0.03 \pm 0.09) \cdot 10^{-4}$
$\Gamma_{833} = K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. $K^0$ )	$(1.1 \pm 0.4 \pm 0.4) \cdot 10^{-6}$
$\Gamma_{910} = 2\pi^- \pi^+ \eta \nu_\tau$ ( $\eta \rightarrow 3\pi^0$ ) (ex. $K^0$ )	$(8.27 \pm 0.88 \pm 0.81) \cdot 10^{-5}$
$\Gamma_{911} = \pi^- 2\pi^0 \eta \nu_\tau$ ( $\eta \rightarrow \pi^+ \pi^- \pi^0$ ) (ex. $K^0$ )	$(4.57 \pm 0.77 \pm 0.50) \cdot 10^{-5}$
$\Gamma_{920} = \pi^- f_1 \nu_\tau$ ( $f_1 \rightarrow 2\pi^- 2\pi^+$ )	$(5.20 \pm 0.31 \pm 0.37) \cdot 10^{-5}$
$\Gamma_{930} = 2\pi^- \pi^+ \eta \nu_\tau$ ( $\eta \rightarrow \pi^+ \pi^- \pi^0$ ) (ex. $K^0$ )	$(5.39 \pm 0.27 \pm 0.41) \cdot 10^{-5}$
$\Gamma_{944} = 2\pi^- \pi^+ \eta \nu_\tau$ ( $\eta \rightarrow \gamma \gamma$ ) (ex. $K^0$ )	$(8.26 \pm 0.35 \pm 0.51) \cdot 10^{-5}$
LEES 12Y ( <i>BABAR</i> ) [37]	
$\Gamma_{47} = \pi^- K_S^0 K_S^0 \nu_\tau$	$(2.31 \pm 0.04 \pm 0.08) \cdot 10^{-4}$
$\Gamma_{50} = \pi^- \pi^0 K_S^0 K_S^0 \nu_\tau$	$(1.60 \pm 0.20 \pm 0.22) \cdot 10^{-5}$
FUJIKAWA 08 (Belle) [27]	
$\Gamma_{13} = h^- \pi^0 \nu_\tau$	$0.2567 \pm 1 \cdot 10^{-4} \pm 0.0039$
INAMI 09 (Belle) [60]	
$\Gamma_{126} = \pi^- \pi^0 \eta \nu_\tau$	$0.00135 \pm 3 \cdot 10^{-5} \pm 7 \cdot 10^{-5}$
$\Gamma_{128} = K^- \eta \nu_\tau$	$0.000158 \pm 5 \cdot 10^{-6} \pm 9 \cdot 10^{-6}$
$\Gamma_{130} = K^- \pi^0 \eta \nu_\tau$	$4.6 \cdot 10^{-5} \pm 1.1 \cdot 10^{-5} \pm 4 \cdot 10^{-6}$
$\Gamma_{132} = \pi^- \bar{K}^0 \eta \nu_\tau$	$8.8 \cdot 10^{-5} \pm 1.4 \cdot 10^{-5} \pm 6 \cdot 10^{-6}$
LEE 10 (Belle) [45]	

Table 17 – continued from previous page

Reference / Branching Fraction	Value
$\Gamma_{60} = \pi^- \pi^+ \pi^- \nu_\tau$ (ex. $K^0$ )	$0.0842 \pm 0^{+0.0026}_{-0.0025}$
$\Gamma_{85} = K^- \pi^+ \pi^- \nu_\tau$ (ex. $K^0$ )	$0.0033 \pm 1 \cdot 10^{-5} {}^{+0.00016}_{-0.00017}$
$\Gamma_{93} = \pi^- K^- K^+ \nu_\tau$	$0.00155 \pm 1 \cdot 10^{-5} {}^{+6 \cdot 10^{-5}}_{-5 \cdot 10^{-5}}$
$\Gamma_{96} = K^- K^- K^+ \nu_\tau$	$3.29 \cdot 10^{-5} \pm 1.7 \cdot 10^{-6} {}^{+1.9 \cdot 10^{-6}}_{-2.0 \cdot 10^{-6}}$
RYU 14vpc (Belle) [7]	
$\Gamma_{35} = \pi^- \bar{K}^0 \nu_\tau$	$8.32 \cdot 10^{-3} \pm 0.3\% \pm 1.8\%$
$\Gamma_{37} = K^- K^0 \nu_\tau$	$14.8 \cdot 10^{-4} \pm 0.9\% \pm 3.7\%$
$\Gamma_{40} = \pi^- \bar{K}^0 \pi^0 \nu_\tau$	$3.86 \cdot 10^{-3} \pm 0.8\% \pm 3.5\%$
$\Gamma_{42} = K^- \pi^0 K^0 \nu_\tau$	$14.96 \cdot 10^{-4} \pm 1.3\% \pm 4.9\%$
$\Gamma_{47} = \pi^- K_S^0 K_S^0 \nu_\tau$	$2.33 \cdot 10^{-4} \pm 1.4\% \pm 4.0\%$
$\Gamma_{50} = \pi^- \pi^0 K_S^0 K_S^0 \nu_\tau$	$2.00 \cdot 10^{-5} \pm 10.8\% \pm 10.1\%$
BEHREND 89B (CELLO) [38]	
$\Gamma_{54} = h^- h^- h^+ \geq 0$ neutrals $\geq 0 K_L^0 \nu_\tau$	$0.15 \pm 0.004 \pm 0.003$
ANASTASSOV 01 (CLEO) [49]	
$\Gamma_{78} = h^- h^- h^+ 3\pi^0 \nu_\tau$	$0.00022 \pm 3 \cdot 10^{-5} \pm 4 \cdot 10^{-5}$
$\Gamma_{104} = 3h^- 2h^+ \pi^0 \nu_\tau$ (ex. $K^0$ )	$0.00017 \pm 2 \cdot 10^{-5} \pm 2 \cdot 10^{-5}$
ANASTASSOV 97 (CLEO) [16]	
$\frac{\Gamma_3}{\Gamma_5} = \frac{\mu^- \bar{\nu}_\mu \nu_\tau}{e^- \bar{\nu}_e \nu_\tau}$	$0.9777 \pm 0.0063 \pm 0.0087$
$\Gamma_5 = e^- \bar{\nu}_e \nu_\tau$	$0.1776 \pm 0.0006 \pm 0.0017$
$\Gamma_8 = h^- \nu_\tau$	$0.1152 \pm 0.0005 \pm 0.0012$
ARTUSO 92 (CLEO) [61]	
$\Gamma_{126} = \pi^- \pi^0 \eta \nu_\tau$	$0.0017 \pm 0.0002 \pm 0.0002$
ARTUSO 94 (CLEO) [28]	
$\Gamma_{13} = h^- \pi^0 \nu_\tau$	$0.2587 \pm 0.0012 \pm 0.0042$
BALEST 95C (CLEO) [43]	
$\Gamma_{57} = h^- h^- h^+ \nu_\tau$ (ex. $K^0$ )	$0.0951 \pm 0.0007 \pm 0.002$
$\Gamma_{66} = h^- h^- h^+ \pi^0 \nu_\tau$ (ex. $K^0$ )	$0.0423 \pm 0.0006 \pm 0.0022$
