HFLAV-Tau 2018 report

Sw. Banerjee, University of Louisville, USA
M. Chrząszcz, IFJ PAN, Kraków, Poland and CERN, Geneva, Switzerland
K. Hayasaka, Niigata University, Japan
H. Hayashii, Nara Women's University, Japan
A. Lusiani, Scuola Normale Superiore and INFN Pisa, Italy
M. Roney, University of Victoria, Canada
B. Shwartz, Budker Institute of Nuclear Physics, Russia

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Contents

1 Introduction

This section reports averages and elaborations of τ branching fractions, and combinations of upper limits on τ branching fractions to lepton-flavour-violating τ decay modes.

Branching fractions averages are obtained with a fit of τ branching fractions measurements aimed at optimally exploiting the available experimental information and described in Section 2. The fit results are used in Section 3 to test the lepton-flavour universality of the charged-current weak interaction. The "universality-improved" [1] branching fraction $\mathcal{B}_e = \mathcal{B}(\tau \to e\nu\overline{\nu})$ and the ratio between the hadronic branching fraction and $\mathcal{B}_e$, are obtained in Section 4. The value of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{us}|$ from τ decays is given in Section 5. Combinations of upper limits on lepton-flavour-violating τ branching fractions are computed in Section 6. All results are obtained from inputs available through the end of 2018.

2 Branching fraction fit

A fit of the available experimental measurements is used to determine the τ branching fractions, together with their uncertainties and correlations.

All relevant published statistical and systematic correlations among the measurements are used. In addition, for a selection of measurements, particularly the most precise and the most recent ones, the documented systematic uncertainty contributions are examined to consider systematic dependencies from external parameters. We use the standard HFLAV procedures to account for the updated values and uncertainties of the external parameters and for the correlations induced on different measurements with a systematic dependence from the same external parameter.

Both the measurements and the fitted quantities consist of either τ decay branching fractions, labelled as $\mathcal{B}_i$, or ratios of two τ decay branching fractions, labelled as $\mathcal{B}_i/\mathcal{B}_j$. Some branching fractions are sums of other branching fractions, for instance $\mathcal{B}_8 = \mathcal{B}(\tau \to h^-\nu_\tau)$ is the sum of $\mathcal{B}_9 = \mathcal{B}(\tau \to \pi^-\nu_\tau)$ and $\mathcal{B}_{10} = \mathcal{B}(\tau \to K^-\nu_\tau)$. The symbol h is used to mean either a π or a K. The fit χ^2 is minimized while respecting a list of constraints on the fitted quantities:

- quantities corresponding to ratios like $\mathcal{B}_i/\mathcal{B}_j$ must be equal to the ratio of the respective quantities $\mathcal{B}_i$ and $\mathcal{B}_j$;
- quantities corresponding to branching fractions that are sum of other branching fractions must be equal to the sum of the quantities corresponding to the summed branching fractions.

In some cases, constraints describe approximate relations that nevertheless hold within the present experimental precision. For instance, the constraint $\mathcal{B}(\tau \to K^-K^-K^+\nu_\tau) = \mathcal{B}(\tau \to K^-\phi\nu_\tau) \times \mathcal{B}(\phi \to K^+K^-)$ is justified within the current experimental evidence. Section 2.7 lists all equations relating one quantity to other quantities.

2.1 Technical implementation of the fit procedure

The fit computes the quantities q_i by minimizing a χ^2 while respecting a series of equality constraints on the q_i. The χ^2 is computed using the measurements x_i and their covariance matrix V_{ij} as

$$\chi^2 = (x_i - A_{ik}q_k)^t V_{ij}^{-1}(x_j - A_{jl}q_l) \ , \tag{1}$$

where the model matrix A_{ij} is used to get the vector of the predicted measurements x'_i from the vector of the fit parameters q_j as $x'_i = A_{ij}q_j$. In this particular implementation, the measurements are grouped according to the measured quantity, and all quantities with at least one measurement correspond to a fit parameter. Therefore, the matrix A_{ij} has one row per measurement x_i and one column per fitted quantity q_j, with unity coefficients for the rows and column that identify a measurement x_i of the quantity q_j. In summary, the χ^2 given in Eq. (1) is minimized subject to the constraints

$$f_r(q_s) - c_r = 0 \ , \tag{2}$$

where Eq. (2) corresponds to the constraint equations, written as a set of "constraint expressions" that are equated to zero. Using the method of Lagrange multipliers, a set of equations is obtained by taking the derivatives with respect to the fitted quantities q_k and the Lagrange multipliers λ_r of the sum of the χ^2 and the constraint expressions multiplied by the Lagrange multipliers λ_r, one for each constraint:

$$\min \left[h = (A_{ik}q_k - x_i)^t V_{ij}^{-1}(A_{jl}q_l - x_j) + 2\lambda_r(f_r(q_s) - c_r) \right] \tag{3}$$

$$(\partial/\partial q_k, \partial/\partial\lambda_r)\, h = 0 \ . \tag{4}$$

Equation (4) defines a set of equations for the vector of the unknowns (q_k, λ_r), some of which may be non-linear, in case of non-linear constraints. An iterative minimization procedure approximates at each step the non-linear constraint expressions by their first order Taylor expansion around the current values of the fitted quantities, $\overline{q}_s$:

$$f_r(q_s) - c_r \simeq f_r(\overline{q}_s) + \left.\frac{\partial f_r(q_s)}{\partial q_s}\right|_{\overline{\boldsymbol{q}}_{\boldsymbol{s}}} (q_s - \overline{q}_s) - c_r \ , \tag{5}$$

which can be written as

$$B_{rs} q_s - c'_r \ , \tag{6}$$

where c'_r are the resulting constant known terms, independent of q_s at first order. After linearization, the differentiation by q_k and λ_r is trivial and leads to a set of linear equations

$$A^t_{ki} V^{-1}_{ij} A_{jl} q_l + B^t_{kr} \lambda_r = A^t_{ki} V^{-1}_{ij} x_j \tag{7}$$

$$B_{rs} q_s = c'_r \ , \tag{8}$$

which can be expressed as:

$$F_{ij} u_j = v_i \ , \tag{9}$$

where $u_j = (q_k, \lambda_r)$ and v_i is the vector of the known constant terms running over the index k and then r in the right terms of Eq. (7) and Eq. (8). Solving the equation set in Eq. (9) gives the fitted quantities and their covariance matrix, using the measurements and their covariance matrix. The fit procedure starts by computing the linear approximation of the non-linear constraint expressions around the quantities seed values. With an iterative procedure, the unknowns are updated at each step by solving the equations and the equations are then linearized around the updated values, until the RMS average of relative variation of the fitted unknowns is reduced below 10^{-12}.

2.2 Fit results

Although the fit treats all quantities in the same way, for the purpose of describing the results we select a set of 47 “basis quantities” from which all remaining quantities can be calculated using the definitions listed in Section 2.7.

The fit output consists of 136 fitted quantities that correspond to either branching fractions or ratios of branching fractions. The fitted quantities values and uncertainties are listed in Table 3. The off-diagonal correlation terms between the basis quantities are listed in Section 2.6.

Furthermore we define (see Section 2.7) $\mathcal{B}_{110} = \mathcal{B}(\tau^- \to X_s^- \nu_\tau)$, the total branching fraction of the τ decays to final states with the strangeness quantum number equal to one, and $\mathcal{B}_{\text{All}}$, the branching fraction of the τ into any measured final state, which should be equal to 1 within the experimental uncertainty. We define the unitarity residual as $\mathcal{B}_{998} = 1 - \mathcal{B}_{\text{All}}$.

The fit has $\chi^2/\text{d.o.f.} = 142/129$, corresponding to a confidence level $\text{CL} = 20.13\%$. We use a total of 176 measurements to fit the above mentioned 136 quantities subjected to 89 constraints. Although the unitarity constraint is not applied, the fit is statistically consistent with unitarity, where the residual is $\mathcal{B}_{998} = 1 - \mathcal{B}_{\text{All}} = (0.0274 \pm 0.1026) \cdot 10^{-2}$.

A scale factor of 5.44 has been applied to the published uncertainties of the two severely inconsistent measurements of $\mathcal{B}_{96} = \tau \to KKK\nu$ by *BABAR* and Belle. The scale factor has been determined using the PDG procedure, *i.e.*, to the proper size in order to obtain a reduced χ^2 equal to 1 when fitting just the two $\mathcal{B}_{96}$ measurements.

2.3 Changes with respect to the previous report

The following changes have been introduced with respect to the previous HFLAV report [2].

We added the *BABAR* 2018 result [3] for the τ branching fraction

$$\mathcal{B}_{37} = K^- K^0 \nu_\tau \quad (14.78 \pm 0.22 \pm 0.40) \cdot 10^{-4} \ ,$$

and the 2018 *BABAR* preliminary results [4] for the τ branching fractions

$$\begin{array}{ll}
\mathcal{B}_{10} = K^-\nu_\tau & (7.17 \pm 0.031 \pm 0.21) \cdot 10^{-3} \\
\mathcal{B}_{16} = K^-\pi^0\nu_\tau & (5.05 \pm 0.02 \pm 0.15) \cdot 10^{-3} \\
\mathcal{B}_{23} = K^-2\pi^0\nu_\tau \text{ (ex. } K^0) & (6.15 \pm 0.12 \pm 0.34) \cdot 10^{-4} \\
\mathcal{B}_{27} = \pi^-3\pi^0\nu_\tau \text{ (ex. } K^0) & (1.168 \pm 0.006 \pm 0.038) \cdot 10^{-2} \\
\mathcal{B}_{28} = K^-3\pi^0\nu_\tau \text{ (ex. } K^0, \eta) & (1.25 \pm 0.16 \pm 0.24) \cdot 10^{-4} \\
\mathcal{B}_{809} = \pi^-4\pi^0\nu_\tau \text{ (ex. } K^0, \eta) & (9.02 \pm 0.40 \pm 0.65) \cdot 10^{-4} \ .
\end{array}$$

The above $\mathcal{B}_{16}$ result supersedes the previous *BABAR* result in Ref. [5].

The parameters used to update the measurements' systematic biases and the parameters appearing in the constraint equations in Section 2.7 have been updated to the PDG 2018 averages [6].

2.4 Differences between the HFLAV 2018 fit and the PDG 2018 fit

As is standard for the PDG branching fraction fits, the PDG 2018 τ branching fraction fit is unitarity constrained, while the HFLAV 2018 fit is unconstrained.

The HFLAV-Tau fit uses an elaboration of the measurements reported on the main ALEPH paper on τ branching fractions [7] to obtain branching fractions to inclusive final states with "hadrons" (where a hadron is either a pion or a kaon), since this set of results is closer to the actual experimental measurements and facilitates a more appropriate and comprehensive treatment of the experimental results correlations. The PDG 2018 fit on the other hand continues to use – as in the past editions – the published ALEPH measurements of branching fractions to esclusive final states with pions [7].

As in 2016, HFLAV uses the ALEPH estimate for $\mathcal{B}_{805} = \mathcal{B}(\tau \to a_1^-(\to \pi^-\gamma)\nu_\tau)$, which is not a direct measurement, and the PDG 2018 fit uses the PDG average of $\mathcal{B}(a_1 \to \pi\gamma)$ as a parameter and defines $\mathcal{B}_{805} = \mathcal{B}(a_1 \to \pi\gamma) \times \mathcal{B}(\tau \to 3\pi\nu)$. As a consequence, the PDG fit procedure does not take into account the large uncertainty on $\mathcal{B}(a_1 \to \pi\gamma)$, resulting in an underestimated fit uncertainty on $\mathcal{B}_{805}$. Therefore, in this case an appropriate correction has been applied after the fit.

Finally, the HFLAV 2018 τ branching fraction fit includes measurements that appeared after the deadline for inclusion in the PDG, and preliminary measurements that are not included in the PDG.

2.5 Branching ratio fit results and experimental inputs

Table 3 reports the τ branching ratio fit results and experimental inputs.

Table 3: HFLAV 2018 branching fractions fit results.

τ lepton branching fraction	**Experiment**	**Reference**
$\mathcal{B}_1 = (\text{particles})^- \geq 0\,\text{neutrals} \geq 0\, K^0\ \nu_\tau$		
0.8521 ± 0.0011	average	
$\mathcal{B}_2 = (\text{particles})^- \geq 0\,\text{neutrals} \geq 0\, K^0_L\ \nu_\tau$		
0.8455 ± 0.0010	average	
$\mathcal{B}_3 = \mu^-\overline{\nu}_\mu\nu_\tau$		
0.17392 ± 0.00039	average	
$0.17319 \pm 0.00070 \pm 0.00032$	ALEPH	[7]
$0.17325 \pm 0.00095 \pm 0.00077$	DELPHI	[8]
$0.17342 \pm 0.00110 \pm 0.00067$	L3	[9]
$0.17340 \pm 0.00090 \pm 0.00060$	OPAL	[10]
$\frac{\mathcal{B}_3}{\mathcal{B}_5} = \frac{\mu^-\overline{\nu}_\mu\nu_\tau}{e^-\overline{\nu}_e\nu_\tau}$		
0.9761 ± 0.0028	average	
$0.9970 \pm 0.0350 \pm 0.0400$	ARGUS	[11]
$0.9796 \pm 0.0016 \pm 0.0036$	*BABAR*	[12]

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τ lepton branching fraction	Experiment	Reference
$0.9777 \pm 0.0063 \pm 0.0087$	CLEO	[13]
$\mathcal{B}_5 = e^- \overline{\nu}_e \nu_\tau$		
0.17817 ± 0.00041	average	
$0.17837 \pm 0.00072 \pm 0.00036$	ALEPH	[7]
$0.17760 \pm 0.00060 \pm 0.00170$	CLEO	[13]
$0.17877 \pm 0.00109 \pm 0.00110$	DELPHI	[8]
$0.17806 \pm 0.00104 \pm 0.00076$	L3	[9]
$0.17810 \pm 0.00090 \pm 0.00060$	OPAL	[14]
$\mathcal{B}_7 = h^- \geq 0\, K_L^0\, \nu_\tau$		
0.12019 ± 0.00053	average	
$0.12400 \pm 0.00700 \pm 0.00700$	DELPHI	[15]
$0.12470 \pm 0.00260 \pm 0.00430$	L3	[16]
$0.12100 \pm 0.00700 \pm 0.00500$	OPAL	[17]
$\mathcal{B}_8 = h^- \nu_\tau$		
0.11502 ± 0.00053	average	
$0.11524 \pm 0.00070 \pm 0.00078$	ALEPH	[7]
$0.11520 \pm 0.00050 \pm 0.00120$	CLEO	[13]
$0.11571 \pm 0.00120 \pm 0.00114$	DELPHI	[18]
$0.11980 \pm 0.00130 \pm 0.00160$	OPAL	[19]
$\frac{\mathcal{B}_8}{\mathcal{B}_5} = \frac{h^- \nu_\tau}{e^- \overline{\nu}_e \nu_\tau}$		
0.6456 ± 0.0033	average	
$\mathcal{B}_9 = \pi^- \nu_\tau$		
0.10804 ± 0.00052	average	
$\frac{\mathcal{B}_9}{\mathcal{B}_5} = \frac{\pi^- \nu_\tau}{e^- \overline{\nu}_e \nu_\tau}$		
0.6064 ± 0.0032	average	
$0.5945 \pm 0.0014 \pm 0.0061$	*BABAR*	[12]
$\mathcal{B}_{10} = K^- \nu_\tau$		
$(0.6986 \pm 0.0085) \cdot 10^{-2}$	average	
$(0.6960 \pm 0.0250 \pm 0.0140) \cdot 10^{-2}$	ALEPH	[20]
$(0.7170 \pm 0.0031 \pm 0.0210) \cdot 10^{-2}$	*BABAR*	[4]
$(0.6600 \pm 0.0700 \pm 0.0900) \cdot 10^{-2}$	CLEO	[21]
$(0.8500 \pm 0.1800 \pm 0.0000) \cdot 10^{-2}$	DELPHI	[22]
$(0.6580 \pm 0.0270 \pm 0.0290) \cdot 10^{-2}$	OPAL	[23]
$\frac{\mathcal{B}_{10}}{\mathcal{B}_5} = \frac{K^- \nu_\tau}{e^- \overline{\nu}_e \nu_\tau}$		
$(3.921 \pm 0.048) \cdot 10^{-2}$	average	
$(3.882 \pm 0.032 \pm 0.057) \cdot 10^{-2}$	*BABAR*	[12]
$\frac{\mathcal{B}_{10}}{\mathcal{B}_9} = \frac{K^- \nu_\tau}{\pi^- \nu_\tau}$		
$(6.467 \pm 0.084) \cdot 10^{-2}$	average	
$\mathcal{B}_{11} = h^- \geq 1$ neutrals ν_τ		
0.36996 ± 0.00094	average	
$\mathcal{B}_{12} = h^- \geq 1\, \pi^0\, \nu_\tau$ (ex. K^0)		

Table 3 – continued from previous page

τ lepton branching fraction	Experiment	Reference
0.36495 ± 0.00094	average	
$\mathcal{B}_{13} = h^- \pi^0 \nu_\tau$		
0.25938 ± 0.00090	average	
$0.25924 \pm 0.00097 \pm 0.00085$	ALEPH	[7]
$0.25670 \pm 0.00010 \pm 0.00390$	Belle	[24]
$0.25870 \pm 0.00120 \pm 0.00420$	CLEO	[25]
$0.25740 \pm 0.00201 \pm 0.00138$	DELPHI	[18]
$0.25050 \pm 0.00350 \pm 0.00500$	L3	[16]
$0.25890 \pm 0.00170 \pm 0.00290$	OPAL	[19]
$\mathcal{B}_{14} = \pi^- \pi^0 \nu_\tau$		
0.25447 ± 0.00091	average	
$\mathcal{B}_{16} = K^- \pi^0 \nu_\tau$		
$(0.4904 \pm 0.0092) \cdot 10^{-2}$	average	
$(0.4440 \pm 0.0260 \pm 0.0240) \cdot 10^{-2}$	ALEPH	[20]
$(0.5050 \pm 0.0020 \pm 0.0150) \cdot 10^{-2}$	*BABAR*	[4]
$(0.5100 \pm 0.1000 \pm 0.0700) \cdot 10^{-2}$	CLEO	[21]
$(0.4710 \pm 0.0590 \pm 0.0230) \cdot 10^{-2}$	OPAL	[26]
$\mathcal{B}_{17} = h^- \geq 2\,\pi^0\,\nu_\tau$		
0.10793 ± 0.00091	average	
$0.09910 \pm 0.00310 \pm 0.00270$	OPAL	[19]
$\mathcal{B}_{18} = h^- 2\pi^0 \nu_\tau$		
$(9.421 \pm 0.092) \cdot 10^{-2}$	average	
$\mathcal{B}_{19} = h^- 2\pi^0 \nu_\tau$ (ex. K^0)		
$(9.270 \pm 0.092) \cdot 10^{-2}$	average	
$(9.295 \pm 0.084 \pm 0.088) \cdot 10^{-2}$	ALEPH	[7]
$(9.498 \pm 0.320 \pm 0.275) \cdot 10^{-2}$	DELPHI	[18]
$(8.880 \pm 0.370 \pm 0.420) \cdot 10^{-2}$	L3	[16]
$\frac{\mathcal{B}_{19}}{\mathcal{B}_{13}} = \frac{h^- 2\pi^0 \nu_\tau \text{ (ex. } K^0)}{h^- \pi^0 \nu_\tau}$		
0.3574 ± 0.0042	average	
$0.3420 \pm 0.0060 \pm 0.0160$	CLEO	[27]
$\mathcal{B}_{20} = \pi^- 2\pi^0 \nu_\tau$ (ex. K^0)		
$(9.211 \pm 0.092) \cdot 10^{-2}$	average	
$\mathcal{B}_{23} = K^- 2\pi^0 \nu_\tau$ (ex. K^0)		
$(0.0585 \pm 0.0027) \cdot 10^{-2}$	average	
$(0.0560 \pm 0.0200 \pm 0.0150) \cdot 10^{-2}$	ALEPH	[20]
$(0.0615 \pm 0.0012 \pm 0.0034) \cdot 10^{-2}$	*BABAR*	[4]
$(0.0900 \pm 0.1000 \pm 0.0300) \cdot 10^{-2}$	CLEO	[21]
$\mathcal{B}_{24} = h^- \geq 3\,\pi^0\,\nu_\tau$		
$(1.372 \pm 0.034) \cdot 10^{-2}$	average	
$\mathcal{B}_{25} = h^- \geq 3\,\pi^0\,\nu_\tau$ (ex. K^0)		
$(1.288 \pm 0.034) \cdot 10^{-2}$	average	