$\frac{\Gamma_{150}}{\Gamma_{66}} = \frac{h^- \omega \nu_\tau}{h^- h^- h^+ \pi^0 \nu_\tau}$ (ex. $K^0$ )	$0.464 \pm 0.016 \pm 0.017$
BARINGER 87 (CLEO) [65]	
$\Gamma_{150} = h^- \omega \nu_\tau$	$0.016 \pm 0.0027 \pm 0.0041$
BARTEL 96 (CLEO) [63]	
$\Gamma_{128} = K^- \eta \nu_\tau$	$(2.6 \pm 0.5 \pm 0.5) \cdot 10^{-4}$
BATTLE 94 (CLEO) [24]	
$\Gamma_{10} = K^- \nu_\tau$	$0.0066 \pm 0.0007 \pm 0.0009$
$\Gamma_{16} = K^- \pi^0 \nu_\tau$	$0.0051 \pm 0.001 \pm 0.0007$
$\Gamma_{23} = K^- 2\pi^0 \nu_\tau$ (ex. $K^0$ )	$0.0009 \pm 0.001 \pm 0.0003$
$\Gamma_{31} = K^- \geq 0 \pi^0 \geq 0 K^0 \geq 0 \gamma \nu_\tau$	$0.017 \pm 0.0012 \pm 0.0019$
BISHAI 99 (CLEO) [64]	
$\Gamma_{130} = K^- \pi^0 \eta \nu_\tau$	$(1.77 \pm 0.56 \pm 0.71) \cdot 10^{-4}$
$\Gamma_{132} = \pi^- \bar{K}^0 \eta \nu_\tau$	$(2.2 \pm 0.70 \pm 0.22) \cdot 10^{-4}$
BORTOLETTO 93 (CLEO) [48]	
$\frac{\Gamma_{76}}{\Gamma_{54}} = \frac{h^- h^- h^+ 2\pi^0 \nu_\tau}{h^- h^- h^+ \geq 0 \text{ neutrals} \geq 0 K_L^0 \nu_\tau}$ (ex. $K^0$ )	$0.034 \pm 0.002 \pm 0.003$
$\frac{\Gamma_{152}}{\Gamma_{76}} = \frac{h^- \omega \pi^0 \nu_\tau}{h^- h^- h^+ 2\pi^0 \nu_\tau}$ (ex. $K^0$ )	$0.81 \pm 0.06 \pm 0.06$
COAN 96 (CLEO) [33]	

Table 17 – continued from previous page

Reference / Branching Fraction	Value
$\Gamma_{34} = h^- \bar{K}^0 \nu_\tau$	$0.00855 \pm 0.00036 \pm 0.00073$
$\Gamma_{37} = K^- K^0 \nu_\tau$	$0.00151 \pm 0.00021 \pm 0.00022$
$\Gamma_{39} = h^- \bar{K}^0 \pi^0 \nu_\tau$	$0.00562 \pm 0.0005 \pm 0.00048$
$\Gamma_{42} = K^- \pi^0 K^0 \nu_\tau$	$0.00145 \pm 0.00036 \pm 0.0002$
$\Gamma_{47} = \pi^- K_S^0 K_S^0 \nu_\tau$	$0.00023 \pm 5 \cdot 10^{-5} \pm 3 \cdot 10^{-5}$
EDWARDS 00A (CLEO) [47]	
$\Gamma_{69} = \pi^- \pi^+ \pi^- \pi^0 \nu_\tau$ (ex. $K^0$ )	$0.0419 \pm 0.001 \pm 0.0021$
GIBAUT 94B (CLEO) [55]	
$\Gamma_{102} = 3h^- 2h^+ \geq 0$ neutrals $\nu_\tau$ (ex. $K^0$ )	$0.00097 \pm 5 \cdot 10^{-5} \pm 0.00011$
$\Gamma_{103} = 3h^- 2h^+ \nu_\tau$ (ex. $K^0$ )	$0.00077 \pm 5 \cdot 10^{-5} \pm 9 \cdot 10^{-5}$
PROCARIO 93 (CLEO) [31]	
$\Gamma_{19} = \frac{h^- 2\pi^0 \nu_\tau}{h^- \pi^0 \nu_\tau}$ (ex. $K^0$ )	$0.342 \pm 0.006 \pm 0.016$
$\Gamma_{13} = \frac{h^- 3\pi^0 \nu_\tau}{h^- \pi^0 \nu_\tau}$	$0.044 \pm 0.003 \pm 0.005$
$\Gamma_{26} = \frac{h^- 4\pi^0 \nu_\tau}{h^- \pi^0 \nu_\tau}$ (ex. $K^0$ )	$0.0016 \pm 0.0005 \pm 0.0005$
RICHICHI 99 (CLEO) [50]	
$\Gamma_{80} = \frac{K^- \pi^- h^+ \nu_\tau}{\pi^- \pi^+ \pi^- \nu_\tau}$ (ex. $K^0$ )	$0.0544 \pm 0.0021 \pm 0.0053$
$\Gamma_{60} = \frac{K^- \pi^- h^+ \pi^0 \nu_\tau}{\pi^- \pi^+ \pi^- \pi^0 \nu_\tau}$ (ex. $K^0$ )	$0.0261 \pm 0.0045 \pm 0.0042$
$\Gamma_{81} = \frac{K^- \pi^- h^+ \pi^0 \nu_\tau}{\pi^- \pi^+ \pi^- \pi^0 \nu_\tau}$ (ex. $K^0$ )	$0.016 \pm 0.0015 \pm 0.003$
$\Gamma_{69} = \frac{\pi^- K^- K^+ \nu_\tau}{\pi^- \pi^+ \pi^- \nu_\tau}$	$0.0079 \pm 0.0044 \pm 0.0016$
ARMS 05 (CLEO3) [53]	
$\Gamma_{88} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. $K^0$ )	$0.00074 \pm 8 \cdot 10^{-5} \pm 0.00011$
$\Gamma_{94} = \pi^- K^- K^+ \pi^0 \nu_\tau$	$(5.5 \pm 1.4 \pm 1.2) \cdot 10^{-5}$
$\Gamma_{151} = K^- \omega \nu_\tau$	$(4.1 \pm 0.6 \pm 0.7) \cdot 10^{-4}$
BRIERE 03 (CLEO3) [46]	
$\Gamma_{60} = \pi^- \pi^+ \pi^- \nu_\tau$ (ex. $K^0$ )	$0.0913 \pm 0.0005 \pm 0.0046$
$\Gamma_{85} = K^- \pi^+ \pi^- \nu_\tau$ (ex. $K^0$ )	$0.00384 \pm 0.00014 \pm 0.00038$
$\Gamma_{93} = \pi^- K^- K^+ \nu_\tau$	$0.00155 \pm 6 \cdot 10^{-5} \pm 9 \cdot 10^{-5}$
ABDALLAH 06A (DELPHI) [21]	
$\Gamma_8 = h^- \nu_\tau$	$0.11571 \pm 0.0012 \pm 0.00114$