Table 3 – continued from previous page

τ lepton branching fraction	Experiment	Reference
$(1.403 \pm 0.214 \pm 0.224) \cdot 10^{-2}$	DELPHI	[18]
$\mathcal{B}_{26} = h^- 3\pi^0 \nu_\tau$		
$(1.236 \pm 0.030) \cdot 10^{-2}$	average	
$(1.082 \pm 0.071 \pm 0.059) \cdot 10^{-2}$	ALEPH	[7]
$(1.700 \pm 0.240 \pm 0.380) \cdot 10^{-2}$	L3	[16]
$\frac{\mathcal{B}_{26}}{\mathcal{B}_{13}} = \frac{h^- 3\pi^0 \nu_\tau}{h^- \pi^0 \nu_\tau}$		
$(4.764 \pm 0.118) \cdot 10^{-2}$	average	
$(4.400 \pm 0.300 \pm 0.500) \cdot 10^{-2}$	CLEO	[27]
$\mathcal{B}_{27} = \pi^- 3\pi^0 \nu_\tau$ (ex. K^0)		
$(1.1381 \pm 0.0292) \cdot 10^{-2}$	average	
$(1.1680 \pm 0.0060 \pm 0.0380) \cdot 10^{-2}$	*BABAR*	[4]
$\mathcal{B}_{28} = K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)		
$(1.127 \pm 0.263) \cdot 10^{-4}$	average	
$(3.700 \pm 2.100 \pm 1.100) \cdot 10^{-4}$	ALEPH	[20]
$(1.250 \pm 0.160 \pm 0.240) \cdot 10^{-4}$	*BABAR*	[4]
$\mathcal{B}_{29} = h^- 4\pi^0 \nu_\tau$ (ex. K^0)		
$(0.1333 \pm 0.0071) \cdot 10^{-2}$	average	
$(0.1600 \pm 0.0500 \pm 0.0500) \cdot 10^{-2}$	CLEO	[27]
$\mathcal{B}_{30} = h^- 4\pi^0 \nu_\tau$ (ex. K^0, η)		
$(0.0864 \pm 0.0067) \cdot 10^{-2}$	average	
$(0.1120 \pm 0.0370 \pm 0.0350) \cdot 10^{-2}$	ALEPH	[7]
$\mathcal{B}_{31} = K^- \geq 0\,\pi^0 \geq 0\,K^0 \geq 0\,\gamma \nu_\tau$		
$(1.568 \pm 0.018) \cdot 10^{-2}$	average	
$(1.700 \pm 0.120 \pm 0.190) \cdot 10^{-2}$	CLEO	[21]
$(1.540 \pm 0.240 \pm 0.000) \cdot 10^{-2}$	DELPHI	[22]
$(1.528 \pm 0.039 \pm 0.040) \cdot 10^{-2}$	OPAL	[23]
$\mathcal{B}_{32} = K^- \geq 1\,(\pi^0 \text{ or } K^0 \text{ or } \gamma)\nu_\tau$		
$(0.8729 \pm 0.0141) \cdot 10^{-2}$	average	
$\mathcal{B}_{33} = K_S^0(\text{particles})^- \nu_\tau$		
$(0.9366 \pm 0.0292) \cdot 10^{-2}$	average	
$(0.9700 \pm 0.0580 \pm 0.0620) \cdot 10^{-2}$	ALEPH	[28]
$(0.9700 \pm 0.0900 \pm 0.0600) \cdot 10^{-2}$	OPAL	[29]
$\mathcal{B}_{34} = h^- \overline{K}^0 \nu_\tau$		
$(0.9860 \pm 0.0138) \cdot 10^{-2}$	average	
$(0.8550 \pm 0.0360 \pm 0.0730) \cdot 10^{-2}$	CLEO	[30]
$\mathcal{B}_{35} = \pi^- \overline{K}^0 \nu_\tau$		
$(0.8378 \pm 0.0139) \cdot 10^{-2}$	average	
$(0.9280 \pm 0.0450 \pm 0.0340) \cdot 10^{-2}$	ALEPH	[20]
$(0.8320 \pm 0.0025 \pm 0.0150) \cdot 10^{-2}$	Belle	[31]
$(0.9500 \pm 0.1500 \pm 0.0600) \cdot 10^{-2}$	L3	[32]
$(0.9330 \pm 0.0680 \pm 0.0490) \cdot 10^{-2}$	OPAL	[33]
$\mathcal{B}_{37} = K^- K^0 \nu_\tau$		

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τ lepton branching fraction	Experiment	Reference
$(0.1483 \pm 0.0034) \cdot 10^{-2}$	average	
$(0.1580 \pm 0.0420 \pm 0.0170) \cdot 10^{-2}$	ALEPH	[28]
$(0.1620 \pm 0.0210 \pm 0.0110) \cdot 10^{-2}$	ALEPH	[20]
$(0.1478 \pm 0.0022 \pm 0.0040) \cdot 10^{-2}$	*BABAR*	[3]
$(0.1480 \pm 0.0013 \pm 0.0055) \cdot 10^{-2}$	Belle	[31]
$(0.1510 \pm 0.0210 \pm 0.0220) \cdot 10^{-2}$	CLEO	[30]
$\mathcal{B}_{38} = K^- K^0 \geq 0\,\pi^0\,\nu_\tau$		
$(0.2977 \pm 0.0073) \cdot 10^{-2}$	average	
$(0.3300 \pm 0.0550 \pm 0.0390) \cdot 10^{-2}$	OPAL	[33]
$\mathcal{B}_{39} = h^- \overline{K}^0 \pi^0 \nu_\tau$		
$(0.5302 \pm 0.0134) \cdot 10^{-2}$	average	
$(0.5620 \pm 0.0500 \pm 0.0480) \cdot 10^{-2}$	CLEO	[30]
$\mathcal{B}_{40} = \pi^- \overline{K}^0 \pi^0 \nu_\tau$		
$(0.3807 \pm 0.0129) \cdot 10^{-2}$	average	
$(0.2940 \pm 0.0730 \pm 0.0370) \cdot 10^{-2}$	ALEPH	[28]
$(0.3470 \pm 0.0530 \pm 0.0370) \cdot 10^{-2}$	ALEPH	[20]
$(0.3860 \pm 0.0031 \pm 0.0135) \cdot 10^{-2}$	Belle	[31]
$(0.4100 \pm 0.1200 \pm 0.0300) \cdot 10^{-2}$	L3	[32]
$\mathcal{B}_{42} = K^- \pi^0 K^0 \nu_\tau$		
$(0.1494 \pm 0.0070) \cdot 10^{-2}$	average	
$(0.1520 \pm 0.0760 \pm 0.0210) \cdot 10^{-2}$	ALEPH	[28]
$(0.1430 \pm 0.0250 \pm 0.0150) \cdot 10^{-2}$	ALEPH	[20]
$(0.1496 \pm 0.0019 \pm 0.0073) \cdot 10^{-2}$	Belle	[31]
$(0.1450 \pm 0.0360 \pm 0.0200) \cdot 10^{-2}$	CLEO	[30]
$\mathcal{B}_{43} = \pi^- \overline{K}^0 \geq 1\,\pi^0\,\nu_\tau$		
$(0.4042 \pm 0.0260) \cdot 10^{-2}$	average	
$(0.3240 \pm 0.0740 \pm 0.0660) \cdot 10^{-2}$	OPAL	[33]
$\mathcal{B}_{44} = \pi^- \overline{K}^0 2\pi^0 \nu_\tau$ (ex. K^0)		
$(2.346 \pm 2.306) \cdot 10^{-4}$	average	
$(2.600 \pm 2.400 \pm 0.000) \cdot 10^{-4}$	ALEPH	[34]
$\mathcal{B}_{46} = \pi^- K^0 \overline{K}^0 \nu_\tau$		
$(0.1516 \pm 0.0247) \cdot 10^{-2}$	average	
$\mathcal{B}_{47} = \pi^- K^0_S K^0_S \nu_\tau$		
$(2.342 \pm 0.065) \cdot 10^{-4}$	average	
$(2.600 \pm 1.000 \pm 0.500) \cdot 10^{-4}$	ALEPH	[28]
$(2.310 \pm 0.040 \pm 0.080) \cdot 10^{-4}$	*BABAR*	[35]
$(2.330 \pm 0.033 \pm 0.093) \cdot 10^{-4}$	Belle	[31]
$(2.300 \pm 0.500 \pm 0.300) \cdot 10^{-4}$	CLEO	[30]
$\mathcal{B}_{48} = \pi^- K^0_S K^0_L \nu_\tau$		
$(0.1048 \pm 0.0247) \cdot 10^{-2}$	average	
$(0.1010 \pm 0.0230 \pm 0.0130) \cdot 10^{-2}$	ALEPH	[28]
$\mathcal{B}_{49} = \pi^- K^0 \overline{K}^0 \pi^0 \nu_\tau$		

Table 3 – continued from previous page

τ lepton branching fraction	Experiment	Reference
$(3.543 \pm 1.193) \cdot 10^{-4}$	average	
$\mathcal{B}_{50} = \pi^- \pi^0 K^0_S K^0_S \nu_\tau$		
$(1.816 \pm 0.207) \cdot 10^{-5}$	average	
$(1.600 \pm 0.200 \pm 0.220) \cdot 10^{-5}$	*BABAR*	[35]
$(2.000 \pm 0.216 \pm 0.202) \cdot 10^{-5}$	Belle	[31]
$\mathcal{B}_{51} = \pi^- \pi^0 K^0_S K^0_L \nu_\tau$		
$(3.179 \pm 1.192) \cdot 10^{-4}$	average	
$(3.100 \pm 1.100 \pm 0.500) \cdot 10^{-4}$	ALEPH	[28]
$\mathcal{B}_{53} = \overline{K}^0 h^- h^- h^+ \nu_\tau$		
$(2.220 \pm 2.024) \cdot 10^{-4}$	average	
$(2.300 \pm 1.900 \pm 0.700) \cdot 10^{-4}$	ALEPH	[28]
$\mathcal{B}_{54} = h^- h^- h^+ \geq 0\,\text{neutrals} \geq 0\,K^0_L\,\nu_\tau$		
0.15206 ± 0.00061	average	
$0.15000 \pm 0.00400 \pm 0.00300$	CELLO	[36]
$0.14400 \pm 0.00600 \pm 0.00300$	L3	[37]
$0.15100 \pm 0.00800 \pm 0.00600$	TPC	[38]
$\mathcal{B}_{55} = h^- h^- h^+ \geq 0\,\text{neutrals}\,\nu_\tau$ (ex. K^0)		
0.14558 ± 0.00056	average	
$0.14556 \pm 0.00105 \pm 0.00076$	L3	[39]
$0.14960 \pm 0.00090 \pm 0.00220$	OPAL	[40]
$\mathcal{B}_{56} = h^- h^- h^+ \nu_\tau$		
$(9.769 \pm 0.053) \cdot 10^{-2}$	average	
$\mathcal{B}_{57} = h^- h^- h^+ \nu_\tau$ (ex. K^0)		
$(9.428 \pm 0.053) \cdot 10^{-2}$	average	
$(9.510 \pm 0.070 \pm 0.200) \cdot 10^{-2}$	CLEO	[41]
$(9.317 \pm 0.090 \pm 0.082) \cdot 10^{-2}$	DELPHI	[18]
$\frac{\mathcal{B}_{57}}{\mathcal{B}_{55}} = \frac{h^- h^- h^+ \nu_\tau \text{ (ex. } K^0)}{h^- h^- h^+ \geq 0\,\text{neutrals}\,\nu_\tau \text{ (ex. } K^0)}$		
0.6476 ± 0.0029	average	
$0.6600 \pm 0.0040 \pm 0.0140$	OPAL	[40]
$\mathcal{B}_{58} = h^- h^- h^+ \nu_\tau$ (ex. K^0, ω)		
$(9.397 \pm 0.053) \cdot 10^{-2}$	average	
$(9.469 \pm 0.062 \pm 0.073) \cdot 10^{-2}$	ALEPH	[7]
$\mathcal{B}_{59} = \pi^- \pi^+ \pi^- \nu_\tau$		
$(9.279 \pm 0.051) \cdot 10^{-2}$	average	
$\mathcal{B}_{60} = \pi^- \pi^+ \pi^- \nu_\tau$ (ex. K^0)		
$(8.990 \pm 0.051) \cdot 10^{-2}$	average	
$(8.830 \pm 0.010 \pm 0.130) \cdot 10^{-2}$	*BABAR*	[42]
$(8.420 \pm 0.000^{+0.260}_{-0.250}) \cdot 10^{-2}$	Belle	[43]
$(9.130 \pm 0.050 \pm 0.460) \cdot 10^{-2}$	CLEO3	[44]
$\mathcal{B}_{62} = \pi^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)		
$(8.960 \pm 0.051) \cdot 10^{-2}$	average	

Table 3 – continued from previous page

τ lepton branching fraction	Experiment	Reference
$\mathcal{B}_{63} = h^- h^- h^+ \geq 1 \,\text{neutrals}\, \nu_\tau$		
$(5.327 \pm 0.049) \cdot 10^{-2}$	average	
$\mathcal{B}_{64} = h^- h^- h^+ \geq 1 \,\pi^0\, \nu_\tau$ (ex. K^0)		
$(5.122 \pm 0.049) \cdot 10^{-2}$	average	
$\mathcal{B}_{65} = h^- h^- h^+ \pi^0 \nu_\tau$		
$(4.791 \pm 0.052) \cdot 10^{-2}$	average	
$\mathcal{B}_{66} = h^- h^- h^+ \pi^0 \nu_\tau$ (ex. K^0)		
$(4.607 \pm 0.051) \cdot 10^{-2}$	average	
$(4.734 \pm 0.059 \pm 0.049) \cdot 10^{-2}$	ALEPH	[7]
$(4.230 \pm 0.060 \pm 0.220) \cdot 10^{-2}$	CLEO	[41]
$(4.545 \pm 0.106 \pm 0.103) \cdot 10^{-2}$	DELPHI	[18]
$\mathcal{B}_{67} = h^- h^- h^+ \pi^0 \nu_\tau$ (ex. K^0, ω)		
$(2.821 \pm 0.070) \cdot 10^{-2}$	average	
$\mathcal{B}_{68} = \pi^- \pi^+ \pi^- \pi^0 \nu_\tau$		
$(4.652 \pm 0.053) \cdot 10^{-2}$	average	
$\mathcal{B}_{69} = \pi^- \pi^+ \pi^- \pi^0 \nu_\tau$ (ex. K^0)		
$(4.520 \pm 0.052) \cdot 10^{-2}$	average	
$(4.190 \pm 0.100 \pm 0.210) \cdot 10^{-2}$	CLEO	[45]
$\mathcal{B}_{70} = \pi^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω)		
$(2.770 \pm 0.071) \cdot 10^{-2}$	average	
$\mathcal{B}_{74} = h^- h^- h^+ \geq 2 \,\pi^0\, \nu_\tau$ (ex. K^0)		
$(0.5148 \pm 0.0311) \cdot 10^{-2}$	average	
$(0.5610 \pm 0.0680 \pm 0.0950) \cdot 10^{-2}$	DELPHI	[18]
$\mathcal{B}_{75} = h^- h^- h^+ 2\pi^0 \nu_\tau$		
$(0.5037 \pm 0.0309) \cdot 10^{-2}$	average	
$\mathcal{B}_{76} = h^- h^- h^+ 2\pi^0 \nu_\tau$ (ex. K^0)		
$(0.4937 \pm 0.0309) \cdot 10^{-2}$	average	
$(0.4350 \pm 0.0300 \pm 0.0350) \cdot 10^{-2}$	ALEPH	[7]
$\dfrac{\mathcal{B}_{76}}{\mathcal{B}_{54}} = \dfrac{h^- h^- h^+ 2\pi^0 \nu_\tau \text{ (ex. } K^0)}{h^- h^- h^+ \geq 0 \,\text{neutrals} \geq 0\, K_L^0\, \nu_\tau}$		
$(3.247 \pm 0.202) \cdot 10^{-2}$	average	
$(3.400 \pm 0.200 \pm 0.300) \cdot 10^{-2}$	CLEO	[46]
$\mathcal{B}_{77} = h^- h^- h^+ 2\pi^0 \nu_\tau$ (ex. K^0, ω, η)		
$(9.812 \pm 3.555) \cdot 10^{-4}$	average	
$\mathcal{B}_{78} = h^- h^- h^+ 3\pi^0 \nu_\tau$		
$(2.114 \pm 0.299) \cdot 10^{-4}$	average	
$(2.200 \pm 0.300 \pm 0.400) \cdot 10^{-4}$	CLEO	[47]
$\mathcal{B}_{79} = K^- h^- h^+ \geq 0 \,\text{neutrals}\, \nu_\tau$		
$(0.6293 \pm 0.0140) \cdot 10^{-2}$	average	
$\mathcal{B}_{80} = K^- \pi^- h^+ \nu_\tau$ (ex. K^0)		
$(0.4361 \pm 0.0072) \cdot 10^{-2}$	average	