$\Gamma_{13} = h^- \pi^0 \nu_\tau$	$0.2574 \pm 0.00201 \pm 0.00138$
$\Gamma_{19} = h^- 2\pi^0 \nu_\tau$ (ex. $K^0$ )	$0.09498 \pm 0.0032 \pm 0.00275$
$\Gamma_{25} = h^- \geq 3\pi^0 \nu_\tau$ (ex. $K^0$ )	$0.01403 \pm 0.00214 \pm 0.00224$
$\Gamma_{57} = h^- h^- h^+ \nu_\tau$ (ex. $K^0$ )	$0.09317 \pm 0.0009 \pm 0.00082$
$\Gamma_{66} = h^- h^- h^+ \pi^0 \nu_\tau$ (ex. $K^0$ )	$0.04545 \pm 0.00106 \pm 0.00103$
$\Gamma_{74} = h^- h^- h^+ \geq 2\pi^0 \nu_\tau$ (ex. $K^0$ )	$0.00561 \pm 0.00068 \pm 0.00095$
$\Gamma_{103} = 3h^- 2h^+ \nu_\tau$ (ex. $K^0$ )	$0.00097 \pm 0.00015 \pm 5 \cdot 10^{-5}$
$\Gamma_{104} = 3h^- 2h^+ \pi^0 \nu_\tau$ (ex. $K^0$ )	$0.00016 \pm 0.00012 \pm 6 \cdot 10^{-5}$
ABREU 92N (DELPHI) [18]	
$\Gamma_7 = h^- \geq 0 K_L^0 \nu_\tau$	$0.124 \pm 0.007 \pm 0.007$
ABREU 94K (DELPHI) [25]	
$\Gamma_{10} = K^- \nu_\tau$	$0.0085 \pm 0.0018$
$\Gamma_{31} = K^- \geq 0 \pi^0 \geq 0 K^0 \geq 0 \gamma \nu_\tau$	$0.0154 \pm 0.0024$
ABREU 99X (DELPHI) [11]	
$\Gamma_3 = \mu^- \bar{\nu}_\mu \nu_\tau$	$0.17325 \pm 0.00095 \pm 0.00077$

**Table 17 – continued from previous page**

Reference / Branching Fraction	Value
$\Gamma_5 = e^- \bar{\nu}_e \nu_\tau$	$0.17877 \pm 0.00109 \pm 0.0011$
BYLSMA 87 (HRS) [56]	
$\Gamma_{102} = 3h^- 2h^+ \geq 0$ neutrals $\nu_\tau$ (ex. $K^0$ )	$0.00102 \pm 0.00029$
$\Gamma_{103} = 3h^- 2h^+ \nu_\tau$ (ex. $K^0$ )	$0.00051 \pm 0.0002$
ACCIARRI 01F (L3) [12]	
$\Gamma_3 = \mu^- \bar{\nu}_\mu \nu_\tau$	$0.17342 \pm 0.0011 \pm 0.00067$
$\Gamma_5 = e^- \bar{\nu}_e \nu_\tau$	$0.17806 \pm 0.00104 \pm 0.00076$
ACCIARRI 95 (L3) [19]	
$\Gamma_7 = h^- \geq 0 K_L^0 \nu_\tau$	$0.1247 \pm 0.0026 \pm 0.0043$
$\Gamma_{13} = h^- \pi^0 \nu_\tau$	$0.2505 \pm 0.0035 \pm 0.005$
$\Gamma_{19} = h^- 2\pi^0 \nu_\tau$ (ex. $K^0$ )	$0.0888 \pm 0.0037 \pm 0.0042$
$\Gamma_{26} = h^- 3\pi^0 \nu_\tau$	$0.017 \pm 0.0024 \pm 0.0038$
ACCIARRI 95F (L3) [34]	
$\Gamma_{35} = \pi^- \bar{K}^0 \nu_\tau$	$0.0095 \pm 0.0015 \pm 0.0006$
$\Gamma_{40} = \pi^- \bar{K}^0 \pi^0 \nu_\tau$	$0.0041 \pm 0.0012 \pm 0.0003$
ACHARD 01D (L3) [41]	
$\Gamma_{55} = h^- h^- h^+ \geq 0$ neutrals $\nu_\tau$ (ex. $K^0$ )	$0.14556 \pm 0.00105 \pm 0.00076$
$\Gamma_{102} = 3h^- 2h^+ \geq 0$ neutrals $\nu_\tau$ (ex. $K^0$ )	$0.0017 \pm 0.00022 \pm 0.00026$
ADEVA 91F (L3) [39]	
$\Gamma_{54} = h^- h^- h^+ \geq 0$ neutrals $\geq 0 K_L^0 \nu_\tau$	$0.144 \pm 0.006 \pm 0.003$
ABBIENDI 00C (OPAL) [35]	
$\Gamma_{35} = \pi^- \bar{K}^0 \nu_\tau$	$0.00933 \pm 0.00068 \pm 0.00049$
$\Gamma_{38} = K^- K^0 \geq 0 \pi^0 \nu_\tau$	$0.0033 \pm 0.00055 \pm 0.00039$
$\Gamma_{43} = \pi^- \bar{K}^0 \geq 1 \pi^0 \nu_\tau$	$0.00324 \pm 0.00074 \pm 0.00066$
ABBIENDI 00D (OPAL) [54]	
$\Gamma_{92} = \pi^- K^- K^+ \geq 0$ neutrals $\nu_\tau$	$0.00159 \pm 0.00053 \pm 0.0002$
ABBIENDI 01J (OPAL) [26]	
$\Gamma_{10} = K^- \nu_\tau$	$0.00658 \pm 0.00027 \pm 0.00029$
$\Gamma_{31} = K^- \geq 0 \pi^0 \geq 0 K^0 \geq 0 \gamma \nu_\tau$	$0.01528 \pm 0.00039 \pm 0.0004$
ABBIENDI 03 (OPAL) [13]	
$\Gamma_3 = \mu^- \bar{\nu}_\mu \nu_\tau$	$0.1734 \pm 0.0009 \pm 0.0006$
ABBIENDI 04J (OPAL) [30]	
$\Gamma_{16} = K^- \pi^0 \nu_\tau$	$0.00471 \pm 0.00059 \pm 0.00023$
$\Gamma_{85} = K^- \pi^+ \pi^- \nu_\tau$ (ex. $K^0$ )	$0.00415 \pm 0.00053 \pm 0.0004$
ABBIENDI 99H (OPAL) [17]	
$\Gamma_5 = e^- \bar{\nu}_e \nu_\tau$	$0.1781 \pm 0.0009 \pm 0.0006$
ACKERSTAFF 98M (OPAL) [22]	
$\Gamma_8 = h^- \nu_\tau$	$0.1198 \pm 0.0013 \pm 0.0016$
$\Gamma_{13} = h^- \pi^0 \nu_\tau$	$0.2589 \pm 0.0017 \pm 0.0029$
$\Gamma_{17} = h^- \geq 2 \pi^0 \nu_\tau$	$0.0991 \pm 0.0031 \pm 0.0027$
ACKERSTAFF 99E (OPAL) [58]	
$\Gamma_{103} = 3h^- 2h^+ \nu_\tau$ (ex. $K^0$ )	$0.00091 \pm 0.00014 \pm 6 \cdot 10^{-5}$
$\Gamma_{104} = 3h^- 2h^+ \pi^0 \nu_\tau$ (ex. $K^0$ )	$0.00027 \pm 0.00018 \pm 9 \cdot 10^{-5}$
AKERS 94G (OPAL) [32]	
$\Gamma_{33} = K_S^0 (\text{particles})^- \nu_\tau$	$0.0097 \pm 0.0009 \pm 0.0006$
AKERS 95Y (OPAL) [42]	

**Table 17 – continued from previous page**

Reference / Branching Fraction	Value
$\Gamma_{55} = h^- h^- h^+ \geq 0 \text{ neutrals } \nu_\tau \text{ (ex. } K^0)$	$0.1496 \pm 0.0009 \pm 0.0022$
$\frac{\Gamma_{57}}{\Gamma_{55}} = \frac{h^- h^- h^+ \nu_\tau \text{ (ex. } K^0)}{h^- h^- h^+ \geq 0 \text{ neutrals } \nu_\tau \text{ (ex. } K^0)}$	$0.66 \pm 0.004 \pm 0.014$
ALEXANDER 91D (OPAL) [20]	
$\Gamma_7 = h^- \geq 0 K_L^0 \nu_\tau$	$0.121 \pm 0.007 \pm 0.005$
AIHARA 87B (TPC) [40]	
$\Gamma_{54} = h^- h^- h^+ \geq 0 \text{ neutrals } \geq 0 K_L^0 \nu_\tau$	$0.151 \pm 0.008 \pm 0.006$
BAUER 94 (TPC) [51]	
$\Gamma_{82} = K^- \pi^- \pi^+ \geq 0 \text{ neutrals } \nu_\tau$	$0.0058^{+0.0015}_{-0.0013} \pm 0.0012$
$\Gamma_{92} = \pi^- K^- K^+ \geq 0 \text{ neutrals } \nu_\tau$	$0.0015^{+0.0009}_{-0.0007} \pm 0.0003$

## References

- [1] Heavy Flavor Averaging Group, D. Asner *et al.*, “Averages of  $b$ -hadron,  $c$ -hadron, and  $\tau$ -lepton properties,” 2010.
- [2] Particle Data Group, C. Patrignani *et al.*, “Review of particle physics,” *Chin. Phys. C40* (2016) 100001.
- [3] Heavy Flavor Averaging Group, Y. Amhis *et al.*, “Averages of  $b$ -hadron,  $c$ -hadron, and  $\tau$ -lepton properties as of early 2012,” 2012.
- [4] Heavy Flavor Averaging Group, Y. Amhis *et al.*, “Averages of  $b$ -hadron,  $c$ -hadron, and  $\tau$ -lepton properties as of summer 2014,” 2014.
- [5] *BABAR* Collaboration, B. Aubert *et al.*, “Measurement of  $B(\tau^- \rightarrow \bar{K}^0 \pi^- \nu_\tau)$  using the *BABAR* detector,” *Nucl. Phys. Proc. Suppl.* **189** (2009) 193, [arXiv:0808.1121 \[hep-ex\]](https://arxiv.org/abs/0808.1121).
- [6] *BABAR* Collaboration, S. Paramesvaran, “Selected topics in tau physics from *BABAR*,” 2009.
- [7] Belle Collaboration, S. Ryu *et al.*, “Measurements of Branching Fractions of  $\tau$  Lepton Decays with one or more  $K_S^0$ ,” *Phys. Rev. D89* (2014) 072009, [arXiv:1402.5213 \[hep-ex\]](https://arxiv.org/abs/1402.5213).
- [8] ALEPH Collaboration, R. Barate *et al.*, “ $K0(S)$  production in tau decays,” *Eur. Phys. J. C4* (1998) 29. <http://cdsweb.cern.ch/record/346304>.
- [9] Particle Data Group, K. Olive *et al.*, “Review of particle physics,” *Chin. Phys. C38* (2014) 090001.
- [10] ALEPH Collaboration, S. Schael *et al.*, “Branching ratios and spectral functions of tau decays: Final ALEPH measurements and physics implications,” *Phys. Rept.* **421** (2005) 191, [arXiv:hep-ex/0506072 \[hep-ex\]](https://arxiv.org/abs/hep-ex/0506072). HFLAV-tau uses measurements of  $\tau \rightarrow hX$  and  $\tau \rightarrow KX$  and obtains  $\tau \rightarrow \pi X$  by difference; the measurement of  $\mathcal{B}(\tau^- \rightarrow 3h^- 2h^+ \pi^0 \nu_\tau \text{ (ex. } K^0\text{)})$  has been read as  $(2.1 \pm 0.7 \pm 0.6) \times 10^{-4}$  whereas PDG11 uses  $(2.1 \pm 0.7 \pm 0.9) \times 10^{-4}$ .
- [11] DELPHI Collaboration, P. Abreu *et al.*, “Measurements of the leptonic branching fractions of the tau,” *Eur. Phys. J. C10* (1999) 201.
- [12] L3 Collaboration, M. Acciarri *et al.*, “Measurement of the  $\tau$  branching fractions into leptons,” *Phys. Lett. B507* (2001) 47, [arXiv:hep-ex/0102023 \[hep-ex\]](https://arxiv.org/abs/hep-ex/0102023).