Table 3 – continued from previous page

τ lepton branching fraction	Experiment	Reference
$\frac{\mathcal{B}_{80}}{\mathcal{B}_{60}} = \frac{K^- \pi^- h^+ \nu_\tau \text{ (ex. } K^0)}{\pi^- \pi^+ \pi^- \nu_\tau \text{ (ex. } K^0)}$		
$(4.851 \pm 0.080) \cdot 10^{-2}$	average	
$(5.440 \pm 0.210 \pm 0.530) \cdot 10^{-2}$	CLEO	[48]
$\mathcal{B}_{81} = K^- \pi^- h^+ \pi^0 \nu_\tau \text{ (ex. } K^0)$		
$(8.727 \pm 1.177) \cdot 10^{-4}$	average	
$\frac{\mathcal{B}_{81}}{\mathcal{B}_{69}} = \frac{K^- \pi^- h^+ \pi^0 \nu_\tau \text{ (ex. } K^0)}{\pi^- \pi^+ \pi^- \pi^0 \nu_\tau \text{ (ex. } K^0)}$		
$(1.931 \pm 0.266) \cdot 10^{-2}$	average	
$(2.610 \pm 0.450 \pm 0.420) \cdot 10^{-2}$	CLEO	[48]
$\mathcal{B}_{82} = K^- \pi^- \pi^+ \geq 0 \text{ neutrals } \nu_\tau$		
$(0.4779 \pm 0.0137) \cdot 10^{-2}$	average	
$(0.5800^{+0.1500}_{-0.1300} \pm 0.1200) \cdot 10^{-2}$	TPC	[49]
$\mathcal{B}_{83} = K^- \pi^- \pi^+ \geq 0\, \pi^0\, \nu_\tau \text{ (ex. } K^0)$		
$(0.3741 \pm 0.0135) \cdot 10^{-2}$	average	
$\mathcal{B}_{84} = K^- \pi^- \pi^+ \nu_\tau$		
$(0.3442 \pm 0.0068) \cdot 10^{-2}$	average	
$\mathcal{B}_{85} = K^- \pi^+ \pi^- \nu_\tau \text{ (ex. } K^0)$		
$(0.2929 \pm 0.0067) \cdot 10^{-2}$	average	
$(0.2140 \pm 0.0370 \pm 0.0290) \cdot 10^{-2}$	ALEPH	[50]
$(0.2730 \pm 0.0020 \pm 0.0090) \cdot 10^{-2}$	*BABAR*	[42]
$(0.3300 \pm 0.0010^{+0.0160}_{-0.0170}) \cdot 10^{-2}$	Belle	[43]
$(0.3840 \pm 0.0140 \pm 0.0380) \cdot 10^{-2}$	CLEO3	[44]
$(0.4150 \pm 0.0530 \pm 0.0400) \cdot 10^{-2}$	OPAL	[26]
$\frac{\mathcal{B}_{85}}{\mathcal{B}_{60}} = \frac{K^- \pi^+ \pi^- \nu_\tau \text{ (ex. } K^0)}{\pi^- \pi^+ \pi^- \nu_\tau \text{ (ex. } K^0)}$		
$(3.258 \pm 0.074) \cdot 10^{-2}$	average	
$\mathcal{B}_{87} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$		
$(0.1329 \pm 0.0119) \cdot 10^{-2}$	average	
$\mathcal{B}_{88} = K^- \pi^- \pi^+ \pi^0 \nu_\tau \text{ (ex. } K^0)$		
$(8.116 \pm 1.168) \cdot 10^{-4}$	average	
$(6.100 \pm 3.900 \pm 1.800) \cdot 10^{-4}$	ALEPH	[50]
$(7.400 \pm 0.800 \pm 1.100) \cdot 10^{-4}$	CLEO3	[51]
$\mathcal{B}_{89} = K^- \pi^- \pi^+ \pi^0 \nu_\tau \text{ (ex. } K^0, \eta)$		
$(7.762 \pm 1.168) \cdot 10^{-4}$	average	
$\mathcal{B}_{92} = \pi^- K^- K^+ \geq 0 \text{ neutrals } \nu_\tau$		
$(0.1493 \pm 0.0033) \cdot 10^{-2}$	average	
$(0.1590 \pm 0.0530 \pm 0.0200) \cdot 10^{-2}$	OPAL	[52]
$(0.1500^{+0.0900}_{-0.0700} \pm 0.0300) \cdot 10^{-2}$	TPC	[49]
$\mathcal{B}_{93} = \pi^- K^- K^+ \nu_\tau$		
$(0.1431 \pm 0.0027) \cdot 10^{-2}$	average	
$(0.1630 \pm 0.0210 \pm 0.0170) \cdot 10^{-2}$	ALEPH	[50]
$(0.1346 \pm 0.0010 \pm 0.0036) \cdot 10^{-2}$	*BABAR*	[42]

Table 3 – continued from previous page

τ lepton branching fraction	Experiment	Reference
$(0.1550 \pm 0.0010^{+0.0060}_{-0.0050}) \cdot 10^{-2}$	Belle	[43]
$(0.1550 \pm 0.0060 \pm 0.0090) \cdot 10^{-2}$	CLEO3	[44]
$\frac{\mathcal{B}_{93}}{\mathcal{B}_{60}} = \frac{\pi^- K^- K^+ \nu_\tau}{\pi^- \pi^+ \pi^- \nu_\tau \text{ (ex. } K^0)}$		
$(1.592 \pm 0.030) \cdot 10^{-2}$	average	
$(1.600 \pm 0.150 \pm 0.300) \cdot 10^{-2}$	CLEO	[48]
$\mathcal{B}_{94} = \pi^- K^- K^+ \pi^0 \nu_\tau$		
$(0.611 \pm 0.183) \cdot 10^{-4}$	average	
$(7.500 \pm 2.900 \pm 1.500) \cdot 10^{-4}$	ALEPH	[50]
$(0.550 \pm 0.140 \pm 0.120) \cdot 10^{-4}$	CLEO3	[51]
$\frac{\mathcal{B}_{94}}{\mathcal{B}_{69}} = \frac{\pi^- K^- K^+ \pi^0 \nu_\tau}{\pi^- \pi^+ \pi^- \pi^0 \nu_\tau \text{ (ex. } K^0)}$		
$(0.1353 \pm 0.0405) \cdot 10^{-2}$	average	
$(0.7900 \pm 0.4400 \pm 0.1600) \cdot 10^{-2}$	CLEO	[48]
$\mathcal{B}_{96} = K^- K^- K^+ \nu_\tau$		
$(2.169 \pm 0.800) \cdot 10^{-5}$	average	
$(1.578 \pm 0.130 \pm 0.123) \cdot 10^{-5}$	*BABAR*	[42]
$(3.290 \pm 0.170^{+0.190}_{-0.200}) \cdot 10^{-5}$	Belle	[43]
$\mathcal{B}_{102} = 3h^- 2h^+ \geq 0\,\text{neutrals}\,\nu_\tau \text{ (ex. } K^0)$		
$(0.0990 \pm 0.0037) \cdot 10^{-2}$	average	
$(0.0970 \pm 0.0050 \pm 0.0110) \cdot 10^{-2}$	CLEO	[53]
$(0.1020 \pm 0.0290 \pm 0.0000) \cdot 10^{-2}$	HRS	[54]
$(0.1700 \pm 0.0220 \pm 0.0260) \cdot 10^{-2}$	L3	[39]
$\mathcal{B}_{103} = 3h^- 2h^+ \nu_\tau \text{ (ex. } K^0)$		
$(8.260 \pm 0.314) \cdot 10^{-4}$	average	
$(7.200 \pm 0.900 \pm 1.200) \cdot 10^{-4}$	ALEPH	[7]
$(6.400 \pm 2.300 \pm 1.000) \cdot 10^{-4}$	ARGUS	[55]
$(7.700 \pm 0.500 \pm 0.900) \cdot 10^{-4}$	CLEO	[53]
$(9.700 \pm 1.500 \pm 0.500) \cdot 10^{-4}$	DELPHI	[18]
$(5.100 \pm 2.000 \pm 0.000) \cdot 10^{-4}$	HRS	[54]
$(9.100 \pm 1.400 \pm 0.600) \cdot 10^{-4}$	OPAL	[56]
$\mathcal{B}_{104} = 3h^- 2h^+ \pi^0 \nu_\tau \text{ (ex. } K^0)$		
$(1.641 \pm 0.114) \cdot 10^{-4}$	average	
$(2.100 \pm 0.700 \pm 0.900) \cdot 10^{-4}$	ALEPH	[7]
$(1.700 \pm 0.200 \pm 0.200) \cdot 10^{-4}$	CLEO	[47]
$(1.600 \pm 1.200 \pm 0.600) \cdot 10^{-4}$	DELPHI	[18]
$(2.700 \pm 1.800 \pm 0.900) \cdot 10^{-4}$	OPAL	[56]
$\mathcal{B}_{106} = (5\pi)^- \nu_\tau$		
$(0.7532 \pm 0.0356) \cdot 10^{-2}$	average	
$\mathcal{B}_{110} = X_s^- \nu_\tau$		
$(2.931 \pm 0.041) \cdot 10^{-2}$	average	
$\mathcal{B}_{126} = \pi^- \pi^0 \eta \nu_\tau$		
$(0.1386 \pm 0.0072) \cdot 10^{-2}$	average	
$(0.1800 \pm 0.0400 \pm 0.0200) \cdot 10^{-2}$	ALEPH	[57]

Table 3 – continued from previous page

τ lepton branching fraction	Experiment	Reference
$(0.1350 \pm 0.0030 \pm 0.0070) \cdot 10^{-2}$	Belle	[58]
$(0.1700 \pm 0.0200 \pm 0.0200) \cdot 10^{-2}$	CLEO	[59]
$\mathcal{B}_{128} = K^- \eta \nu_\tau$		
$(1.543 \pm 0.080) \cdot 10^{-4}$	average	
$(2.900^{+1.300}_{-1.200} \pm 0.700) \cdot 10^{-4}$	ALEPH	[57]
$(1.420 \pm 0.110 \pm 0.070) \cdot 10^{-4}$	*BABAR*	[60]
$(1.580 \pm 0.050 \pm 0.090) \cdot 10^{-4}$	Belle	[58]
$(2.600 \pm 0.500 \pm 0.500) \cdot 10^{-4}$	CLEO	[61]
$\mathcal{B}_{130} = K^- \pi^0 \eta \nu_\tau$		
$(0.483 \pm 0.116) \cdot 10^{-4}$	average	
$(0.460 \pm 0.110 \pm 0.040) \cdot 10^{-4}$	Belle	[58]
$(1.770 \pm 0.560 \pm 0.710) \cdot 10^{-4}$	CLEO	[62]
$\mathcal{B}_{132} = \pi^- \overline{K}^0 \eta \nu_\tau$		
$(0.936 \pm 0.149) \cdot 10^{-4}$	average	
$(0.880 \pm 0.140 \pm 0.060) \cdot 10^{-4}$	Belle	[58]
$(2.200 \pm 0.700 \pm 0.220) \cdot 10^{-4}$	CLEO	[62]
$\mathcal{B}_{136} = \pi^- \pi^+ \pi^- \eta \nu_\tau$ (ex. K^0)		
$(2.196 \pm 0.129) \cdot 10^{-4}$	average	
$\mathcal{B}_{149} = h^- \omega \geq 0\,\text{neutrals}\,\nu_\tau$		
$(2.402 \pm 0.075) \cdot 10^{-2}$	average	
$\mathcal{B}_{150} = h^- \omega \nu_\tau$		
$(1.996 \pm 0.064) \cdot 10^{-2}$	average	
$(1.910 \pm 0.070 \pm 0.060) \cdot 10^{-2}$	ALEPH	[57]
$(1.600 \pm 0.270 \pm 0.410) \cdot 10^{-2}$	CLEO	[63]
$\frac{\mathcal{B}_{150}}{\mathcal{B}_{66}} = \frac{h^- \omega \nu_\tau}{h^- h^- h^+ \pi^0 \nu_\tau \text{ (ex. } K^0\text{)}}$		
0.4331 ± 0.0139	average	
$0.4310 \pm 0.0330 \pm 0.0000$	ALEPH	[64]
$0.4640 \pm 0.0160 \pm 0.0170$	CLEO	[41]
$\mathcal{B}_{151} = K^- \omega \nu_\tau$		
$(4.100 \pm 0.922) \cdot 10^{-4}$	average	
$(4.100 \pm 0.600 \pm 0.700) \cdot 10^{-4}$	CLEO3	[51]
$\mathcal{B}_{152} = h^- \pi^0 \omega \nu_\tau$		
$(0.4066 \pm 0.0419) \cdot 10^{-2}$	average	
$(0.4300 \pm 0.0600 \pm 0.0500) \cdot 10^{-2}$	ALEPH	[57]
$\frac{\mathcal{B}_{152}}{\mathcal{B}_{54}} = \frac{h^- \omega \pi^0 \nu_\tau}{h^- h^- h^+ \geq 0\,\text{neutrals} \geq 0\, K_L^0\, \nu_\tau}$		
$(2.674 \pm 0.275) \cdot 10^{-2}$	average	
$\frac{\mathcal{B}_{152}}{\mathcal{B}_{76}} = \frac{h^- \omega \pi^0 \nu_\tau}{h^- h^- h^+ 2\pi^0 \nu_\tau \text{ (ex. } K^0\text{)}}$		
0.8236 ± 0.0757	average	
$0.8100 \pm 0.0600 \pm 0.0600$	CLEO	[46]
$\mathcal{B}_{167} = K^- \phi \nu_\tau$		

Table 3 – continued from previous page

τ lepton branching fraction	Experiment	Reference
$(4.409 \pm 1.626) \cdot 10^{-5}$	average	
$\mathcal{B}_{168} = K^- \phi \nu_\tau$ $(\phi \to K^+ K^-)$		
$(2.169 \pm 0.800) \cdot 10^{-5}$	average	
$\mathcal{B}_{169} = K^- \phi \nu_\tau$ $(\phi \to K^0_S K^0_L)$		
$(1.499 \pm 0.553) \cdot 10^{-5}$	average	
$\mathcal{B}_{800} = \pi^- \omega \nu_\tau$		
$(1.955 \pm 0.065) \cdot 10^{-2}$	average	
$\mathcal{B}_{802} = K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)		
$(0.2923 \pm 0.0067) \cdot 10^{-2}$	average	
$\mathcal{B}_{803} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)		
$(4.105 \pm 1.429) \cdot 10^{-4}$	average	
$\mathcal{B}_{804} = \pi^- K^0_L K^0_L \nu_\tau$		
$(2.342 \pm 0.065) \cdot 10^{-4}$	average	
$\mathcal{B}_{805} = a_1^- (\to \pi^- \gamma) \nu_\tau$		
$(4.000 \pm 2.000) \cdot 10^{-4}$	average	
$(4.000 \pm 2.000 \pm 0.000) \cdot 10^{-4}$	ALEPH	[7]
$\mathcal{B}_{806} = \pi^- \pi^0 K^0_L K^0_L \nu_\tau$		
$(1.816 \pm 0.207) \cdot 10^{-5}$	average	
$\mathcal{B}_{809} = \pi^- 4\pi^0 \nu_\tau$ (ex. K^0, η)		
$(8.640 \pm 0.670) \cdot 10^{-4}$	average	
$(9.020 \pm 0.400 \pm 0.650) \cdot 10^{-4}$	*BABAR*	[4]
$\mathcal{B}_{810} = 2\pi^- \pi^+ 3\pi^0 \nu_\tau$ (ex. K^0)		
$(1.931 \pm 0.298) \cdot 10^{-4}$	average	
$\mathcal{B}_{811} = \pi^- 2\pi^0 \omega \nu_\tau$ (ex. K^0)		
$(7.139 \pm 1.586) \cdot 10^{-5}$	average	
$(7.300 \pm 1.200 \pm 1.200) \cdot 10^{-5}$	*BABAR*	[65]
$\mathcal{B}_{812} = 2\pi^- \pi^+ 3\pi^0 \nu_\tau$ (ex. K^0, η, ω, f_1)		
$(1.325 \pm 2.682) \cdot 10^{-5}$	average	
$(1.000 \pm 0.800 \pm 3.000) \cdot 10^{-5}$	*BABAR*	[65]
$\mathcal{B}_{820} = 3\pi^- 2\pi^+ \nu_\tau$ (ex. K^0, ω)		
$(8.242 \pm 0.313) \cdot 10^{-4}$	average	
$\mathcal{B}_{821} = 3\pi^- 2\pi^+ \nu_\tau$ (ex. K^0, ω, f_1)		
$(7.719 \pm 0.295) \cdot 10^{-4}$	average	
$(7.680 \pm 0.040 \pm 0.400) \cdot 10^{-4}$	*BABAR*	[65]
$\mathcal{B}_{822} = K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)		
$(0.594 \pm 1.208) \cdot 10^{-6}$	average	
$(0.600 \pm 0.500 \pm 1.100) \cdot 10^{-6}$	*BABAR*	[65]
$\mathcal{B}_{830} = 3\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)		
$(1.630 \pm 0.113) \cdot 10^{-4}$	average	
$\mathcal{B}_{831} = 2\pi^- \pi^+ \omega \nu_\tau$ (ex. K^0)		

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τ lepton branching fraction	Experiment	Reference
$(8.400 \pm 0.624) \cdot 10^{-5}$	average	
$(8.400 \pm 0.400 \pm 0.600) \cdot 10^{-5}$	*BABAR*	[65]
$\mathcal{B}_{832} = 3\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0, η, ω, f_1)		
$(3.775 \pm 0.874) \cdot 10^{-5}$	average	
$(3.600 \pm 0.300 \pm 0.900) \cdot 10^{-5}$	*BABAR*	[65]
$\mathcal{B}_{833} = K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)		
$(1.108 \pm 0.566) \cdot 10^{-6}$	average	
$(1.100 \pm 0.400 \pm 0.400) \cdot 10^{-6}$	*BABAR*	[65]
$\mathcal{B}_{910} = 2\pi^- \pi^+ \eta \nu_\tau$ ($\eta \to 3\pi^0$) (ex. K^0)		
$(7.176 \pm 0.422) \cdot 10^{-5}$	average	
$(8.270 \pm 0.880 \pm 0.810) \cdot 10^{-5}$	*BABAR*	[65]
$\mathcal{B}_{911} = \pi^- 2\pi^0 \eta \nu_\tau$ ($\eta \to \pi^+ \pi^- \pi^0$) (ex. K^0)		
$(4.444 \pm 0.867) \cdot 10^{-5}$	average	
$(4.570 \pm 0.770 \pm 0.500) \cdot 10^{-5}$	*BABAR*	[65]
$\mathcal{B}_{920} = \pi^- f_1 \nu_\tau$ ($f_1 \to 2\pi^- 2\pi^+$)		
$(5.225 \pm 0.444) \cdot 10^{-5}$	average	
$(5.200 \pm 0.310 \pm 0.370) \cdot 10^{-5}$	*BABAR*	[65]
$\mathcal{B}_{930} = 2\pi^- \pi^+ \eta \nu_\tau$ ($\eta \to \pi^+ \pi^- \pi^0$) (ex. K^0)		
$(5.033 \pm 0.296) \cdot 10^{-5}$	average	
$(5.390 \pm 0.270 \pm 0.410) \cdot 10^{-5}$	*BABAR*	[65]
$\mathcal{B}_{944} = 2\pi^- \pi^+ \eta \nu_\tau$ ($\eta \to \gamma\gamma$) (ex. K^0)		
$(8.654 \pm 0.509) \cdot 10^{-5}$	average	
$(8.260 \pm 0.350 \pm 0.510) \cdot 10^{-5}$	*BABAR*	[65]
$\mathcal{B}_{945} = \pi^- 2\pi^0 \eta \nu_\tau$		
$(1.939 \pm 0.378) \cdot 10^{-4}$	average	
$\mathcal{B}_{998} = 1 - \mathcal{B}_{\text{All}}$		
$(0.0274 \pm 0.1026) \cdot 10^{-2}$	average	

2.6 Correlation terms between basis branching fractions uncertainties

The following tables report the correlation coefficients between basis quantities that were obtained from the τ branching fractions fit, in percent.

Table 4: Basis quantities correlation coefficients in percent, subtable 1.