- [13] OPAL Collaboration, G. Abbiendi *et al.*, “A Measurement of the  $\tau \rightarrow \mu \bar{\nu}_\mu \bar{\nu}_\tau$  branching ratios,” *Phys. Lett. B551* (2003) 35, [arXiv:hep-ex/0211066 \[hep-ex\]](https://arxiv.org/abs/hep-ex/0211066).
- [14] ARGUS Collaboration, H. Albrecht *et al.*, “Measurement of exclusive one prong and inclusive three prong branching ratios of the tau lepton,” *Z. Phys. C53* (1992) 367.
- [15] *BABAR* Collaboration, B. Aubert *et al.*, “Measurements of Charged Current Lepton Universality and  $|V_{us}|$  using Tau Lepton Decays to  $e^- \bar{\nu}_e \tau^- \bar{\nu}_\tau$ ,  $\mu^- \bar{\nu}_\mu \tau^- \bar{\nu}_\tau$ ,  $\pi^- \bar{\nu}_\pi \tau^- \bar{\nu}_\tau$  and  $K^- \bar{\nu}_K \tau^- \bar{\nu}_\tau$ ,” *Phys. Rev. Lett.* **105** (2010) 051602, [arXiv:0912.0242 \[hep-ex\]](https://arxiv.org/abs/0912.0242).
- [16] CLEO Collaboration, A. Anastassov *et al.*, “Experimental test of lepton universality in tau decay,” *Phys. Rev. D55* (1997) 2559. Erratum *ibid. D58*, 119903, (1998).
- [17] OPAL Collaboration, G. Abbiendi *et al.*, “A Measurement of the  $\tau \rightarrow e^- \bar{\nu}_e \tau^- \bar{\nu}_\tau$  branching ratio,” *Phys. Lett. B447* (1999) 134, [arXiv:hep-ex/9812017 \[hep-ex\]](https://arxiv.org/abs/hep-ex/9812017).
- [18] DELPHI Collaboration, P. Abreu *et al.*, “A Study of the decays of tau leptons produced on the Z resonance at LEP,” *Z. Phys. C55* (1992) 555.
- [19] L3 Collaboration, M. Acciarri *et al.*, “Measurement of exclusive branching fractions of hadronic one space prong tau decays,” *Phys. Lett. B345* (1995) 93.
- [20] OPAL Collaboration, G. Alexander *et al.*, “Measurement of branching ratios and tau polarization from  $\tau \rightarrow e^- \bar{\nu}_e \tau^- \bar{\nu}_\tau$ ,  $\tau \rightarrow \mu^- \bar{\nu}_\mu \tau^- \bar{\nu}_\tau$ , and  $\tau \rightarrow \pi^- (\bar{K}^-) \bar{\nu}_\pi (\bar{K}^-) \tau^- \bar{\nu}_\tau$  neutrino decays at LEP,” *Phys. Lett. B266* (1991) 201.
- [21] DELPHI Collaboration, J. Abdallah *et al.*, “A Measurement of the tau hadronic branching ratios,” *Eur. Phys. J. C46* (2006) 1, [arXiv:hep-ex/0603044 \[hep-ex\]](https://arxiv.org/abs/hep-ex/0603044).

- [22] OPAL Collaboration, K. Ackerstaff *et al.*, “Measurement of the one prong hadronic tau branching ratios at LEP,” *Eur. Phys. J. C4* (1998) 193, [arXiv:hep-ex/9801029 \[hep-ex\]](#).
- [23] ALEPH Collaboration, R. Barate *et al.*, “One prong tau decays with kaons,” *Eur. Phys. J. C10* (1999) 1, [arXiv:hep-ex/9903014 \[hep-ex\]](#).
- [24] CLEO Collaboration, M. Battle *et al.*, “Measurement of Cabibbo suppressed decays of the tau lepton,” *Phys. Rev. Lett.* **73** (1994) 1079, [arXiv:hep-ph/9403329 \[hep-ph\]](#).
- [25] DELPHI Collaboration, P. Abreu *et al.*, “Charged kaon production in tau decays at LEP,” *Phys. Lett. B334* (1994) 435.
- [26] OPAL Collaboration, G. Abbiendi *et al.*, “A Study of one prong tau decays with a charged kaon,” *Eur. Phys. J. C19* (2001) 653, [arXiv:hep-ex/0009017 \[hep-ex\]](#).
- [27] Belle Collaboration, M. Fujikawa *et al.*, “High-Statistics Study of the  $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$  Decay,” *Phys. Rev. D78* (2008) 072006, [arXiv:0805.3773 \[hep-ex\]](#).
- [28] CLEO Collaboration, M. Artuso *et al.*, “A Measurement of the branching fraction Beta ( $\tau^- \rightarrow h^- \pi^0 \nu_\tau$  tau-neutrino),” *Phys. Rev. Lett.* **72** (1994) 3762, [arXiv:hep-ph/9404310 \[hep-ph\]](#).
- [29] *BABAR* Collaboration, B. Aubert *et al.*, “Measurement of the  $\tau^- \rightarrow K^- \pi^0 \nu_\tau$  branching fraction,” *Phys. Rev. D76* (2007) 051104, [arXiv:0707.2922 \[hep-ex\]](#).
- [30] OPAL Collaboration, G. Abbiendi *et al.*, “Measurement of the strange spectral function in hadronic tau decays,” *Eur. Phys. J. C35* (2004) 437, [arXiv:hep-ex/0406007 \[hep-ex\]](#).
- [31] CLEO Collaboration, M. Procaro *et al.*, “Tau decays with one charged particle plus multiple pi0s,” *Phys. Rev. Lett.* **70** (1993) 1207.
- [32] OPAL Collaboration, R. Akers *et al.*, “Measurements of the inclusive branching ratios of tau leptons to  $K0(s)$  and charged  $K^*(892)$ ,” *Phys. Lett. B339* (1994) 278.
- [33] CLEO Collaboration, T. Coan *et al.*, “Decays of tau leptons to final states containing  $K(s)0$  mesons,” *Phys. Rev. D53* (1996) 6037.
- [34] L3 Collaboration, M. Acciarri *et al.*, “One prong tau decays with neutral kaons,” *Phys. Lett. B352* (1995) 487.
- [35] OPAL Collaboration, G. Abbiendi *et al.*, “Tau decays with neutral kaons,” *Eur. Phys. J. C13* (2000) 213, [arXiv:hep-ex/9911029 \[hep-ex\]](#).
- [36] ALEPH Collaboration, R. Barate *et al.*, “Study of tau decays involving kaons, spectral functions and determination of the strange quark mass,” *Eur. Phys. J. C11* (1999) 599, [arXiv:hep-ex/9903015 \[hep-ex\]](#).
- [37] *BABAR* Collaboration, J. P. Lees *et al.*, “The branching fraction of  $\tau^- \rightarrow \pi^- K_S^0 K_S^0(\pi^0) \nu_\tau$  decays,” *Phys. Rev. D86* (2012) 092013, [arXiv:1208.0376 \[hep-ex\]](#).
- [38] CELLO Collaboration, H. Behrend *et al.*, “Tau production and decay with the CELLO detector at PETRA,” *Phys. Lett. B222* (1989) 163.
- [39] L3 Collaboration, B. Adeva *et al.*, “Decay properties of tau leptons measured at the Z0 resonance,” *Phys. Lett. B265* (1991) 451.
- [40] TPC/Two Gamma Collaboration, H. Aihara *et al.*, “Measurement of  $\tau$  branching ratios,” *Phys. Rev. D35* (1987) 1553.
- [41] L3 Collaboration, P. Achard *et al.*, “Measurement of the topological branching fractions of the  $\tau$  lepton at LEP,” *Phys. Lett. B519* (2001) 189, [arXiv:hep-ex/0107055 \[hep-ex\]](#).
- [42] OPAL Collaboration, R. Akers *et al.*, “Measurement of the  $\tau^- \rightarrow h^- h^+ h^- \tau$  neutrino and  $\tau^- \rightarrow h^- h^+ h^- \geq 1 \pi^0 \tau$  neutrino branching ratios,” *Z. Phys. C68* (1995) 555.
- [43] CLEO Collaboration, R. Balest *et al.*, “Measurements of the decays  $\tau^- \rightarrow h^- h^+ h^- \tau$  neutrino and  $\tau^- \rightarrow h^- h^+ h^- \pi^0 \tau$  neutrino,” *Phys. Rev. Lett.* **75** (1995) 3809.
- [44] *BABAR* Collaboration, B. Aubert *et al.*, “Exclusive branching fraction measurements of semileptonic tau decays into three charged hadrons,  $\tau^- \rightarrow \phi \pi^- \nu_\tau$  and  $\tau^- \rightarrow \phi K^- \nu_\tau$ ,” *Phys. Rev. Lett.* **100** (2008) 011801, [arXiv:0707.2981 \[hep-ex\]](#).

- [45] Belle Collaboration, M. Lee *et al.*, “Measurement of the branching fractions and the invariant mass distributions for  $\tau^- \rightarrow h^- h^+ h^- \nu_\tau$  decays,” *Phys. Rev.* D81 (2010) 113007, [arXiv:1001.0083 \[hep-ex\]](#).
- [46] CLEO Collaboration, R. A. Briere *et al.*, “Branching fractions of tau leptons decays to three charged hadrons,” *Phys. Rev. Lett.* 90 (2003) 181802, [arXiv:hep-ex/0302028 \[hep-ex\]](#).
- [47] CLEO Collaboration, K. Edwards *et al.*, “Resonant structure of tau  $\rightarrow$  three pi pi0 neutrino(tau) and tau  $\rightarrow$  omega pi neutrino(tau) decays,” *Phys. Rev.* D61 (2000) 072003, [arXiv:hep-ex/9908024 \[hep-ex\]](#).
- [48] CLEO Collaboration, D. Bortoletto *et al.*, “Measurement of the decay tau-  $\rightarrow$  pi- pi+ pi- 2 pi0 tau-neutrino,” *Phys. Rev. Lett.* 71 (1993) 1791.
- [49] CLEO Collaboration, A. Anastassov *et al.*, “Study of tau decays to six pions and neutrino,” *Phys. Rev. Lett.* 86 (2001) 4467, [arXiv:hep-ex/0010025 \[hep-ex\]](#).
- [50] CLEO Collaboration, S. Richichi *et al.*, “Study of three prong hadronic tau decays with charged kaons,” *Phys. Rev.* D60 (1999) 112002, [arXiv:hep-ex/9810026 \[hep-ex\]](#).
- [51] TPC/Two Gamma Collaboration, D. A. Bauer *et al.*, “Measurement of the kaon content of three prong tau decays,” *Phys. Rev.* D50 (1994) 13.
- [52] ALEPH Collaboration, R. Barate *et al.*, “Three prong tau decays with charged kaons,” *Eur. Phys. J.* C1 (1998) 65.
- [53] CLEO Collaboration, K. E. Arms *et al.*, “Study of tau decays to four-hadron final states with kaons,” *Phys. Rev. Lett.* 94 (2005) 241802, [arXiv:hep-ex/0501042 \[hep-ex\]](#).
- [54] OPAL Collaboration, G. Abbiendi *et al.*, “A Study of three prong tau decays with charged kaons,” *Eur. Phys. J.* C13 (2000) 197, [arXiv:hep-ex/9908013 \[hep-ex\]](#).
- [55] CLEO Collaboration, D. Gibaut *et al.*, “Study of the five charged pion decay of the tau lepton,” *Phys. Rev. Lett.* 73 (1994) 934.
- [56] B. Bylsma *et al.*, “Limit on tau decay to seven charged particles,” *Phys. Rev.* D35 (1987) 2269.
- [57] ARGUS Collaboration, H. Albrecht *et al.*, “An Improved Upper Limit on the tau-neutrino Mass from the Decay tau-  $\rightarrow$  pi- pi- pi- pi+ pi+ tau-neutrino,” *Phys. Lett.* B202 (1988) 149.
- [58] OPAL Collaboration, K. Ackerstaff *et al.*, “Measurement of tau branching ratios to five charged hadrons,” *Eur. Phys. J.* C8 (1999) 183, [arXiv:hep-ex/9808011 \[hep-ex\]](#).
- [59] ALEPH Collaboration, D. Buskulic *et al.*, “A Study of tau decays involving eta and omega mesons,” *Z. Phys.* C74 (1997) 263.
- [60] Belle Collaboration, K. Inami *et al.*, “Precise measurement of hadronic tau-decays with an eta meson,” *Phys. Lett.* B672 (2009) 209, [arXiv:0811.0088 \[hep-ex\]](#).
- [61] CLEO Collaboration, M. Artuso *et al.*, “Measurement of tau decays involving eta mesons,” *Phys. Rev. Lett.* 69 (1992) 3278.
- [62] BABAR Collaboration, P. del Amo Sanchez *et al.*, “Studies of tau-  $\rightarrow$  eta K-nu and tau-  $\rightarrow$  eta pi- nu(tau) at BaBar and a search for a second-class current,” *Phys. Rev.* D83 (2011) 032002, [arXiv:1011.3917 \[hep-ex\]](#).
- [63] CLEO Collaboration, J. E. Bartelt *et al.*, “First observation of the decay tau-  $\rightarrow$  K- eta tau-neutrino,” *Phys. Rev. Lett.* 76 (1996) 4119.