$\mathcal{B}_5$	22													
$\mathcal{B}_9$	6	4												
$\mathcal{B}_{10}$	2	4	2											
$\mathcal{B}_{14}$	-13	-14	-13	-7										
$\mathcal{B}_{16}$	-2	-1	-3	35	-13									
$\mathcal{B}_{20}$	-7	-7	-12	-4	-42	-16								
$\mathcal{B}_{23}$	-3	-2	-5	14	-9	66	-18							
$\mathcal{B}_{27}$	-4	-4	-7	3	-9	61	-23	72						
$\mathcal{B}_{28}$	-2	-1	-3	2	-4	32	-10	28	37					
$\mathcal{B}_{30}$	-3	-3	-6	-1	-6	34	-14	41	52	23				
$\mathcal{B}_{35}$	0	0	0	0	0	0	0	0	0	0	0			
$\mathcal{B}_{37}$	0	-1	1	0	0	0	0	0	0	-1	0	-15		
$\mathcal{B}_{40}$	0	0	0	0	0	0	0	0	-1	0	0	-12	2	
	$\mathcal{B}_3$	$\mathcal{B}_5$	$\mathcal{B}_9$	$\mathcal{B}_{10}$	$\mathcal{B}_{14}$	$\mathcal{B}_{16}$	$\mathcal{B}_{20}$	$\mathcal{B}_{23}$	$\mathcal{B}_{27}$	$\mathcal{B}_{28}$	$\mathcal{B}_{30}$	$\mathcal{B}_{35}$	$\mathcal{B}_{37}$	$\mathcal{B}_{40}$

Table 5: Basis quantities correlation coefficients in percent, subtable 2.

$\mathcal{B}_{42}$	0	0	0	-2	1	-5	1	-4	-4	-2	-2	-1	-15	-20
$\mathcal{B}_{44}$	0	0	0	0	0	0	0	0	0	0	0	-1	0	-4
$\mathcal{B}_{47}$	0	-1	2	1	-1	2	-1	1	1	0	0	-1	2	-4
$\mathcal{B}_{48}$	0	0	0	0	0	0	0	0	0	0	0	-3	0	-2
$\mathcal{B}_{50}$	0	0	0	0	0	0	0	0	0	0	0	1	5	0
$\mathcal{B}_{51}$	0	0	0	0	0	0	0	0	0	0	0	-1	0	-1
$\mathcal{B}_{53}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{B}_{62}$	-4	-5	6	2	-4	1	-11	-1	-2	-2	-3	-1	3	0
$\mathcal{B}_{70}$	-5	-6	-7	-2	-8	-1	-1	-1	-1	0	0	0	-1	0
$\mathcal{B}_{77}$	0	0	-2	0	-2	1	0	1	2	1	1	0	0	0
$\mathcal{B}_{93}$	-1	-1	2	1	-1	1	-2	0	0	0	0	0	1	0
$\mathcal{B}_{94}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{B}_{126}$	0	0	0	0	0	0	-1	0	0	0	0	0	0	0
$\mathcal{B}_{128}$	0	0	1	0	0	0	0	0	0	0	0	0	1	0
	$\mathcal{B}_3$	$\mathcal{B}_5$	$\mathcal{B}_9$	$\mathcal{B}_{10}$	$\mathcal{B}_{14}$	$\mathcal{B}_{16}$	$\mathcal{B}_{20}$	$\mathcal{B}_{23}$	$\mathcal{B}_{27}$	$\mathcal{B}_{28}$	$\mathcal{B}_{30}$	$\mathcal{B}_{35}$	$\mathcal{B}_{37}$	$\mathcal{B}_{40}$

Table 6: Basis quantities correlation coefficients in percent, subtable 3.

$\mathcal{B}_{130}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{B}_{132}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{B}_{136}$	0	0	1	1	0	1	-1	0	0	0	0	0	1	0
$\mathcal{B}_{151}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{B}_{152}$	0	0	-3	0	-2	1	0	1	2	1	2	0	0	0
$\mathcal{B}_{167}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{B}_{800}$	-1	-1	-2	0	-3	0	0	0	0	0	0	0	0	0
$\mathcal{B}_{802}$	-1	-1	0	0	-1	-1	-3	-1	-2	-1	-1	0	0	0
$\mathcal{B}_{803}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{B}_{805}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{B}_{811}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{B}_{812}$	1	1	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{B}_{821}$	0	0	2	1	0	1	-2	0	0	0	0	0	1	0
$\mathcal{B}_{822}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\mathcal{B}_3$	$\mathcal{B}_5$	$\mathcal{B}_9$	$\mathcal{B}_{10}$	$\mathcal{B}_{14}$	$\mathcal{B}_{16}$	$\mathcal{B}_{20}$	$\mathcal{B}_{23}$	$\mathcal{B}_{27}$	$\mathcal{B}_{28}$	$\mathcal{B}_{30}$	$\mathcal{B}_{35}$	$\mathcal{B}_{37}$	$\mathcal{B}_{40}$

Table 7: Basis quantities correlation coefficients in percent, subtable 4.

$\mathcal{B}_{831}$	0	0	1	0	0	0	-1	0	0	0	0	0	1	0
$\mathcal{B}_{832}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{B}_{833}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{B}_{920}$	0	0	1	0	0	0	-1	0	0	0	0	0	0	0
$\mathcal{B}_{945}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\mathcal{B}_{3}$	$\mathcal{B}_{5}$	$\mathcal{B}_{9}$	$\mathcal{B}_{10}$	$\mathcal{B}_{14}$	$\mathcal{B}_{16}$	$\mathcal{B}_{20}$	$\mathcal{B}_{23}$	$\mathcal{B}_{27}$	$\mathcal{B}_{28}$	$\mathcal{B}_{30}$	$\mathcal{B}_{35}$	$\mathcal{B}_{37}$	$\mathcal{B}_{40}$

Table 8: Basis quantities correlation coefficients in percent, subtable 5.

$\mathcal{B}_{44}$	0													
$\mathcal{B}_{47}$	1	0												
$\mathcal{B}_{48}$	-1	-6	0											
$\mathcal{B}_{50}$	6	0	-7	0										
$\mathcal{B}_{51}$	0	-3	0	-6	0									
$\mathcal{B}_{53}$	0	0	0	0	0	0								
$\mathcal{B}_{62}$	-1	0	5	0	1	0	0							
$\mathcal{B}_{70}$	0	0	-1	0	0	0	0	-19						
$\mathcal{B}_{77}$	0	0	0	0	0	0	0	-1	-7					
$\mathcal{B}_{93}$	0	0	2	0	0	0	0	14	-4	0				
$\mathcal{B}_{94}$	0	0	0	0	0	0	0	0	-2	0	0			
$\mathcal{B}_{126}$	0	0	0	0	0	0	0	0	0	-5	0	0		
$\mathcal{B}_{128}$	0	0	1	0	0	0	0	2	0	0	1	0	4	
	$\mathcal{B}_{42}$	$\mathcal{B}_{44}$	$\mathcal{B}_{47}$	$\mathcal{B}_{48}$	$\mathcal{B}_{50}$	$\mathcal{B}_{51}$	$\mathcal{B}_{53}$	$\mathcal{B}_{62}$	$\mathcal{B}_{70}$	$\mathcal{B}_{77}$	$\mathcal{B}_{93}$	$\mathcal{B}_{94}$	$\mathcal{B}_{126}$	$\mathcal{B}_{128}$

Table 9: Basis quantities correlation coefficients in percent, subtable 6.

$\mathcal{B}_{130}$	0	0	0	0	0	0	0	0	0	-1	0	0	1	1
$\mathcal{B}_{132}$	0	0	0	0	0	0	0	0	0	0	0	0	2	1
$\mathcal{B}_{136}$	0	0	1	0	0	0	0	2	-1	0	1	0	0	0
$\mathcal{B}_{151}$	0	0	0	0	0	0	0	0	12	0	0	0	0	0
$\mathcal{B}_{152}$	0	0	0	0	0	0	0	-1	-11	-64	0	0	0	0
$\mathcal{B}_{167}$	0	0	0	0	0	0	0	-1	0	0	1	0	0	0
$\mathcal{B}_{800}$	0	0	0	0	0	0	0	-8	-69	-2	-1	0	0	0
$\mathcal{B}_{802}$	0	0	0	0	0	0	0	16	-6	0	0	0	0	0
$\mathcal{B}_{803}$	0	0	0	0	0	0	0	-1	-19	0	0	-2	0	-1
$\mathcal{B}_{805}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{B}_{811}$	0	0	0	0	0	0	0	0	-1	0	0	0	0	0
$\mathcal{B}_{812}$	0	0	0	0	-1	0	0	-1	-1	0	0	0	0	0
$\mathcal{B}_{821}$	0	0	2	0	0	0	0	3	-1	0	1	0	0	1
$\mathcal{B}_{822}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\mathcal{B}_{42}$	$\mathcal{B}_{44}$	$\mathcal{B}_{47}$	$\mathcal{B}_{48}$	$\mathcal{B}_{50}$	$\mathcal{B}_{51}$	$\mathcal{B}_{53}$	$\mathcal{B}_{62}$	$\mathcal{B}_{70}$	$\mathcal{B}_{77}$	$\mathcal{B}_{93}$	$\mathcal{B}_{94}$	$\mathcal{B}_{126}$	$\mathcal{B}_{128}$

Table 10: Basis quantities correlation coefficients in percent, subtable 7.

$\mathcal{B}_{831}$	0	0	1	0	0	0	0	1	-1	0	1	0	0	0
$\mathcal{B}_{832}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{B}_{833}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{B}_{920}$	0	0	1	0	0	0	0	1	-1	0	1	0	0	0
$\mathcal{B}_{945}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\mathcal{B}_{42}$	$\mathcal{B}_{44}$	$\mathcal{B}_{47}$	$\mathcal{B}_{48}$	$\mathcal{B}_{50}$	$\mathcal{B}_{51}$	$\mathcal{B}_{53}$	$\mathcal{B}_{62}$	$\mathcal{B}_{70}$	$\mathcal{B}_{77}$	$\mathcal{B}_{93}$	$\mathcal{B}_{94}$	$\mathcal{B}_{126}$	$\mathcal{B}_{128}$

Table 11: Basis quantities correlation coefficients in percent, subtable 8.

	$\mathcal{B}_{130}$	$\mathcal{B}_{132}$	$\mathcal{B}_{136}$	$\mathcal{B}_{151}$	$\mathcal{B}_{152}$	$\mathcal{B}_{167}$	$\mathcal{B}_{800}$	$\mathcal{B}_{802}$	$\mathcal{B}_{803}$	$\mathcal{B}_{805}$	$\mathcal{B}_{811}$	$\mathcal{B}_{812}$	$\mathcal{B}_{821}$	$\mathcal{B}_{822}$
$\mathcal{B}_{132}$	0													
$\mathcal{B}_{136}$	0	0												
$\mathcal{B}_{151}$	0	0	0											
$\mathcal{B}_{152}$	0	0	0	0										
$\mathcal{B}_{167}$	0	0	0	0	0									
$\mathcal{B}_{800}$	0	0	0	-14	-3	0								
$\mathcal{B}_{802}$	0	0	0	-2	0	1	-1							
$\mathcal{B}_{803}$	0	0	0	-58	0	0	9	1						
$\mathcal{B}_{805}$	0	0	0	0	0	0	0	0	0					
$\mathcal{B}_{811}$	0	-1	20	0	0	0	0	0	0	0				
$\mathcal{B}_{812}$	0	-2	-8	0	0	0	0	0	0	0	-16			
$\mathcal{B}_{821}$	0	0	46	0	0	0	0	0	0	0	8	-4		
$\mathcal{B}_{822}$	0	0	-1	0	0	0	0	0	0	0	0	0	-1	

Table 12: Basis quantities correlation coefficients in percent, subtable 9.

	$\mathcal{B}_{130}$	$\mathcal{B}_{132}$	$\mathcal{B}_{136}$	$\mathcal{B}_{151}$	$\mathcal{B}_{152}$	$\mathcal{B}_{167}$	$\mathcal{B}_{800}$	$\mathcal{B}_{802}$	$\mathcal{B}_{803}$	$\mathcal{B}_{805}$	$\mathcal{B}_{811}$	$\mathcal{B}_{812}$	$\mathcal{B}_{821}$	$\mathcal{B}_{822}$
$\mathcal{B}_{831}$	0	0	38	0	0	0	0	0	0	0	14	-4	39	-1
$\mathcal{B}_{832}$	0	0	3	0	0	0	0	0	0	0	2	0	3	0
$\mathcal{B}_{833}$	0	0	-1	0	0	0	0	0	0	0	0	0	-1	0
$\mathcal{B}_{920}$	0	0	20	0	0	0	0	0	0	0	3	-2	34	-1
$\mathcal{B}_{945}$	0	-1	25	0	0	0	0	0	0	0	10	-11	10	0

Table 13: Basis quantities correlation coefficients in percent, subtable 10.

	$\mathcal{B}_{831}$	$\mathcal{B}_{832}$	$\mathcal{B}_{833}$	$\mathcal{B}_{920}$	$\mathcal{B}_{945}$
$\mathcal{B}_{832}$	-2				
$\mathcal{B}_{833}$	-1	-1			
$\mathcal{B}_{920}$	17	1	0		
$\mathcal{B}_{945}$	17	2	0	4	

2.7 Equality constraints

The constraints on the τ branching fractions fitted quantities are listed in the following. The constraint equations include as coefficients the values of some non-tau branching fractions, denoted *e.g.*, with the self-describing notation $\mathcal{B}_{K_S\to\pi^0\pi^0}$. Some coefficients are probabilities corresponding to the modulus square of amplitudes describing quantum mixtures of states such as K^0, $\overline{K}^0$, K_S, K_L, denoted with *e.g.*, $\mathcal{B}_{<K^0|K_S>} = |<K^0|K_S>|^2$. All non-tau quantities are taken from the PDG 2018 [6] averages. The fit procedure does not account for their uncertainties, which are generally small with respect to the uncertainties on the τ branching fractions. Please note that, in the following table, when a quantity like $\mathcal{B}_3/\mathcal{B}_5$ appears on the left side of the equation, it represents a fitted quantity, and when it appears on the right side it represents the ratio of two separate fitted quantities.

$$\begin{aligned}
\mathcal{B}_1 = {} & \mathcal{B}_3 + \mathcal{B}_5 + \mathcal{B}_9 + \mathcal{B}_{10} + \mathcal{B}_{14} + \mathcal{B}_{16} \\
& + \mathcal{B}_{20} + \mathcal{B}_{23} + \mathcal{B}_{27} + \mathcal{B}_{28} + \mathcal{B}_{30} + \mathcal{B}_{35} \\
& + \mathcal{B}_{40} + \mathcal{B}_{44} + \mathcal{B}_{37} + \mathcal{B}_{42} + \mathcal{B}_{47} + \mathcal{B}_{48} \\
& + \mathcal{B}_{804} + \mathcal{B}_{50} + \mathcal{B}_{51} + \mathcal{B}_{806} + \mathcal{B}_{126} \cdot \mathcal{B}_{\eta\to\text{neutral}} \\
& + \mathcal{B}_{128} \cdot \mathcal{B}_{\eta\to\text{neutral}} + \mathcal{B}_{130} \cdot \mathcal{B}_{\eta\to\text{neutral}} + \mathcal{B}_{132} \cdot \mathcal{B}_{\eta\to\text{neutral}} \\
& + \mathcal{B}_{800} \cdot \mathcal{B}_{\omega\to\pi^0\gamma} + \mathcal{B}_{151} \cdot \mathcal{B}_{\omega\to\pi^0\gamma} + \mathcal{B}_{152} \cdot \mathcal{B}_{\omega\to\pi^0\gamma} \\
& + \mathcal{B}_{167} \cdot \mathcal{B}_{\phi\to K_S K_L}
\end{aligned}$$

$$\begin{aligned}\mathcal{B}_2 = {} & \mathcal{B}_3 + \mathcal{B}_5 + \mathcal{B}_9 + \mathcal{B}_{10} + \mathcal{B}_{14} + \mathcal{B}_{16} \\
& + \mathcal{B}_{20} + \mathcal{B}_{23} + \mathcal{B}_{27} + \mathcal{B}_{28} + \mathcal{B}_{30} + \mathcal{B}_{35} \cdot (\mathcal{B}_{<\overline{K}^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0} \\
& + \mathcal{B}_{<\overline{K}^0|K_L>}) + \mathcal{B}_{40} \cdot (\mathcal{B}_{<\overline{K}^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0} + \mathcal{B}_{<\overline{K}^0|K_L>}) + \mathcal{B}_{44} \cdot (\mathcal{B}_{<\overline{K}^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0} \\
& + \mathcal{B}_{<\overline{K}^0|K_L>}) + \mathcal{B}_{37} \cdot (\mathcal{B}_{<\overline{K}^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0} + \mathcal{B}_{<\overline{K}^0|K_L>}) + \mathcal{B}_{42} \cdot (\mathcal{B}_{<\overline{K}^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0} \\
& + \mathcal{B}_{<\overline{K}^0|K_L>}) + \mathcal{B}_{47} \cdot (\mathcal{B}_{K_S\to\pi^0\pi^0} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) + \mathcal{B}_{48} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0} \\
& + \mathcal{B}_{804} + \mathcal{B}_{50} \cdot (\mathcal{B}_{K_S\to\pi^0\pi^0} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) + \mathcal{B}_{51} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0} \\
& + \mathcal{B}_{806} + \mathcal{B}_{126} \cdot \mathcal{B}_{\eta\to\text{neutral}} + \mathcal{B}_{128} \cdot \mathcal{B}_{\eta\to\text{neutral}} + \mathcal{B}_{130} \cdot \mathcal{B}_{\eta\to\text{neutral}} \\
& + \mathcal{B}_{132} \cdot (\mathcal{B}_{\eta\to\text{neutral}} \cdot (\mathcal{B}_{<\overline{K}^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0} + \mathcal{B}_{<\overline{K}^0|K_L>})) + \mathcal{B}_{800} \cdot \mathcal{B}_{\omega\to\pi^0\gamma} \\
& + \mathcal{B}_{151} \cdot \mathcal{B}_{\omega\to\pi^0\gamma} + \mathcal{B}_{152} \cdot \mathcal{B}_{\omega\to\pi^0\gamma} + \mathcal{B}_{167} \cdot (\mathcal{B}_{\phi\to K_S K_L} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0})\end{aligned}$$

$$\frac{\mathcal{B}_3}{\mathcal{B}_5} = \frac{\mathcal{B}_3}{\mathcal{B}_5}$$

$$\begin{aligned}\mathcal{B}_7 = {} & \mathcal{B}_{35} \cdot \mathcal{B}_{<\overline{K}^0|K_L>} + \mathcal{B}_9 + \mathcal{B}_{804} + \mathcal{B}_{37} \cdot \mathcal{B}_{<K^0|K_L>} \\
& + \mathcal{B}_{10}\end{aligned}$$

$$\mathcal{B}_8 = \mathcal{B}_9 + \mathcal{B}_{10}$$

$$\frac{\mathcal{B}_8}{\mathcal{B}_5} = \frac{\mathcal{B}_8}{\mathcal{B}_5}$$

$$\frac{\mathcal{B}_9}{\mathcal{B}_5} = \frac{\mathcal{B}_9}{\mathcal{B}_5}$$

$$\frac{\mathcal{B}_{10}}{\mathcal{B}_5} = \frac{\mathcal{B}_{10}}{\mathcal{B}_5}$$

$$\frac{\mathcal{B}_{10}}{\mathcal{B}_9} = \frac{\mathcal{B}_{10}}{\mathcal{B}_9}$$