- [64] CLEO Collaboration, M. Bishai *et al.*, “First observation of the decay tau-  $\rightarrow$  K\*- eta tau-neutrino,” *Phys. Rev. Lett.* 82 (1999) 281, [arXiv:hep-ex/9809012 \[hep-ex\]](#).
- [65] CLEO Collaboration, P. S. Baringer *et al.*, “Production of eta and omega mesons in tau decay and a search for second class currents,” *Phys. Rev. Lett.* 59 (1987) 1993.
- [66] ALEPH Collaboration, D. Buskulic *et al.*, “Tau hadronic branching ratios,” *Z. Phys.* C70 (1996) 579.
- [67] BABAR Collaboration, J. P. Lees *et al.*, “Study of high-multiplicity 3-prong and 5-prong tau decays at BABAR,” *Phys. Rev.* D86 (2012) 092010, [arXiv:1209.2734 \[hep-ex\]](#).
- [68] Belle Collaboration, K. Belous *et al.*, “Measurement of the  $\tau$ -lepton lifetime at Belle,” *Phys. Rev. Lett.* 112 (2014) 031801, [arXiv:1310.8503 \[hep-ex\]](#).

- [69] W. Marciano and A. Sirlin, “Electroweak Radiative Corrections to tau Decay,” *Phys. Rev. Lett.* **61** (1988) 1815.
- [70] A. Pich, “Precision Tau Physics,” *Prog. Part. Nucl. Phys.* **75** (2014) 41, [arXiv:1310.7922 \[hep-ph\]](#).
- [71] A. Ferroglio, C. Greub, A. Sirlin, and Z. Zhang, “Contributions of the W-boson propagator to  $\mu$  and  $\tau$  leptonic decay rates,” *Phys. Rev. D* **88** (2013) 033012, [arXiv:1307.6900 \[hep-ph\]](#).
- [72] M. Fael, L. Mercolli, and M. Passera, “W-propagator corrections to  $\mu$  and  $\tau$  leptonic decays,” *Phys. Rev. D* **88** (2013) 093011, [arXiv:1310.1081 \[hep-ph\]](#).
- [73] W. J. Marciano and A. Sirlin, “Radiative corrections to  $\pi_{\ell 2}$  decays,” *Phys. Rev. Lett.* **71** (1993) 3629.
- [74] R. Decker and M. Finkemeier, “Radiative corrections to the decay  $\tau \rightarrow \pi(K)\nu_{\tau}$ ,” *Phys. Lett. B* **334** (1994) 199.
- [75] R. Decker and M. Finkemeier, “Short and long distance effects in the decay  $\tau \rightarrow \pi\nu_{\tau}(\gamma)$ ,” *Nucl. Phys. B* **438** (1995) 17, [arXiv:hep-ph/9403385](#).
- [76] R. Decker and M. Finkemeier, “Radiative corrections to the decay  $\tau \rightarrow \pi\nu_{\tau}$ ,” *Nucl. Phys. Proc. Suppl.* **40** (1995) 453, [arXiv:hep-ph/9411316 \[hep-ph\]](#).
- [77] M. Davier, A. Hocker, and Z. Zhang, “The Physics of hadronic tau decays,” *Rev. Mod. Phys.* **78** (2006) 1043, [arXiv:hep-ph/0507078 \[hep-ph\]](#).
- [78] FlaviaNet working group on kaon decays, M. Antonelli *et al.*, “An Evaluation of  $|V_{us}|$  and precise tests of the Standard Model from world data on leptonic and semileptonic kaon decays,” *Eur. Phys. J. C* **69** (2010) 399, [arXiv:1005.2323 \[hep-ph\]](#).
- [79] E. Gamiz, M. Jamin, A. Pich, J. Prades, and F. Schwab, “Determination of  $m(s)$  and  $|V_{us}|$  from hadronic tau decays,” *JHEP* **01** (2003) 060, [arXiv:hep-ph/0212230 \[hep-ph\]](#).
- [80] E. Gamiz, M. Jamin, A. Pich, J. Prades, and F. Schwab, “ $|V_{us}|$  and  $m(s)$  from hadronic tau decays,” *Phys. Rev. Lett.* **94** (2005) 011803, [arXiv:hep-ph/0408044 \[hep-ph\]](#).
- [81] E. Gamiz, M. Jamin, A. Pich, J. Prades, and F. Schwab, “ $|V_{us}|$  and  $m(s)$  from hadronic tau decays,” *Nucl. Phys. Proc. Suppl.* **169** (2007) 85, [arXiv:hep-ph/0612154](#).
- [82] E. Gamiz, M. Jamin, A. Pich, J. Prades, and F. Schwab, “Theoretical progress on the  $V_{us}$  determination from tau decays,” *PoS KAON* (2008) 008, [arXiv:0709.0282 \[hep-ph\]](#).
- [83] K. Maltman, “A Critical look at  $V_{us}$  determinations from hadronic  $\tau$  decay data,” *Nucl. Phys. Proc. Suppl.* **218** (2011) 146, [arXiv:1011.6391 \[hep-ph\]](#).
- [84] J. C. Hardy and I. S. Towner, “Superallowed  $0^+ \rightarrow 0^+$  nuclear  $\beta$  decays: 2014 critical survey, with precise results for  $V_{ud}$  and CKM unitarity,” *Phys. Rev. C* **91** (2015) 025501, [arXiv:1411.5987 \[nucl-ex\]](#).
- [85] FLAG working group, S. Aoki *et al.*, “Review of lattice results concerning low-energy particle physics,” *Eur. Phys. J. C* **77** (2017) 112, [arXiv:1607.00299 \[hep-lat\]](#). see also <http://itpwiki.unibe.ch/flag/>.
- [86] V. Cirigliano and H. Neufeld, “A note on isospin violation in  $\Pi 2(\text{gamma})$  decays,” *Phys. Lett. B* **700** (2011) 7, [arXiv:1102.0563 \[hep-ph\]](#).
- [87] W. J. Marciano, “Precise determination of  $|V_{us}|$  from lattice calculations of pseudoscalar decay constants,” *Phys. Rev. Lett.* **93** (2004) 231803, [arXiv:hep-ph/0402299](#).
- [88] M. Jamin, A. Pich, and J. Portoles, “What can be learned from the Belle spectrum for the decay  $\tau^- \rightarrow \nu_{\tau} K_S \pi^-$ ,” *Phys. Lett. B* **664** (2008) 78, [arXiv:0803.1786 \[hep-ph\]](#).
- [89] M. Antonelli, V. Cirigliano, A. Lusiani, and E. Passemar, “Predicting the  $\tau$  strange branching ratios and implications for  $V_{us}$ ,” *JHEP* **10** (2013) 070, [arXiv:1304.8134 \[hep-ph\]](#).