$$\begin{aligned}\mathcal{B}_{11} = {} & \mathcal{B}_{14} + \mathcal{B}_{16} + \mathcal{B}_{20} + \mathcal{B}_{23} + \mathcal{B}_{27} + \mathcal{B}_{28} \\
& + \mathcal{B}_{30} + \mathcal{B}_{35} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) + \mathcal{B}_{37} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) \\
& + \mathcal{B}_{40} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) + \mathcal{B}_{42} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) \\
& + \mathcal{B}_{47} \cdot (\mathcal{B}_{K_S\to\pi^0\pi^0} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) + \mathcal{B}_{50} \cdot (\mathcal{B}_{K_S\to\pi^0\pi^0} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) \\
& + \mathcal{B}_{126} \cdot \mathcal{B}_{\eta\to\text{neutral}} + \mathcal{B}_{128} \cdot \mathcal{B}_{\eta\to\text{neutral}} + \mathcal{B}_{130} \cdot \mathcal{B}_{\eta\to\text{neutral}} \\
& + \mathcal{B}_{132} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0} \cdot \mathcal{B}_{\eta\to\text{neutral}}) + \mathcal{B}_{151} \cdot \mathcal{B}_{\omega\to\pi^0\gamma} \\
& + \mathcal{B}_{152} \cdot \mathcal{B}_{\omega\to\pi^0\gamma} + \mathcal{B}_{800} \cdot \mathcal{B}_{\omega\to\pi^0\gamma}\end{aligned}$$

$$\begin{aligned}\mathcal{B}_{12} = {} & \mathcal{B}_{128} \cdot \mathcal{B}_{\eta\to3\pi^0} + \mathcal{B}_{30} + \mathcal{B}_{23} + \mathcal{B}_{28} + \mathcal{B}_{14} \\
& + \mathcal{B}_{16} + \mathcal{B}_{20} + \mathcal{B}_{27} + \mathcal{B}_{126} \cdot \mathcal{B}_{\eta\to3\pi^0} + \mathcal{B}_{130} \cdot \mathcal{B}_{\eta\to3\pi^0}\end{aligned}$$

$$\mathcal{B}_{13} = \mathcal{B}_{14} + \mathcal{B}_{16}$$

$$\begin{aligned}\mathcal{B}_{17} = {} & \mathcal{B}_{128} \cdot \mathcal{B}_{\eta\to3\pi^0} + \mathcal{B}_{30} + \mathcal{B}_{23} + \mathcal{B}_{28} + \mathcal{B}_{35} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) \\
& + \mathcal{B}_{40} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) + \mathcal{B}_{42} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) \\
& + \mathcal{B}_{20} + \mathcal{B}_{27} + \mathcal{B}_{47} \cdot (\mathcal{B}_{K_S\to\pi^0\pi^0} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) + \mathcal{B}_{50} \cdot (\mathcal{B}_{K_S\to\pi^0\pi^0} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) \\
& + \mathcal{B}_{126} \cdot \mathcal{B}_{\eta\to3\pi^0} + \mathcal{B}_{37} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) + \mathcal{B}_{130} \cdot \mathcal{B}_{\eta\to3\pi^0}\end{aligned}$$

$$\mathcal{B}_{18} = \mathcal{B}_{23} + \mathcal{B}_{35} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) + \mathcal{B}_{20} + \mathcal{B}_{37} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0})$$

$$\mathcal{B}_{19} = \mathcal{B}_{23} + \mathcal{B}_{20}$$

$$\frac{\mathcal{B}_{19}}{\mathcal{B}_{13}} = \frac{\mathcal{B}_{19}}{\mathcal{B}_{13}}$$

$$\begin{aligned}\mathcal{B}_{24} = {} & \mathcal{B}_{27} + \mathcal{B}_{28} + \mathcal{B}_{30} + \mathcal{B}_{40} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) \\
& + \mathcal{B}_{42} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) + \mathcal{B}_{47} \cdot (\mathcal{B}_{K_S\to\pi^0\pi^0} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) \\
& + \mathcal{B}_{50} \cdot (\mathcal{B}_{K_S\to\pi^0\pi^0} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) + \mathcal{B}_{126} \cdot \mathcal{B}_{\eta\to3\pi^0} + \mathcal{B}_{128} \cdot \mathcal{B}_{\eta\to3\pi^0} \\
& + \mathcal{B}_{130} \cdot \mathcal{B}_{\eta\to3\pi^0} + \mathcal{B}_{132} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0} \cdot \mathcal{B}_{\eta\to3\pi^0})\end{aligned}$$

$$\begin{aligned}\mathcal{B}_{25} = {} & \mathcal{B}_{128} \cdot \mathcal{B}_{\eta\to3\pi^0} + \mathcal{B}_{30} + \mathcal{B}_{28} + \mathcal{B}_{27} + \mathcal{B}_{126} \cdot \mathcal{B}_{\eta\to3\pi^0} \\
& + \mathcal{B}_{130} \cdot \mathcal{B}_{\eta\to3\pi^0}\end{aligned}$$

$$\begin{aligned}\mathcal{B}_{26} = {} & \mathcal{B}_{128} \cdot \mathcal{B}_{\eta\to3\pi^0} + \mathcal{B}_{28} + \mathcal{B}_{40} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) \\ & + \mathcal{B}_{42} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) + \mathcal{B}_{27}\end{aligned}$$

$$\frac{\mathcal{B}_{26}}{\mathcal{B}_{13}} = \frac{\mathcal{B}_{26}}{\mathcal{B}_{13}}$$

$$\mathcal{B}_{29} = \mathcal{B}_{30} + \mathcal{B}_{126} \cdot \mathcal{B}_{\eta\to3\pi^0} + \mathcal{B}_{130} \cdot \mathcal{B}_{\eta\to3\pi^0}$$

$$\begin{aligned}\mathcal{B}_{31} = {} & \mathcal{B}_{128} \cdot \mathcal{B}_{\eta\to\text{neutral}} + \mathcal{B}_{23} + \mathcal{B}_{28} + \mathcal{B}_{42} + \mathcal{B}_{16} \\ & + \mathcal{B}_{37} + \mathcal{B}_{10} + \mathcal{B}_{167} \cdot (\mathcal{B}_{\phi\to K_S K_L} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0})\end{aligned}$$

$$\begin{aligned}\mathcal{B}_{32} = {} & \mathcal{B}_{16} + \mathcal{B}_{23} + \mathcal{B}_{28} + \mathcal{B}_{37} + \mathcal{B}_{42} + \mathcal{B}_{128} \cdot \mathcal{B}_{\eta\to\text{neutral}} \\ & + \mathcal{B}_{130} \cdot \mathcal{B}_{\eta\to\text{neutral}} + \mathcal{B}_{167} \cdot (\mathcal{B}_{\phi\to K_S K_L} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0})\end{aligned}$$

$$\begin{aligned}\mathcal{B}_{33} = {} & \mathcal{B}_{35} \cdot \mathcal{B}_{<\overline{K}^0|K_S>} + \mathcal{B}_{40} \cdot \mathcal{B}_{<\overline{K}^0|K_S>} + \mathcal{B}_{42} \cdot \mathcal{B}_{<K^0|K_S>} \\ & + \mathcal{B}_{47} + \mathcal{B}_{48} + \mathcal{B}_{50} + \mathcal{B}_{51} + \mathcal{B}_{37} \cdot \mathcal{B}_{<K^0|K_S>} \\ & + \mathcal{B}_{132} \cdot (\mathcal{B}_{<\overline{K}^0|K_S>} \cdot \mathcal{B}_{\eta\to\text{neutral}}) + \mathcal{B}_{44} \cdot \mathcal{B}_{<\overline{K}^0|K_S>} + \mathcal{B}_{167} \cdot \mathcal{B}_{\phi\to K_S K_L}\end{aligned}$$

$$\mathcal{B}_{34} = \mathcal{B}_{35} + \mathcal{B}_{37}$$

$$\mathcal{B}_{38} = \mathcal{B}_{42} + \mathcal{B}_{37}$$

$$\mathcal{B}_{39} = \mathcal{B}_{40} + \mathcal{B}_{42}$$

$$\mathcal{B}_{43} = \mathcal{B}_{40} + \mathcal{B}_{44}$$

$$\mathcal{B}_{46} = \mathcal{B}_{48} + \mathcal{B}_{47} + \mathcal{B}_{804}$$

$$\mathcal{B}_{49} = \mathcal{B}_{50} + \mathcal{B}_{51} + \mathcal{B}_{806}$$

$$\begin{aligned}\mathcal{B}_{54} = {} & \mathcal{B}_{35} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^+\pi^-}) + \mathcal{B}_{37} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^+\pi^-}) \\ & + \mathcal{B}_{40} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^+\pi^-}) + \mathcal{B}_{42} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^+\pi^-}) \\ & + \mathcal{B}_{47} \cdot (2 \cdot \mathcal{B}_{K_S\to\pi^+\pi^-} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) + \mathcal{B}_{48} \cdot \mathcal{B}_{K_S\to\pi^+\pi^-} \\ & + \mathcal{B}_{50} \cdot (2 \cdot \mathcal{B}_{K_S\to\pi^+\pi^-} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0}) + \mathcal{B}_{51} \cdot \mathcal{B}_{K_S\to\pi^+\pi^-} \\ & + \mathcal{B}_{53} \cdot (\mathcal{B}_{<\overline{K}^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0} + \mathcal{B}_{<\overline{K}^0|K_L>}) + \mathcal{B}_{62} + \mathcal{B}_{70} \\ & + \mathcal{B}_{77} + \mathcal{B}_{78} + \mathcal{B}_{93} + \mathcal{B}_{94} + \mathcal{B}_{126} \cdot \mathcal{B}_{\eta\to\text{charged}} \\ & + \mathcal{B}_{128} \cdot \mathcal{B}_{\eta\to\text{charged}} + \mathcal{B}_{130} \cdot \mathcal{B}_{\eta\to\text{charged}} + \mathcal{B}_{132} \cdot (\mathcal{B}_{<\overline{K}^0|K_L>} \cdot \mathcal{B}_{\eta\to\pi^+\pi^-\pi^0} \\ & + \mathcal{B}_{<\overline{K}^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^0\pi^0} \cdot \mathcal{B}_{\eta\to\pi^+\pi^-\pi^0} + \mathcal{B}_{<\overline{K}^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^+\pi^-} \cdot \mathcal{B}_{\eta\to3\pi^0}) \\ & + \mathcal{B}_{151} \cdot (\mathcal{B}_{\omega\to\pi^+\pi^-\pi^0} + \mathcal{B}_{\omega\to\pi^+\pi^-}) + \mathcal{B}_{152} \cdot (\mathcal{B}_{\omega\to\pi^+\pi^-\pi^0} + \mathcal{B}_{\omega\to\pi^+\pi^-}) \\ & + \mathcal{B}_{167} \cdot (\mathcal{B}_{\phi\to K^+K^-} + \mathcal{B}_{\phi\to K_S K_L} \cdot \mathcal{B}_{K_S\to\pi^+\pi^-}) + \mathcal{B}_{802} + \mathcal{B}_{803} \\ & + \mathcal{B}_{800} \cdot (\mathcal{B}_{\omega\to\pi^+\pi^-\pi^0} + \mathcal{B}_{\omega\to\pi^+\pi^-})\end{aligned}$$

$$\begin{aligned}\mathcal{B}_{55} = {} & \mathcal{B}_{128} \cdot \mathcal{B}_{\eta\to\text{charged}} + \mathcal{B}_{152} \cdot (\mathcal{B}_{\omega\to\pi^+\pi^-\pi^0} + \mathcal{B}_{\omega\to\pi^+\pi^-}) + \mathcal{B}_{78} \\ & + \mathcal{B}_{77} + \mathcal{B}_{94} + \mathcal{B}_{62} + \mathcal{B}_{70} + \mathcal{B}_{93} + \mathcal{B}_{126} \cdot \mathcal{B}_{\eta\to\text{charged}} \\ & + \mathcal{B}_{802} + \mathcal{B}_{803} + \mathcal{B}_{800} \cdot (\mathcal{B}_{\omega\to\pi^+\pi^-\pi^0} + \mathcal{B}_{\omega\to\pi^+\pi^-}) + \mathcal{B}_{151} \cdot (\mathcal{B}_{\omega\to\pi^+\pi^-\pi^0} \\ & + \mathcal{B}_{\omega\to\pi^+\pi^-}) + \mathcal{B}_{130} \cdot \mathcal{B}_{\eta\to\text{charged}} + \mathcal{B}_{168}\end{aligned}$$

$$\begin{aligned}\mathcal{B}_{56} = {} & \mathcal{B}_{35} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^+\pi^-}) + \mathcal{B}_{62} + \mathcal{B}_{93} + \mathcal{B}_{37} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^+\pi^-}) \\ & + \mathcal{B}_{802} + \mathcal{B}_{800} \cdot \mathcal{B}_{\omega\to\pi^+\pi^-} + \mathcal{B}_{151} \cdot \mathcal{B}_{\omega\to\pi^+\pi^-} + \mathcal{B}_{168}\end{aligned}$$

$$\begin{aligned}\mathcal{B}_{57} = {} & \mathcal{B}_{62} + \mathcal{B}_{93} + \mathcal{B}_{802} + \mathcal{B}_{800} \cdot \mathcal{B}_{\omega\to\pi^+\pi^-} + \mathcal{B}_{151} \cdot \mathcal{B}_{\omega\to\pi^+\pi^-} \\ & + \mathcal{B}_{167} \cdot \mathcal{B}_{\phi\to K^+K^-}\end{aligned}$$

$$\frac{\mathcal{B}_{57}}{\mathcal{B}_{55}} = \frac{\mathcal{B}_{57}}{\mathcal{B}_{55}}$$

$$\mathcal{B}_{58} = \mathcal{B}_{62} + \mathcal{B}_{93} + \mathcal{B}_{802} + \mathcal{B}_{167} \cdot \mathcal{B}_{\phi\to K^+K^-}$$

$$\mathcal{B}_{59} = \mathcal{B}_{35} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^+\pi^-}) + \mathcal{B}_{62} + \mathcal{B}_{800} \cdot \mathcal{B}_{\omega\to\pi^+\pi^-}$$

$$\mathcal{B}_{60} = \mathcal{B}_{62} + \mathcal{B}_{800} \cdot \mathcal{B}_{\omega\to\pi^+\pi^-}$$

$$\begin{aligned}\mathcal{B}_{63} = {} & \mathcal{B}_{40} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S \to \pi^+\pi^-}) + \mathcal{B}_{42} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S \to \pi^+\pi^-}) \\ & + \mathcal{B}_{47} \cdot (2 \cdot \mathcal{B}_{K_S \to \pi^+\pi^-} \cdot \mathcal{B}_{K_S \to \pi^0\pi^0}) + \mathcal{B}_{50} \cdot (2 \cdot \mathcal{B}_{K_S \to \pi^+\pi^-} \cdot \mathcal{B}_{K_S \to \pi^0\pi^0}) \\ & + \mathcal{B}_{70} + \mathcal{B}_{77} + \mathcal{B}_{78} + \mathcal{B}_{94} + \mathcal{B}_{126} \cdot \mathcal{B}_{\eta \to \text{charged}} \\ & + \mathcal{B}_{128} \cdot \mathcal{B}_{\eta \to \text{charged}} + \mathcal{B}_{130} \cdot \mathcal{B}_{\eta \to \text{charged}} + \mathcal{B}_{132} \cdot (\mathcal{B}_{<\overline{K}^0|K_S>} \cdot \mathcal{B}_{K_S \to \pi^+\pi^-} \cdot \mathcal{B}_{\eta \to \text{neutral}} \\ & + \mathcal{B}_{<\overline{K}^0|K_S>} \cdot \mathcal{B}_{K_S \to \pi^0\pi^0} \cdot \mathcal{B}_{\eta \to \text{charged}}) + \mathcal{B}_{151} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-\pi^0} + \mathcal{B}_{152} \cdot (\mathcal{B}_{\omega \to \pi^+\pi^-\pi^0} \\ & + \mathcal{B}_{\omega \to \pi^+\pi^-}) + \mathcal{B}_{800} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-\pi^0} + \mathcal{B}_{803}\end{aligned}$$

$$\begin{aligned}\mathcal{B}_{64} = {} & \mathcal{B}_{78} + \mathcal{B}_{77} + \mathcal{B}_{94} + \mathcal{B}_{70} + \mathcal{B}_{126} \cdot \mathcal{B}_{\eta \to \pi^+\pi^-\pi^0} \\ & + \mathcal{B}_{128} \cdot \mathcal{B}_{\eta \to \pi^+\pi^-\pi^0} + \mathcal{B}_{130} \cdot \mathcal{B}_{\eta \to \pi^+\pi^-\pi^0} + \mathcal{B}_{800} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-\pi^0} \\ & + \mathcal{B}_{151} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-\pi^0} + \mathcal{B}_{152} \cdot (\mathcal{B}_{\omega \to \pi^+\pi^-\pi^0} + \mathcal{B}_{\omega \to \pi^+\pi^-}) + \mathcal{B}_{803}\end{aligned}$$

$$\begin{aligned}\mathcal{B}_{65} = {} & \mathcal{B}_{40} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S \to \pi^+\pi^-}) + \mathcal{B}_{42} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S \to \pi^+\pi^-}) \\ & + \mathcal{B}_{70} + \mathcal{B}_{94} + \mathcal{B}_{128} \cdot \mathcal{B}_{\eta \to \pi^+\pi^-\pi^0} + \mathcal{B}_{151} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-\pi^0} \\ & + \mathcal{B}_{152} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-} + \mathcal{B}_{800} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-\pi^0} + \mathcal{B}_{803}\end{aligned}$$

$$\begin{aligned}\mathcal{B}_{66} = {} & \mathcal{B}_{70} + \mathcal{B}_{94} + \mathcal{B}_{128} \cdot \mathcal{B}_{\eta \to \pi^+\pi^-\pi^0} + \mathcal{B}_{151} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-\pi^0} \\ & + \mathcal{B}_{152} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-} + \mathcal{B}_{800} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-\pi^0} + \mathcal{B}_{803}\end{aligned}$$

$$\mathcal{B}_{67} = \mathcal{B}_{70} + \mathcal{B}_{94} + \mathcal{B}_{128} \cdot \mathcal{B}_{\eta \to \pi^+\pi^-\pi^0} + \mathcal{B}_{803}$$

$$\begin{aligned}\mathcal{B}_{68} = {} & \mathcal{B}_{40} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S \to \pi^+\pi^-}) + \mathcal{B}_{70} + \mathcal{B}_{152} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-} \\ & + \mathcal{B}_{800} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-\pi^0}\end{aligned}$$

$$\mathcal{B}_{69} = \mathcal{B}_{152} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-} + \mathcal{B}_{70} + \mathcal{B}_{800} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-\pi^0}$$

$$\begin{aligned}\mathcal{B}_{74} = {} & \mathcal{B}_{152} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-\pi^0} + \mathcal{B}_{78} + \mathcal{B}_{77} + \mathcal{B}_{126} \cdot \mathcal{B}_{\eta \to \pi^+\pi^-\pi^0} \\ & + \mathcal{B}_{130} \cdot \mathcal{B}_{\eta \to \pi^+\pi^-\pi^0}\end{aligned}$$

$$\begin{aligned}\mathcal{B}_{75} = {} & \mathcal{B}_{152} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-\pi^0} + \mathcal{B}_{47} \cdot (2 \cdot \mathcal{B}_{K_S \to \pi^+\pi^-} \cdot \mathcal{B}_{K_S \to \pi^0\pi^0}) \\ & + \mathcal{B}_{77} + \mathcal{B}_{126} \cdot \mathcal{B}_{\eta \to \pi^+\pi^-\pi^0} + \mathcal{B}_{130} \cdot \mathcal{B}_{\eta \to \pi^+\pi^-\pi^0}\end{aligned}$$

$$\mathcal{B}_{76} = \mathcal{B}_{152} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-\pi^0} + \mathcal{B}_{77} + \mathcal{B}_{126} \cdot \mathcal{B}_{\eta \to \pi^+\pi^-\pi^0} + \mathcal{B}_{130} \cdot \mathcal{B}_{\eta \to \pi^+\pi^-\pi^0}$$

$$\frac{\mathcal{B}_{76}}{\mathcal{B}_{54}} = \frac{\mathcal{B}_{76}}{\mathcal{B}_{54}}$$

$$\mathcal{B}_{78} = \mathcal{B}_{810} + \mathcal{B}_{50} \cdot (2 \cdot \mathcal{B}_{K_S \to \pi^+\pi^-} \cdot \mathcal{B}_{K_S \to \pi^0\pi^0}) + \mathcal{B}_{132} \cdot (\mathcal{B}_{<\overline{K}^0|K_S>} \cdot \mathcal{B}_{K_S \to \pi^+\pi^-} \cdot \mathcal{B}_{\eta \to 3\pi^0})$$

$$\begin{aligned}\mathcal{B}_{79} = {} & \mathcal{B}_{37} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S \to \pi^+\pi^-}) + \mathcal{B}_{42} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S \to \pi^+\pi^-}) \\ & + \mathcal{B}_{93} + \mathcal{B}_{94} + \mathcal{B}_{128} \cdot \mathcal{B}_{\eta \to \text{charged}} + \mathcal{B}_{151} \cdot (\mathcal{B}_{\omega \to \pi^+\pi^-\pi^0} \\ & + \mathcal{B}_{\omega \to \pi^+\pi^-}) + \mathcal{B}_{168} + \mathcal{B}_{802} + \mathcal{B}_{803}\end{aligned}$$

$$\mathcal{B}_{80} = \mathcal{B}_{93} + \mathcal{B}_{802} + \mathcal{B}_{151} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-}$$

$$\frac{\mathcal{B}_{80}}{\mathcal{B}_{60}} = \frac{\mathcal{B}_{80}}{\mathcal{B}_{60}}$$

$$\mathcal{B}_{81} = \mathcal{B}_{128} \cdot \mathcal{B}_{\eta \to \pi^+\pi^-\pi^0} + \mathcal{B}_{94} + \mathcal{B}_{803} + \mathcal{B}_{151} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-\pi^0}$$

$$\frac{\mathcal{B}_{81}}{\mathcal{B}_{69}} = \frac{\mathcal{B}_{81}}{\mathcal{B}_{69}}$$

$$\begin{aligned}\mathcal{B}_{82} = {} & \mathcal{B}_{128} \cdot \mathcal{B}_{\eta \to \text{charged}} + \mathcal{B}_{42} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S \to \pi^+\pi^-}) + \mathcal{B}_{802} \\ & + \mathcal{B}_{803} + \mathcal{B}_{151} \cdot (\mathcal{B}_{\omega \to \pi^+\pi^-\pi^0} + \mathcal{B}_{\omega \to \pi^+\pi^-}) + \mathcal{B}_{37} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S \to \pi^+\pi^-})\end{aligned}$$

$$\begin{aligned}\mathcal{B}_{83} = {} & \mathcal{B}_{128} \cdot \mathcal{B}_{\eta \to \pi^+\pi^-\pi^0} + \mathcal{B}_{802} + \mathcal{B}_{803} + \mathcal{B}_{151} \cdot (\mathcal{B}_{\omega \to \pi^+\pi^-\pi^0} \\ & + \mathcal{B}_{\omega \to \pi^+\pi^-})\end{aligned}$$

$$\mathcal{B}_{84} = \mathcal{B}_{802} + \mathcal{B}_{151} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-} + \mathcal{B}_{37} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S \to \pi^+\pi^-})$$

$$\mathcal{B}_{85} = \mathcal{B}_{802} + \mathcal{B}_{151} \cdot \mathcal{B}_{\omega \to \pi^+\pi^-}$$

$$\frac{\mathcal{B}_{85}}{\mathcal{B}_{60}} = \frac{\mathcal{B}_{85}}{\mathcal{B}_{60}}$$

$$\begin{aligned}\mathcal{B}_{87} = {} & \mathcal{B}_{42} \cdot (\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{K_S\to\pi^+\pi^-}) + \mathcal{B}_{128} \cdot \mathcal{B}_{\eta\to\pi^+\pi^-\pi^0} + \mathcal{B}_{151} \cdot \mathcal{B}_{\omega\to\pi^+\pi^-\pi^0} \\ & + \mathcal{B}_{803}\end{aligned}$$

$$\mathcal{B}_{88} = \mathcal{B}_{128} \cdot \mathcal{B}_{\eta\to\pi^+\pi^-\pi^0} + \mathcal{B}_{803} + \mathcal{B}_{151} \cdot \mathcal{B}_{\omega\to\pi^+\pi^-\pi^0}$$

$$\mathcal{B}_{89} = \mathcal{B}_{803} + \mathcal{B}_{151} \cdot \mathcal{B}_{\omega\to\pi^+\pi^-\pi^0}$$

$$\mathcal{B}_{92} = \mathcal{B}_{94} + \mathcal{B}_{93}$$

$$\frac{\mathcal{B}_{93}}{\mathcal{B}_{60}} = \frac{\mathcal{B}_{93}}{\mathcal{B}_{60}}$$

$$\frac{\mathcal{B}_{94}}{\mathcal{B}_{69}} = \frac{\mathcal{B}_{94}}{\mathcal{B}_{69}}$$

$$\mathcal{B}_{96} = \mathcal{B}_{167} \cdot \mathcal{B}_{\phi\to K^+K^-}$$

$$\mathcal{B}_{102} = \mathcal{B}_{103} + \mathcal{B}_{104}$$

$$\mathcal{B}_{103} = \mathcal{B}_{820} + \mathcal{B}_{822} + \mathcal{B}_{831} \cdot \mathcal{B}_{\omega\to\pi^+\pi^-}$$

$$\mathcal{B}_{104} = \mathcal{B}_{830} + \mathcal{B}_{833}$$

$$\begin{aligned}\mathcal{B}_{106} = {} & \mathcal{B}_{30} + \mathcal{B}_{44} \cdot \mathcal{B}_{<\overline{K}^0|K_S>} + \mathcal{B}_{47} + \mathcal{B}_{53} \cdot \mathcal{B}_{<K^0|K_S>} \\ & + \mathcal{B}_{77} + \mathcal{B}_{103} + \mathcal{B}_{126} \cdot (\mathcal{B}_{\eta\to 3\pi^0} + \mathcal{B}_{\eta\to\pi^+\pi^-\pi^0}) + \mathcal{B}_{152} \cdot \mathcal{B}_{\omega\to\pi^+\pi^-\pi^0}\end{aligned}$$

$$\begin{aligned}\mathcal{B}_{110} = {} & \mathcal{B}_{10} + \mathcal{B}_{16} + \mathcal{B}_{23} + \mathcal{B}_{28} + \mathcal{B}_{35} + \mathcal{B}_{40} \\ & + \mathcal{B}_{128} + \mathcal{B}_{802} + \mathcal{B}_{803} + \mathcal{B}_{151} + \mathcal{B}_{130} + \mathcal{B}_{132} \\ & + \mathcal{B}_{44} + \mathcal{B}_{53} + \mathcal{B}_{168} + \mathcal{B}_{169} + \mathcal{B}_{822} + \mathcal{B}_{833}\end{aligned}$$

$$\mathcal{B}_{149} = \mathcal{B}_{152} + \mathcal{B}_{800} + \mathcal{B}_{151}$$

$$\mathcal{B}_{150} = \mathcal{B}_{800} + \mathcal{B}_{151}$$

$$\frac{\mathcal{B}_{150}}{\mathcal{B}_{66}} = \frac{\mathcal{B}_{150}}{\mathcal{B}_{66}}$$

$$\frac{\mathcal{B}_{152}}{\mathcal{B}_{54}} = \frac{\mathcal{B}_{152}}{\mathcal{B}_{54}}$$

$$\frac{\mathcal{B}_{152}}{\mathcal{B}_{76}} = \frac{\mathcal{B}_{152}}{\mathcal{B}_{76}}$$

$$\mathcal{B}_{168} = \mathcal{B}_{167} \cdot \mathcal{B}_{\phi\to K^+K^-}$$

$$\mathcal{B}_{169} = \mathcal{B}_{167} \cdot \mathcal{B}_{\phi\to K_S K_L}$$

$$\mathcal{B}_{804} = \mathcal{B}_{47} \cdot ((\mathcal{B}_{<K^0|K_L>} \cdot \mathcal{B}_{<\overline{K}^0|K_L>})/(\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{<\overline{K}^0|K_S>}))$$

$$\mathcal{B}_{806} = \mathcal{B}_{50} \cdot ((\mathcal{B}_{<K^0|K_L>} \cdot \mathcal{B}_{<\overline{K}^0|K_L>})/(\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{<\overline{K}^0|K_S>}))$$

$$\mathcal{B}_{809} = \mathcal{B}_{30}$$

$$\mathcal{B}_{810} = \mathcal{B}_{910} + \mathcal{B}_{911} + \mathcal{B}_{811} \cdot \mathcal{B}_{\omega\to\pi^+\pi^-\pi^0} + \mathcal{B}_{812}$$

$$\mathcal{B}_{820} = \mathcal{B}_{920} + \mathcal{B}_{821}$$

$$\mathcal{B}_{830} = \mathcal{B}_{930} + \mathcal{B}_{831} \cdot \mathcal{B}_{\omega\to\pi^+\pi^-\pi^0} + \mathcal{B}_{832}$$

$$\mathcal{B}_{910} = \mathcal{B}_{136} \cdot \mathcal{B}_{\eta\to 3\pi^0}$$

$$\mathcal{B}_{911} = \mathcal{B}_{945} \cdot \mathcal{B}_{\eta\to\pi^+\pi^-\pi^0}$$

$$\mathcal{B}_{930} = \mathcal{B}_{136} \cdot \mathcal{B}_{\eta\to\pi^+\pi^-\pi^0}$$

$$\mathcal{B}_{944} = \mathcal{B}_{136} \cdot \mathcal{B}_{\eta\to\gamma\gamma}$$

$$
\begin{aligned}
\mathcal{B}_{\text{All}} = {} & \mathcal{B}_{3} + \mathcal{B}_{5} + \mathcal{B}_{9} + \mathcal{B}_{10} + \mathcal{B}_{14} + \mathcal{B}_{16} \\
& + \mathcal{B}_{20} + \mathcal{B}_{23} + \mathcal{B}_{27} + \mathcal{B}_{28} + \mathcal{B}_{30} + \mathcal{B}_{35} \\
& + \mathcal{B}_{37} + \mathcal{B}_{40} + \mathcal{B}_{42} + \mathcal{B}_{47} \cdot (1 + ((\mathcal{B}_{<K^0|K_L>} \cdot \mathcal{B}_{<\overline{K}^0|K_L>})/(\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{<\overline{K}^0|K_S>}))) \\
& + \mathcal{B}_{48} + \mathcal{B}_{62} + \mathcal{B}_{70} + \mathcal{B}_{77} + \mathcal{B}_{811} + \mathcal{B}_{812} \\
& + \mathcal{B}_{93} + \mathcal{B}_{94} + \mathcal{B}_{832} + \mathcal{B}_{833} + \mathcal{B}_{126} + \mathcal{B}_{128} \\
& + \mathcal{B}_{802} + \mathcal{B}_{803} + \mathcal{B}_{800} + \mathcal{B}_{151} + \mathcal{B}_{130} + \mathcal{B}_{132} \\
& + \mathcal{B}_{44} + \mathcal{B}_{53} + \mathcal{B}_{50} \cdot (1 + ((\mathcal{B}_{<K^0|K_L>} \cdot \mathcal{B}_{<\overline{K}^0|K_L>})/(\mathcal{B}_{<K^0|K_S>} \cdot \mathcal{B}_{<\overline{K}^0|K_S>}))) \\
& + \mathcal{B}_{51} + \mathcal{B}_{167} \cdot (\mathcal{B}_{\phi\to K^+K^-} + \mathcal{B}_{\phi\to K_S K_L}) + \mathcal{B}_{152} + \mathcal{B}_{920} \\
& + \mathcal{B}_{821} + \mathcal{B}_{822} + \mathcal{B}_{831} + \mathcal{B}_{136} + \mathcal{B}_{945} + \mathcal{B}_{805}
\end{aligned}
$$

3 Tests of lepton universality

Lepton universality tests probe the Standard Model prediction that the charged weak current interaction has the same coupling for all lepton generations. The precision of such tests has been significantly improved since the 2014 edition by the addition of the Belle τ lifetime measurement [66], while improvements from the τ branching fraction fit are negligible. We compute the universality tests by using ratios of the partial widths of a heavier lepton λ decaying to a lighter lepton ρ [67],

$$
\Gamma(\lambda \to \nu_\lambda \rho \overline{\nu}_\rho (\gamma)) = \frac{\mathcal{B}(\lambda \to \nu_\lambda \rho \overline{\nu}_\rho)}{\tau_\lambda} = \frac{G_\lambda G_\rho m_\lambda^5}{192\pi^3} \, f\left(\frac{m_\rho^2}{m_\lambda^2}\right) R_W^\lambda R_\gamma^\lambda \; ,
$$

where

$$
G_\rho = \frac{g_\rho^2}{4\sqrt{2} M_W^2} \; , \qquad f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x \; ,
$$

$$
R_W^\lambda = 1 + \frac{3}{5}\frac{m_\lambda^2}{M_W^2} + \frac{9}{5}\frac{m_\rho^2}{M_W^2} \; [68, 69, 70], \qquad R_\gamma^\lambda = 1 + \frac{\alpha(m_\lambda)}{2\pi}\left(\frac{25}{4} - \pi^2\right) \; .
$$

We use $R_\gamma^\tau = 1 - 43.2 \cdot 10^{-4}$ and $R_\gamma^\mu = 1 - 42.4 \cdot 10^{-4}$ [67] and M_W from PDG 2018 [6]. We use HFLAV 2018 averages and PDG 2018 for the other quantities. Using pure leptonic processes we obtain

$$
\left(\frac{g_\tau}{g_\mu}\right) = 1.0010 \pm 0.0014 \; , \qquad \left(\frac{g_\tau}{g_e}\right) = 1.0029 \pm 0.0014 \; , \qquad \left(\frac{g_\mu}{g_e}\right) = 1.0018 \pm 0.0014 \; .
$$

Using the expressions for the τ semi-hadronic partial widths, we obtain

$$
\left(\frac{g_\tau}{g_\mu}\right)^2 = \frac{\mathcal{B}(\tau \to h\nu_\tau)}{\mathcal{B}(h \to \mu\overline{\nu}_\mu)} \frac{2 m_h m_\mu^2 \tau_h}{(1 + \delta R_{\tau/h}) m_\tau^3 \tau_\tau} \left(\frac{1 - m_\mu^2/m_h^2}{1 - m_h^2/m_\tau^2}\right)^2 \; ,
$$

where $h = \pi$ or K and the radiative corrections are $\delta R_{\tau/\pi} = (0.16 \pm 0.14)\%$ and $\delta R_{\tau/K} = (0.90 \pm 0.22)\%$ [71, 72, 73, 74]. We measure:

$$
\left(\frac{g_\tau}{g_\mu}\right)_\pi = 0.9958 \pm 0.0026 \; , \qquad \left(\frac{g_\tau}{g_\mu}\right)_K = 0.9879 \pm 0.0063 \; .
$$

Similar tests could be performed with decays to electrons, however they are less precise because the hadron two body decays to electrons are helicity-suppressed. Averaging the three g_τ/g_μ ratios we obtain

$$
\left(\frac{g_\tau}{g_\mu}\right)_{\tau+\pi+K} = 0.9999 \pm 0.0014 \; ,
$$

accounting for correlations. Table 14 reports the correlation coefficients for the fitted coupling ratios.

Table 14: Universality coupling ratios correlation coefficients (%).

$\left(\frac{g_\tau}{g_e}\right)$	51			
$\left(\frac{g_\mu}{g_e}\right)$	-50	49		
$\left(\frac{g_\tau}{g_\mu}\right)_\pi$	23	25	2	
$\left(\frac{g_\tau}{g_\mu}\right)_K$	11	10	-1	6
	$\left(\frac{g_\tau}{g_\mu}\right)$	$\left(\frac{g_\tau}{g_e}\right)$	$\left(\frac{g_\mu}{g_e}\right)$	$\left(\frac{g_\tau}{g_\mu}\right)_\pi$

Since there is 100% correlation between g_τ/g_μ, g_τ/g_e and g_μ/g_e, the correlation matrix is expected to be positive semi-definite, with one eigenvalue equal to zero. Due to numerical inaccuracies, one eigenvalue is expected to be close to zero rather than exactly zero.

4 Universality-improved $\mathcal{B}(\tau \to e\nu\overline{\nu})$ and $\mathrm{R}_{\mathrm{had}}$

We compute two quantities that are used in this report and that have been traditionally used for further elaborations and tests involving the τ branching fractions:

- the "universality-improved" experimental determination of $\mathcal{B}_e = \mathcal{B}(\tau \to e\nu\overline{\nu})$, which relies on the assumption that the Standard Model and lepton universality hold;
- the ratio R_{had} between the total branching fraction of the τ to hadrons, $\mathcal{B}_{\mathrm{had}}$ and the universality-improved $\mathcal{B}_e$, which is the same as the ratio of the two respective partial widths, $\Gamma(\tau \to \mathrm{had})$ and $\Gamma(\tau \to e\nu\overline{\nu})$.

Following Ref. [1], we obtain a more precise experimental determination of $\mathcal{B}_e$ using the τ branching fraction to $\mu\nu\overline{\nu}$, $\mathcal{B}_\mu$, and the τ lifetime. We average:

- the $\mathcal{B}_e$ fit value $\mathcal{B}_5$,
- the $\mathcal{B}_e$ determination from the $\mathcal{B}_\mu = \mathcal{B}(\tau \to \mu\nu\overline{\nu})$ fit value $\mathcal{B}_3$ assuming that $g_\mu/g_e = 1$, hence (see also Section 3)
$$\mathcal{B}_e = \mathcal{B}_\mu \cdot f(m_e^2/m_\tau^2)/f(m_\mu^2/m_\tau^2) \; ,$$
- the $\mathcal{B}_e$ determination from the τ lifetime assuming that $g_\tau/g_\mu = 1$, hence
$$\mathcal{B}_e = \mathcal{B}(\mu \to e\overline{\nu}_e\nu_\mu) \cdot (\tau_\tau/\tau_\mu) \cdot (m_\tau/m_\mu)^5 \cdot f(m_e^2/m_\tau^2)/f(m_e^2/m_\mu^2) \cdot (R_\gamma^\tau R_W^\tau)/(R_\gamma^\mu R_W^\mu) \; ,$$
where $\mathcal{B}(\mu \to e\overline{\nu}_e\nu_\mu) = 1$.