- [90] K. Maltman, R. J. Hudspith, R. Lewis, C. E. Wolfe, and J. Zanotti, “A resolution of the inclusive flavor-breaking sum rule  $\tau |V_{us}|$  puzzle,” [arXiv:1510.06954 \[hep-ph\]](#).
- [91] R. J. Hudspith, R. Lewis, K. Maltman, and J. Zanotti, “A resolution of the inclusive flavor-breaking  $\tau |V_{us}|$  puzzle,” [arXiv:1702.01767 \[hep-ph\]](#).
- [92] *BABAR* Collaboration, B. Aubert *et al.*, “Searches for Lepton Flavor Violation in the Decays  $\tau^{+-} \rightarrow e^{+-}$  gamma and  $\tau^{+-} \rightarrow \mu^{+-}$  gamma,” *Phys. Rev. Lett.* **104** (2010) 021802, [arXiv:0908.2381 \[hep-ex\]](#).

- [93] Belle Collaboration, K. Hayasaka *et al.*, “New search for tau → mu gamma and tau → e gamma decays at Belle,” *Phys. Lett.* **B666** (2008) 16, [arXiv:0705.0650 \[hep-ex\]](#).
- [94] *BABAR* Collaboration, B. Aubert *et al.*, “Search for Lepton Flavor Violating Decays  $\tau^\pm \rightarrow \ell^\pm \pi^0, \ell^\pm \eta, \ell^\pm \eta'$ ,” *Phys. Rev. Lett.* **98** (2007) 061803, [arXiv:hep-ex/0610067 \[hep-ex\]](#).
- [95] Belle Collaboration, Y. Miyazaki *et al.*, “Search for lepton flavor violating tau- decays into l- eta, l- eta-prime and l- pi0,” *Phys. Lett.* **B648** (2007) 341, [arXiv:hep-ex/0703009 \[hep-ex\]](#).
- [96] *BABAR* Collaboration, B. Aubert *et al.*, “Search for Lepton Flavor Violating Decays tau → l- K0(s) with the BABAR Experiment,” *Phys. Rev. D79* (2009) 012004, [arXiv:0812.3804 \[hep-ex\]](#).
- [97] Belle Collaboration, Y. Miyazaki *et al.*, “Search for Lepton Flavor Violating tau- Decays into  $\ell$ -K0s and  $\ell$ -K0sK0s,” *Phys. Lett.* **B692** (2010) 4, [arXiv:1003.1183 \[hep-ex\]](#).
- [98] *BABAR* Collaboration, B. Aubert *et al.*, “Improved limits on lepton flavor violating tau decays to l phi, l rho, l K\* and l anti-K\*,” *Phys. Rev. Lett.* **103** (2009) 021801, [arXiv:0904.0339 \[hep-ex\]](#).
- [99] Belle Collaboration, Y. Miyazaki, “Search for Lepton-Flavor-Violating tau Decays into a Lepton and a Vector Meson,” *Phys. Lett.* **B699** (2011) 251, [arXiv:1101.0755 \[hep-ex\]](#).
- [100] *BABAR* Collaboration, B. Aubert *et al.*, “Search for lepton flavor violating decays tau+- → l+- omega (l = e, mu),” *Phys. Rev. Lett.* **100** (2008) 071802, [arXiv:0711.0980 \[hep-ex\]](#).
- [101] Belle Collaboration, Y. Miyazaki *et al.*, “Search for Lepton-Flavor-Violating tau Decays into Lepton and f0(980) Meson,” *Phys. Lett.* **B672** (2009) 317, [arXiv:0810.3519 \[hep-ex\]](#).
- [102] *BABAR* Collaboration, J. P. Lees *et al.*, “Limits on tau Lepton-Flavor Violating Decays in three charged leptons,” *Phys. Rev. D81* (2010) 111101, [arXiv:1002.4550 \[hep-ex\]](#).
- [103] Belle Collaboration, K. Hayasaka *et al.*, “Search for Lepton Flavor Violating Tau Decays into Three Leptons with 719 Million Produced Tau+Tau- Pairs,” *Phys. Lett.* **B687** (2010) 139, [arXiv:1001.3221 \[hep-ex\]](#).
- [104] ATLAS Collaboration, G. Aad *et al.*, “Probing lepton flavour violation via neutrinoless  $\tau \rightarrow 3\mu$  decays with the ATLAS detector,” *Eur. Phys. J. C76* (2016) 232, [arXiv:1601.03567 \[hep-ex\]](#).
- [105] LHCb Collaboration, R. Aaij *et al.*, “Search for the lepton flavour violating decay  $\tau^- \rightarrow \mu^- \mu^+ \mu^-$ ,” *JHEP* **02** (2015) 121, [arXiv:1409.8548 \[hep-ex\]](#).
- [106] *BABAR* Collaboration, B. Aubert *et al.*, “Search for lepton-flavor and lepton-number violation in the decay  $\tau^- \rightarrow \ell^\mp h^\pm h^-$ ,” *Phys. Rev. Lett.* **95** (2005) 191801, [arXiv:hep-ex/0506066 \[hep-ex\]](#).
- [107] Belle Collaboration, Y. Miyazaki *et al.*, “Search for Lepton-Flavor-Violating and Lepton-Number-Violating  $\tau \rightarrow \ell hh'$  Decay Modes,” *Phys. Lett.* **B719** (2013) 346, [arXiv:1206.5595 \[hep-ex\]](#).
- [108] Belle Collaboration, Y. Miyazaki *et al.*, “Search for lepton and baryon number violating tau- decays into anti-Lambda pi- and Lambda pi-,” *Phys. Lett.* **B632** (2006) 51, [arXiv:hep-ex/0508044 \[hep-ex\]](#).
- [109] LHCb Collaboration, R. Aaij *et al.*, “Searches for violation of lepton flavour and baryon number in tau lepton decays at LHCb,” *Phys. Lett.* **B724** (2013) 36, [arXiv:1304.4518 \[hep-ex\]](#).
- [110] A. L. Read, “Presentation of search results: The CL(s) technique,” *J. Phys. G28* (2002) 2693.
- [111] S. Banerjee, B. Pietrzyk, J. M. Roney, and Z. Was, “Tau and muon pair production cross-sections in electron-positron annihilations at  $\sqrt{s} = 10.58$  GeV,” *Phys. Rev. D77* (2008) 054012, [arXiv:0706.3235 \[hep-ph\]](#).
- [112] CDF Collaboration, T. Junk, “Sensitivity, Exclusion and Discovery with Small Signals, Large Backgrounds, and Large Systematic Uncertainties.” CDF note 8128, 2007.