Accounting for correlations, we obtain

$$\mathcal{B}_e^{\mathrm{uni}} = (17.814 \pm 0.022)\%.$$

We use $\mathcal{B}_e^{\mathrm{uni}}$ to obtain the ratio

$$R_{\mathrm{had}} = \frac{\Gamma(\tau \to \mathrm{hadrons})}{\Gamma(\tau \to e\nu\overline{\nu})} = \frac{\mathcal{B}_{\mathrm{had}}}{\mathcal{B}_e^{\mathrm{uni}}} = 3.6355 \pm 0.0081 \; .$$

We define $\mathcal{B}_{\mathrm{had}}$ as the sum of all *measured* branching fractions to hadrons, which corresponds to the sum of all branching fractions minus the leptonic branching fractions, $\mathcal{B}_{\mathrm{had}} = \mathcal{B}_{\mathrm{All}} - \mathcal{B}_e - \mathcal{B}_\mu = (64.76 \pm 0.10)\%$ (see Section 2 and Table 3 for more details on the definition of $\mathcal{B}_{\mathrm{All}}$). An alternative definition of $\mathcal{B}_{\mathrm{had}}$ uses the unitarity of the sum of all branching fractions, $\mathcal{B}_{\mathrm{had}}^{\mathrm{uni}} = 1 - \mathcal{B}_e - \mathcal{B}_\mu = (64.79 \pm 0.06)\%$, and results in:

$$R_{\mathrm{had}}^{\mathrm{uni}} = \frac{1 - \mathcal{B}_e - \mathcal{B}_\mu}{\mathcal{B}_e^{\mathrm{uni}}} = 3.6370 \pm 0.0075 \; .$$

A third definition of $\mathcal{B}_{\mathrm{had}}$ uses the unitarity of the sum of all branching fractions, the Standard Model prediction $\mathcal{B}_\mu = \mathcal{B}_e \cdot f(m_\mu^2/m_\tau^2)/f(m_e^2/m_\tau^2)$ and $\mathcal{B}_e^{\mathrm{uni}}$ to define $\mathcal{B}_{\mathrm{had}}^{\mathrm{uni,\,SM}} = 1 - \mathcal{B}_e^{\mathrm{uni}} - \mathcal{B}_e^{\mathrm{uni}} \cdot f(m_\mu^2/m_\tau^2)/f(m_e^2/m_\tau^2) = (64.86 \pm 0.04)\%$, and to compute

$$R_{\mathrm{had}}^{\mathrm{uni,\,SM}} = \frac{1 - \mathcal{B}_e^{\mathrm{uni}} - \mathcal{B}_e^{\mathrm{uni}} \cdot f(m_\mu^2/m_\tau^2)/f(m_e^2/m_\tau^2)}{\mathcal{B}_e^{\mathrm{uni}}} = 3.6409 \pm 0.0070 \; .$$

Although $\mathcal{B}_{\mathrm{had}}^{\mathrm{uni}}$ and $\mathcal{B}_{\mathrm{had}}^{\mathrm{uni,\,SM}}$ are more precise than $\mathcal{B}_{\mathrm{had}}$, the precision of $R_{\mathrm{had}}^{\mathrm{uni}}$ and $R_{\mathrm{had}}^{\mathrm{uni,\,SM}}$ is just slightly better than the one of R_{had} because there are larger correlations between $\mathcal{B}_{\mathrm{had}}^{\mathrm{uni}}$, $\mathcal{B}_{\mathrm{had}}^{\mathrm{uni,\,SM}}$ and $\mathcal{B}_e^{\mathrm{uni}}$ than between $\mathcal{B}_{\mathrm{had}}$ and $\mathcal{B}_e^{\mathrm{uni}}$.

5 $|V_{us}|$ measurement

The CKM matrix element magnitude $|V_{us}|$ is most precisely determined from kaon decays [75] (see Figure 1), and its precision is limited by the uncertainties of the lattice QCD estimates of the meson decay constants $f_+^{K\pi}(0)$ and $f_{K\pm}/f_{\pi\pm}$. Using the τ branching fractions, it is possible to determine $|V_{us}|$ in an alternative way [76, 77] that does not depend on lattice QCD and has small theory uncertainties (as discussed in Section 5.1). Moreover, $|V_{us}|$ can be determined using the τ branching fractions similarly to the kaon case, using the same meson decay constants from lattice QCD.

5.1 $|V_{us}|$ from $\mathcal{B}(\tau \to X_s \nu)$

The τ hadronic partial width is the sum of the τ partial widths to strange and to non-strange hadronic final states, $\Gamma_{\text{had}} = \Gamma_s + \Gamma_{\text{VA}}$. The suffix "VA" traditionally denotes the sum of the τ partial widths to non-strange final states, which proceed through either vector or axial-vector currents.

Dividing any partial width Γ_x by the electronic partial width, Γ_e, we obtain partial width ratios R_x (which are equal to the respective branching fraction ratios $\mathcal{B}_x/\mathcal{B}_e$) for which $R_{\text{had}} = R_s + R_{\text{VA}}$. In terms of such ratios, $|V_{us}|$ can be measured as [76, 77]

$$|V_{us}|_{\tau s} = \sqrt{R_s / \left[\frac{R_{\text{VA}}}{|V_{ud}|^2} - \delta R_{\text{theory}} \right]} \ ,$$

where δR_{theory} can be determined in the context of low energy QCD theory, partly relying on experimental low energy scattering data. The literature reports several calculations [78, 79, 80]. In this report we use Ref. [78], whose estimated uncertainty size is intermediate between the two other ones. We use the information in that paper and the PDG 2018 value for the s-quark mass $m_s = 95.00 \pm 6.70$ MeV [6] to calculate $\delta R_{\text{theory}} = 0.242 \pm 0.033$.

We proceed following the same procedure of the 2012 HFLAV report [81]. We sum the relevant τ branching fractions to compute $\mathcal{B}_{\text{VA}}$ and $\mathcal{B}_s$ and we use the universality-improved $\mathcal{B}_e^{\text{uni}}$ (see Section 4) to compute the R_{VA} and R_s ratios. In past determinations of $|V_{us}|$, for example in the 2009 HFLAV report [82], the total hadronic branching fraction has been computed using unitarity as $\mathcal{B}_{\text{had}}^{\text{uni}} = 1 - \mathcal{B}_e - \mathcal{B}_\mu$, obtaining then $\mathcal{B}_s$ from the sum of the strange branching fractions and $\mathcal{B}_{\text{VA}}$ from $\mathcal{B}_{\text{had}}^{\text{uni}} - \mathcal{B}_s$. We prefer to use the more direct experimental determination of $\mathcal{B}_{\text{VA}}$ for two reasons. First, both methods result in comparable uncertainties on $|V_{us}|$, since the better precision on $\mathcal{B}_{\text{had}}^{\text{uni}} = 1 - \mathcal{B}_e - \mathcal{B}_\mu$ is counterbalanced by increased correlations in the expressions $(1 - \mathcal{B}_e - \mathcal{B}_\mu)/\mathcal{B}_e^{\text{univ}}$ and $\mathcal{B}_s/(\mathcal{B}_{\text{had}} - \mathcal{B}_s)$ in the $|V_{us}|$ calculation. Second, if there are unobserved τ hadronic decay modes, they would affect $\mathcal{B}_{\text{VA}}$ and $\mathcal{B}_s$ in a more asymmetric way when using unitarity.

Using the τ branching fraction fit results with their uncertainties and correlations (Section 2), we compute $\mathcal{B}_s = (2.931 \pm 0.041)\%$ (see also Table 15) and $\mathcal{B}_{\text{VA}} = \mathcal{B}_{\text{had}} - \mathcal{B}_s = (61.83 \pm 0.10)\%$, where $\mathcal{B}_{\text{had}}$ has been defined in section 4. PDG 2018 averages are used for non-τ quantities; $|V_{ud}| = 0.97420 \pm 0.00021$ [83, 84].

We obtain $|V_{us}|_{\tau s} = 0.2195 \pm 0.0019$, which is 2.9σ lower than the unitarity CKM prediction $|V_{us}|_{\text{uni}} = 0.22565 \pm 0.00089$, from $(|V_{us}|_{\text{uni}})^2 = 1 - |V_{ud}|^2 - |V_{ub}|^2$. The $|V_{us}|_{\tau s}$ uncertainty includes a systematic error contribution of 0.0011 from the theory uncertainty on δR_{theory}. The 2018 BABAR preliminary results improved the $|V_{us}|$ precision by about 10% and reduced the discrepancy by about 6.5%.

5.2 $|V_{us}|$ from $\mathcal{B}(\tau \to K\nu)/\mathcal{B}(\tau \to \pi\nu)$

We compute $|V_{us}|$ from the ratio of branching fractions $\mathcal{B}(\tau \to K^-\nu_\tau)/\mathcal{B}(\tau \to \pi^-\nu_\tau) = (6.467 \pm 0.084) \cdot 10^{-2}$ from the equation [85]:

$$\frac{\mathcal{B}(\tau \to K^-\nu_\tau)}{\mathcal{B}(\tau \to \pi^-\nu_\tau)} = \frac{f_{K\pm}^2 |V_{us}|^2}{f_{\pi\pm}^2 |V_{ud}|^2} \frac{\left(m_\tau^2 - m_K^2\right)^2}{\left(m_\tau^2 - m_\pi^2\right)^2} \frac{1 + \delta R_{\tau/K}}{1 + \delta R_{\tau/\pi}} (1 + \delta R_{K/\pi})$$

We use $f_{K\pm}/f_{\pi\pm} = 1.1932 \pm 0.0019$ from the FLAG 2019 lattice QCD averages with $N_f = 2 + 1 + 1$ [86, 87, 88, 89],

$$\frac{1 + \delta R_{\tau/K}}{1 + \delta R_{\tau/\pi}} = \frac{1 + (0.90 \pm 0.22)\%}{1 + (0.16 \pm 0.14)\%} \quad [71, 72, 73, 74] \ ,$$

$$1 + \delta R_{K/\pi} = 1 + (-0.69 \pm 0.17)\% \quad [68, 90, 91] \ .$$

The value of $\delta R_{K/\pi}$ in the Spring 2017 HFLAV-Tau report [2] incorrectly included a strong isospin-breaking correction that is not needed when using $f_{K\pm}/f_{\pi\pm}$ rather than its isospin-limit variant. We compute $|V_{us}|_{\tau K/\pi} = 0.2236 \pm 0.0015$, 1.2σ below the CKM unitarity prediction.

Table 15: HFLAV 2018 τ branching fractions to strange final states.

Branching fraction	**HFLAV 2018 fit (%)**
$K^-\nu_\tau$	0.6986 ± 0.0085
$K^-\pi^0\nu_\tau$	0.4904 ± 0.0092
$K^-2\pi^0\nu_\tau$ (ex. K^0)	0.0585 ± 0.0027
$K^-3\pi^0\nu_\tau$ (ex. K^0, η)	0.0113 ± 0.0026
$\pi^-\overline{K}^0\nu_\tau$	0.8378 ± 0.0139
$\pi^-\overline{K}^0\pi^0\nu_\tau$	0.3807 ± 0.0129
$\pi^-\overline{K}^0 2\pi^0\nu_\tau$ (ex. K^0)	0.0235 ± 0.0231
$\overline{K}^0 h^- h^- h^+\nu_\tau$	0.0222 ± 0.0202
$K^-\eta\nu_\tau$	0.0154 ± 0.0008
$K^-\pi^0\eta\nu_\tau$	0.0048 ± 0.0012
$\pi^-\overline{K}^0\eta\nu_\tau$	0.0094 ± 0.0015
$K^-\omega\nu_\tau$	0.0410 ± 0.0092
$K^-\phi\nu_\tau$ ($\phi \to K^+K^-$)	0.0022 ± 0.0008
$K^-\phi\nu_\tau$ ($\phi \to K_S^0 K_L^0$)	0.0015 ± 0.0006
$K^-\pi^-\pi^+\nu_\tau$ (ex. K^0, ω)	0.2923 ± 0.0067
$K^-\pi^-\pi^+\pi^0\nu_\tau$ (ex. K^0, ω, η)	0.0410 ± 0.0143
$K^-2\pi^-2\pi^+\nu_\tau$ (ex. K^0)	0.0001 ± 0.0001
$K^-2\pi^-2\pi^+\pi^0\nu_\tau$ (ex. K^0)	0.0001 ± 0.0001
$X_s^-\nu_\tau$	2.9308 ± 0.0412

5.3 $|V_{us}|$ from $\mathcal{B}(\tau \to K\nu)$

We determine $|V_{us}|$ from the branching fraction $\mathcal{B}(\tau^- \to K^-\nu_\tau)$ using

$$\mathcal{B}(\tau^- \to K^-\nu_\tau) = \frac{G_F^2}{16\pi\hbar} f_{K\pm}^2 |V_{us}|^2 \tau_\tau m_\tau^3 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 (1 + \delta R_{\tau/K})(1 + \delta R_{K\mu 2}) \ .$$

We use $f_{K\pm} = 155.7 \pm 0.3\,\text{MeV}$ from the FLAG 2019 lattice QCD averages with $N_f = 2+1+1$ [86, 87, 92, 88], $\delta R_{\tau/K} = (0.90 \pm 0.22)\%$ [71, 72, 73, 74] and $\delta R_{K\mu 2} = (1.07 \pm 0.21)\%$ [90, 6, 93], which includes short and long-distance radiative corrections. We obtain $|V_{us}|_{\tau K} = 0.2234 \pm 0.0015$, which is 1.3σ below the CKM unitarity prediction. The physical constants have been taken from PDG 2018 (which uses CODATA 2014 [94]).

5.4 $|V_{us}|$ from τ summary

We summarize the $|V_{us}|$ results reporting the values, the discrepancy with respect to the $|V_{us}|$ determination from CKM unitarity, and an illustration of the measurement method:

$$\begin{aligned}
|V_{us}|_{\text{uni}} &= 0.22565 \pm 0.00089 & & [\text{from } \sqrt{1 - |V_{ud}|^2 - |V_{ub}|^2} \quad (\text{CKM unitarity})] \ , \\
|V_{us}|_{\tau s} &= 0.2195 \ \ \pm 0.0019 & -2.9\sigma \quad & [\text{from } \mathcal{B}(\tau^- \to X_s^-\nu_\tau)] \ , \\
|V_{us}|_{\tau K/\pi} &= 0.2236 \ \ \pm 0.0015 & -1.2\sigma \quad & [\text{from } \mathcal{B}(\tau^- \to K^-\nu_\tau)/\mathcal{B}(\tau^- \to \pi^-\nu_\tau)] \ , \\
|V_{us}|_{\tau K} &= 0.2234 \ \ \pm 0.0015 & -1.3\sigma \quad & [\text{from } \mathcal{B}(\tau^- \to K^-\nu_\tau)] \ .
\end{aligned}$$

Averaging the three above $|V_{us}|$ determinations that rely on the τ branching fractions (taking into account all correlations due to the τ HFLAV and other mentioned inputs) we obtain, for $|V_{us}|$ and its discrepancy:

$$|V_{us}|_\tau = 0.2221 \pm 0.0013 \qquad -2.2\sigma \qquad [\text{average of 3 } |V_{us}| \ \tau \text{ measurements}] \ .$$

The correlation between $f_{K\pm}$ and $f_{K\pm}/f_{\pi\pm}$ has been assumed to be zero. Even assuming ±100% correlation, the $|V_{us}|$ uncertainty varies by less than 10%.

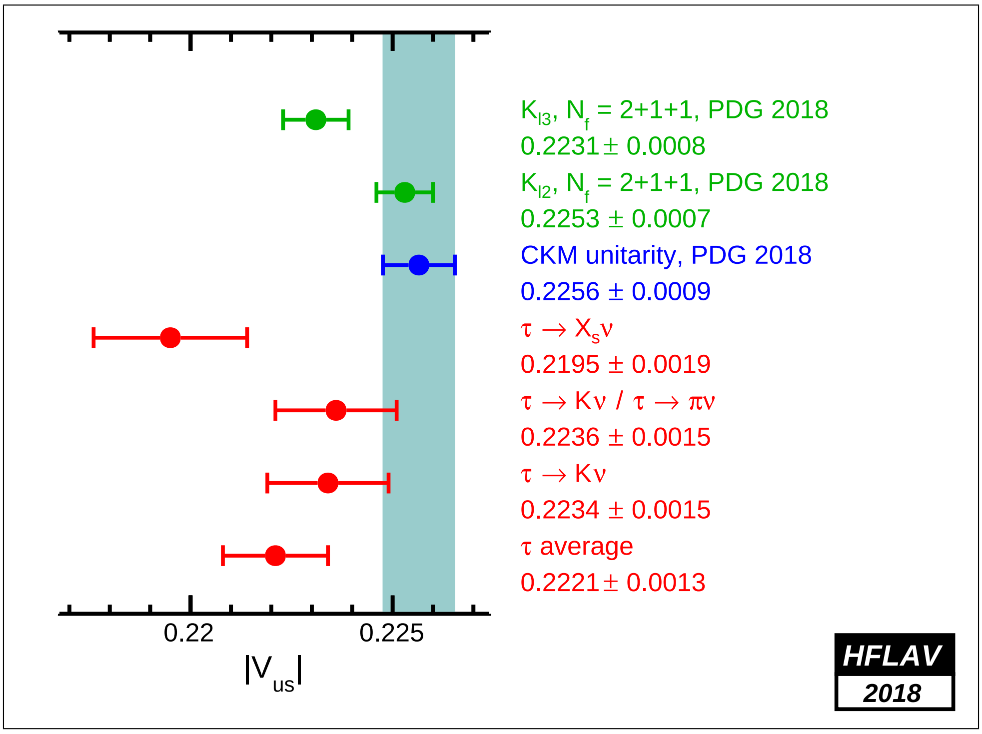

Figure 1: $|V_{us}|$ averages.

All $|V_{us}|$ determinations based on measured τ branching fractions are lower than both the kaon and the CKM-unitarity determinations. This is correlated with the fact that the direct measurements of the three major τ branching fractions to kaons [$\mathcal{B}(\tau \to K^-\nu_\tau)$, $\mathcal{B}(\tau \to K^-\pi^0\nu_\tau)$ and $\mathcal{B}(\tau \to \pi^-\overline{K}^0\nu_\tau)$] are lower than their determinations from the kaon branching fractions into final states with leptons within the SM [68, 95, 96].

Alternative deterrminations of $|V_{us}|$ from $\mathcal{B}(\tau \to X_s\nu)$ [97, 98], based on partially different sets of experimental inputs, report $|V_{us}|$ values consistent with the unitarity determination.

Figure 1 reports the HFLAV $|V_{us}|$ determinations that use the τ branching fractions, compared to two $|V_{us}|$ determinations based on kaon data [6] and to $|V_{us}|$ obtained from $|V_{ud}|$ and the CKM matrix unitarity [6].

6 Combination of upper limits on τ lepton-flavour-violating branching fractions

The Standard Model predicts that the τ lepton-flavour-violating (LFV) branching fractions are too small to be measured with the available experimental precision. We report in Table 16 and Figure 2 the experimental upper limits on these branching fractions that have been published by the B-factories BABAR and Belle and later experiments. We omit previous weaker upper limits (mainly from CLEO) and all preliminary results older than a few years. Presently, no preliminary result is included.

Combining upper limits is a delicate issue, since there is no standard and generally agreed procedure. Furthermore, the τ LFV searches published limits are extracted from the data with a variety of methods, and cannot be directly combined with a uniform procedure. It is however possible to use a single and effective upper limit combination procedure for all modes by re-computing the published upper limits with just one extraction method, using the published information that documents the upper limit determination: number of observed candidates, expected background, signal efficiency and number of analyzed τ decays.

We chose to use the CL$_s$ method [99] to re-compute the τ LFV upper limits, since it is well known and widely used (see the Statistics review of PDG 2018 [6]), and since the limits computed with the CL$_s$ method can be combined in a straightforward way (see below). The CL$_s$ method is based on two hypotheses: signal plus background and background only. We calculate the observed confidence levels for the two hypotheses:

$$\mathrm{CL}_{s+b} = P_{s+b}(Q \le Q_{obs}) = \int_{-\infty}^{Q_{obs}} \frac{dP_{s+b}}{dQ} dQ, \tag{10}$$

$$\mathrm{CL}_{b} = P_{b}(Q \le Q_{obs}) = \int_{-\infty}^{Q_{obs}} \frac{dP_{b}}{dQ} dQ, \tag{11}$$

where CL_{s+b} is the confidence level observed for the signal plus background hypotheses, CL_b is the confidence level observed for the background only hypothesis, $\frac{dP_{s+b}}{dQ}$ and $\frac{dP_b}{dQ}$ are the probability distribution functions (PDFs) for the two corresponding hypothesis and Q is called the test statistic. The CL_s value is defined as the ratio between the confidence level for the signal plus background hypothesis and the confidence level for the background hypothesis:

$$\mathrm{CL}_s = \frac{\mathrm{CL}_{s+b}}{\mathrm{CL}_b}. \tag{12}$$

When multiple results are combined, the PDFs in Eqs. (10) and (11) are the product of the individual PDFs,

$$\mathrm{CL}_s = \frac{\prod_{i=1}^{N} \sum_{n=0}^{n_i} \dfrac{e^{-(s_i+b_i)}(s_i+b_i)^n}{n!}}{\prod_{i=1}^{N} \sum_{n=0}^{n_i} \dfrac{e^{-b_i} b_i^n}{n!}} \frac{\prod_{j=1}^{N} [s_i S_i(x_{ij}) + b_i B_i(x_{ij})]}{\prod_{j=1}^{N} B_i(x_{ij})}, \tag{13}$$

where N is the number of results (or channels), and, for each channel i, n_i is the number of observed candidates, x_{ij} are the values of the discriminating variables (with index j), s_i and b_i are the number of signal and background events and S_i, B_i are the probability distribution functions of the discriminating variables. The discriminating variables x_{ij} are assumed to be uncorrelated. The expected signal s_i is related to the τ lepton branching fraction $\mathcal{B}(\tau \to f_i)$ into the searched final state f_i by $s_i = N_i \epsilon_i \mathcal{B}(\tau \to f_i)$, where N_i is the number of produced τ leptons and ϵ_i is the detection efficiency for observing the decay $\tau \to f_i$. For e^+e^- experiments, $N_i = 2\mathcal{L}_i \sigma_{\tau\tau}$, where $\mathcal{L}_i$ is the integrated luminosity and $\sigma_{\tau\tau}$ is the τ pair production cross section $\sigma(e^+e^- \to \tau^+\tau^-)$ [100]. In experiments where τ leptons are produced in more complex multiple reactions, the effective N_i is typically estimated with Monte Carlo simulations calibrated with related data yields.

The extraction of the upper limits is performed using the code provided by Tom Junk [101]. The systematic uncertainties are modeled in the Monte Carlo toy experiments by convolving the S_i and B_i PDFs with Gaussian distributions corresponding to the nuisance parameters.

Table 16 reports the HFLAV combinations of the τ LFV limits. Since there is negligible gain in combining limits of very different strength, the combinations do not include the CLEO searches and do not include results where the single event sensitivity is more than a factor of 5 lower than the value for the search with the best limit.

Figure 3 reports a graphical representation of the τ LFV limits combinations listed in Table 16. The published information that has been used to obtain these limits is reported in Table 17. In the previous HFLAV reports, the determination of combined limit $\mathcal{B}_{183} = \mu^-\mu^+\mu^-$ erroneously counted twice the systematic uncertainty of the LHCb limit. That has been fixed now, and the combination of the upper limits on $\mathcal{B}_{183} = \mu^-\mu^+\mu^-$ has changed from $< 1.2 \cdot 10^{-8}$ to $< 1.1 \cdot 10^{-8}$.

Table 16: Experimental upper limits on lepton flavour violating τ decays. The modes are grouped according to the properties of their final states. Modes with baryon number violation are labelled with "BNV". The experiment "HFLAV" denotes the combinations of upper limits computed by HFLAV. The references associated with the combination list what upper limits have been used.

Decay mode	**Category**	**90% CL Limit**	**Experiment**	**References**
$\mathcal{B}_{156} = e^-\gamma$	$\ell\gamma$	$3.3 \cdot 10^{-8}$	*BABAR*	[102]
		$1.2 \cdot 10^{-7}$	Belle	[103]
		$5.4 \cdot 10^{-8}$	HFLAV	[103, 102]
$\mathcal{B}_{157} = \mu^-\gamma$		$4.4 \cdot 10^{-8}$	*BABAR*	[102]
		$4.5 \cdot 10^{-8}$	Belle	[103]

Table 16 – continued from previous page

Decay mode	Category	90% CL Limit	Experiment	References
		$5.0 \cdot 10^{-8}$	HFLAV	[103, 102]
$\mathcal{B}_{158} = e^- \pi^0$	ℓP^0	$1.3 \cdot 10^{-7}$	BABAR	[104]
		$8.0 \cdot 10^{-8}$	Belle	[105]
		$4.9 \cdot 10^{-8}$	HFLAV	[105, 104]
$\mathcal{B}_{159} = \mu^- \pi^0$		$1.1 \cdot 10^{-7}$	BABAR	[104]
		$1.2 \cdot 10^{-7}$	Belle	[105]
		$3.6 \cdot 10^{-8}$	HFLAV	[105, 104]
$\mathcal{B}_{160} = e^- K_S^0$		$3.3 \cdot 10^{-8}$	BABAR	[106]
		$2.6 \cdot 10^{-8}$	Belle	[107]
		$1.4 \cdot 10^{-8}$	HFLAV	[107, 106]
$\mathcal{B}_{161} = \mu^- K_S^0$		$4.0 \cdot 10^{-8}$	BABAR	[106]
		$2.3 \cdot 10^{-8}$	Belle	[107]
		$1.5 \cdot 10^{-8}$	HFLAV	[107, 106]
$\mathcal{B}_{162} = e^- \eta$		$1.6 \cdot 10^{-7}$	BABAR	[104]
		$9.2 \cdot 10^{-8}$	Belle	[105]
		$5.5 \cdot 10^{-8}$	HFLAV	[105, 104]
$\mathcal{B}_{163} = \mu^- \eta$		$1.5 \cdot 10^{-7}$	BABAR	[104]
		$6.5 \cdot 10^{-8}$	Belle	[105]
		$3.8 \cdot 10^{-8}$	HFLAV	[105, 104]
$\mathcal{B}_{172} = e^- \eta'(958)$		$2.4 \cdot 10^{-7}$	BABAR	[104]
		$1.6 \cdot 10^{-7}$	Belle	[105]
		$9.9 \cdot 10^{-8}$	HFLAV	[105, 104]
$\mathcal{B}_{173} = \mu^- \eta'(958)$		$1.4 \cdot 10^{-7}$	BABAR	[104]
		$1.3 \cdot 10^{-7}$	Belle	[105]
		$6.3 \cdot 10^{-8}$	HFLAV	[105, 104]
$\mathcal{B}_{164} = e^- \rho^0$	ℓV^0	$4.6 \cdot 10^{-8}$	BABAR	[108]
		$1.8 \cdot 10^{-8}$	Belle	[109]
		$1.5 \cdot 10^{-8}$	HFLAV	[109, 108]
$\mathcal{B}_{165} = \mu^- \rho^0$		$2.6 \cdot 10^{-8}$	BABAR	[108]
		$1.2 \cdot 10^{-8}$	Belle	[109]
		$1.5 \cdot 10^{-8}$	HFLAV	[109, 108]
$\mathcal{B}_{166} = e^- \omega$		$1.1 \cdot 10^{-7}$	BABAR	[110]
		$4.8 \cdot 10^{-8}$	Belle	[109]
		$3.3 \cdot 10^{-8}$	HFLAV	[109, 110]
$\mathcal{B}_{167} = \mu^- \omega$		$1.0 \cdot 10^{-7}$	BABAR	[110]
		$4.7 \cdot 10^{-8}$	Belle	[109]
		$4.0 \cdot 10^{-8}$	HFLAV	[109, 110]
$\mathcal{B}_{168} = e^- K^*(892)$		$5.9 \cdot 10^{-8}$	BABAR	[108]
		$3.2 \cdot 10^{-8}$	Belle	[109]
		$2.3 \cdot 10^{-8}$	HFLAV	[109, 108]
$\mathcal{B}_{169} = \mu^- K^*(892)$		$1.7 \cdot 10^{-7}$	BABAR	[108]
		$7.2 \cdot 10^{-8}$	Belle	[109]
		$6.0 \cdot 10^{-8}$	HFLAV	[109, 108]
$\mathcal{B}_{170} = e^- \overline{K}^*(892)$		$4.6 \cdot 10^{-8}$	BABAR	[108]
		$3.4 \cdot 10^{-8}$	Belle	[109]
		$2.2 \cdot 10^{-8}$	HFLAV	[109, 108]
$\mathcal{B}_{171} = \mu^- \overline{K}^*(892)$		$7.3 \cdot 10^{-8}$	BABAR	[108]
		$7.0 \cdot 10^{-8}$	Belle	[109]
		$4.2 \cdot 10^{-8}$	HFLAV	[109, 108]
$\mathcal{B}_{176} = e^- \phi$		$3.1 \cdot 10^{-8}$	BABAR	[108]
		$3.1 \cdot 10^{-8}$	Belle	[109]
		$2.0 \cdot 10^{-8}$	HFLAV	[109, 108]
$\mathcal{B}_{177} = \mu^- \phi$		$1.9 \cdot 10^{-7}$	BABAR	[108]

Table 16 – continued from previous page

Decay mode	Category	90% CL Limit	Experiment	References
		$8.4 \cdot 10^{-8}$	Belle	[109]
		$6.8 \cdot 10^{-8}$	HFLAV	[109, 108]
$\mathcal{B}_{174} = e^- f_0(980)$	ℓS^0	$3.2 \cdot 10^{-8}$	Belle	[111]
$\mathcal{B}_{175} = \mu^- f_0(980)$		$3.4 \cdot 10^{-8}$	Belle	[111]
$\mathcal{B}_{178} = e^- e^+ e^-$	$\ell\ell\ell$	$2.9 \cdot 10^{-8}$	*BABAR*	[112]
		$2.7 \cdot 10^{-8}$	Belle	[113]
		$1.4 \cdot 10^{-8}$	HFLAV	[113, 112]
$\mathcal{B}_{179} = e^- \mu^+ \mu^-$		$3.2 \cdot 10^{-8}$	*BABAR*	[112]
		$2.7 \cdot 10^{-8}$	Belle	[113]
		$1.6 \cdot 10^{-8}$	HFLAV	[113, 112]
$\mathcal{B}_{180} = \mu^- e^+ \mu^-$		$2.6 \cdot 10^{-8}$	*BABAR*	[112]
		$1.7 \cdot 10^{-8}$	Belle	[113]
		$9.8 \cdot 10^{-9}$	HFLAV	[113, 112]
$\mathcal{B}_{181} = \mu^- e^+ e^-$		$2.2 \cdot 10^{-8}$	*BABAR*	[112]
		$1.8 \cdot 10^{-8}$	Belle	[113]
		$1.1 \cdot 10^{-8}$	HFLAV	[113, 112]
$\mathcal{B}_{182} = e^- \mu^+ e^-$		$1.8 \cdot 10^{-8}$	*BABAR*	[112]
		$1.5 \cdot 10^{-8}$	Belle	[113]
		$8.4 \cdot 10^{-9}$	HFLAV	[113, 112]
$\mathcal{B}_{183} = \mu^- \mu^+ \mu^-$		$3.8 \cdot 10^{-7}$	ATLAS	[114]
		$3.3 \cdot 10^{-8}$	*BABAR*	[112]
		$2.1 \cdot 10^{-8}$	Belle	[113]
		$4.6 \cdot 10^{-8}$	LHCb	[115]
		$1.1 \cdot 10^{-8}$	HFLAV	[113, 112, 115]
$\mathcal{B}_{184} = e^- \pi^+ \pi^-$	ℓhh	$1.2 \cdot 10^{-7}$	*BABAR*	[116]
		$2.3 \cdot 10^{-8}$	Belle	[117]
$\mathcal{B}_{185} = e^+ \pi^- \pi^-$		$2.7 \cdot 10^{-7}$	*BABAR*	[116]
		$2.0 \cdot 10^{-8}$	Belle	[117]
$\mathcal{B}_{186} = \mu^- \pi^+ \pi^-$		$2.9 \cdot 10^{-7}$	*BABAR*	[116]
		$2.1 \cdot 10^{-8}$	Belle	[117]
$\mathcal{B}_{187} = \mu^+ \pi^- \pi^-$		$7.0 \cdot 10^{-8}$	*BABAR*	[116]
		$3.9 \cdot 10^{-8}$	Belle	[117]
$\mathcal{B}_{188} = e^- \pi^+ K^-$		$3.2 \cdot 10^{-7}$	*BABAR*	[116]
		$3.7 \cdot 10^{-8}$	Belle	[117]
$\mathcal{B}_{189} = e^- K^+ \pi^-$		$1.7 \cdot 10^{-7}$	*BABAR*	[116]
		$3.1 \cdot 10^{-8}$	Belle	[117]
$\mathcal{B}_{190} = e^+ \pi^- K^-$		$1.8 \cdot 10^{-7}$	*BABAR*	[116]
		$3.2 \cdot 10^{-8}$	Belle	[117]
$\mathcal{B}_{191} = e^- K_S^0 K_S^0$		$7.1 \cdot 10^{-8}$	Belle	[107]
$\mathcal{B}_{192} = e^- K^+ K^-$		$1.4 \cdot 10^{-7}$	*BABAR*	[116]
		$3.4 \cdot 10^{-8}$	Belle	[117]
$\mathcal{B}_{193} = e^+ K^- K^-$		$1.5 \cdot 10^{-7}$	*BABAR*	[116]
		$3.3 \cdot 10^{-8}$	Belle	[117]
$\mathcal{B}_{194} = \mu^- \pi^+ K^-$		$2.6 \cdot 10^{-7}$	*BABAR*	[116]
		$8.6 \cdot 10^{-8}$	Belle	[117]
$\mathcal{B}_{195} = \mu^- K^+ \pi^-$		$3.2 \cdot 10^{-7}$	*BABAR*	[116]
		$4.5 \cdot 10^{-8}$	Belle	[117]
$\mathcal{B}_{196} = \mu^+ \pi^- K^-$		$2.2 \cdot 10^{-7}$	*BABAR*	[116]
		$4.8 \cdot 10^{-8}$	Belle	[117]
$\mathcal{B}_{197} = \mu^- K_S^0 K_S^0$		$8.0 \cdot 10^{-8}$	Belle	[107]
$\mathcal{B}_{198} = \mu^- K^+ K^-$		$2.5 \cdot 10^{-7}$	*BABAR*	[116]
		$4.4 \cdot 10^{-8}$	Belle	[117]
$\mathcal{B}_{199} = \mu^+ K^- K^-$		$4.8 \cdot 10^{-7}$	*BABAR*	[116]

Table 16 – continued from previous page

Decay mode	Category	90% CL Limit	Experiment	References
		$4.7 \cdot 10^{-8}$	Belle	[117]
$\mathcal{B}_{211} = \pi^-\Lambda$	BNV	$7.2 \cdot 10^{-8}$	Belle	[118]
$\mathcal{B}_{212} = \pi^-\overline{\Lambda}$		$1.4 \cdot 10^{-7}$	Belle	[118]
$\mathcal{B}_{215} = p\mu^-\mu^-$		$4.4 \cdot 10^{-7}$	LHCb	[119]
$\mathcal{B}_{216} = \overline{p}\mu^+\mu^-$		$3.3 \cdot 10^{-7}$	LHCb	[119]

Table 17: Published information that has been used to re-compute upper limits with the CL_s method, *i.e.* the number of τ leptons produced, the signal detection efficiency and its uncertainty, the number of expected background events and its uncertainty, and the number of observed events. The uncertainty on the efficiency includes the minor uncertainty contribution on the number of τ leptons (typically originating on the uncertainties on the integrated luminosity and on the production cross-section). The additional limit used in the combinations (from LHCb) has been originally determined with the CL_s method.

Decay mode	Exp.	Ref.	N_τ (millions)	efficiency (%)	$\mathrm{N_{bkg}}$	$\mathrm{N_{obs}}$
$\mathcal{B}_{156} = e^-\gamma$	*BABAR*	[102]	963	3.90 ± 0.30	1.60 ± 0.40	0
$\mathcal{B}_{156} = e^-\gamma$	Belle	[103]	983	3.00 ± 0.10	5.14 ± 3.30	5
$\mathcal{B}_{157} = \mu^-\gamma$	*BABAR*	[102]	963	6.10 ± 0.50	3.60 ± 0.70	2
$\mathcal{B}_{157} = \mu^-\gamma$	Belle	[103]	983	5.07 ± 0.20	13.90 ± 5.00	10
$\mathcal{B}_{158} = e^-\pi^0$	*BABAR*	[104]	339	2.83 ± 0.25	0.17 ± 0.04	0
$\mathcal{B}_{158} = e^-\pi^0$	Belle	[105]	401	3.93 ± 0.18	0.20 ± 0.20	0
$\mathcal{B}_{159} = \mu^-\pi^0$	*BABAR*	[104]	339	4.75 ± 0.37	1.33 ± 0.15	1
$\mathcal{B}_{159} = \mu^-\pi^0$	Belle	[105]	401	4.53 ± 0.20	0.58 ± 0.34	1
$\mathcal{B}_{160} = e^-K^0_S$	*BABAR*	[106]	862	9.10 ± 1.73	0.59 ± 0.25	1
$\mathcal{B}_{160} = e^-K^0_S$	Belle	[107]	1274	10.20 ± 0.67	0.18 ± 0.18	0
$\mathcal{B}_{161} = \mu^-K^0_S$	*BABAR*	[106]	862	6.14 ± 0.20	0.30 ± 0.18	1
$\mathcal{B}_{161} = \mu^-K^0_S$	Belle	[107]	1274	10.70 ± 0.73	0.35 ± 0.21	0
$\mathcal{B}_{162} = e^-\eta$	*BABAR*	[104]	339	2.12 ± 0.20	0.22 ± 0.05	0
$\mathcal{B}_{162} = e^-\eta$	Belle	[105]	401	2.87 ± 0.20	0.78 ± 0.78	0
$\mathcal{B}_{163} = \mu^-\eta$	*BABAR*	[104]	339	3.59 ± 0.41	0.75 ± 0.08	1
$\mathcal{B}_{163} = \mu^-\eta$	Belle	[105]	401	4.08 ± 0.28	0.64 ± 0.04	0
$\mathcal{B}_{172} = e^-\eta'(958)$	*BABAR*	[104]	339	1.53 ± 0.16	0.12 ± 0.03	0
$\mathcal{B}_{172} = e^-\eta'(958)$	Belle	[105]	401	1.59 ± 0.13	0.01 ± 0.41	0
$\mathcal{B}_{173} = \mu^-\eta'(958)$	*BABAR*	[104]	339	2.18 ± 0.26	0.49 ± 0.26	0
$\mathcal{B}_{173} = \mu^-\eta'(958)$	Belle	[105]	401	2.47 ± 0.20	0.23 ± 0.46	0
$\mathcal{B}_{164} = e^-\rho^0$	*BABAR*	[108]	829	7.31 ± 0.20	1.32 ± 0.17	1
$\mathcal{B}_{164} = e^-\rho^0$	Belle	[109]	1554	7.58 ± 0.41	0.29 ± 0.15	0
$\mathcal{B}_{165} = \mu^-\rho^0$	*BABAR*	[108]	829	4.52 ± 0.40	2.04 ± 0.19	0
$\mathcal{B}_{165} = \mu^-\rho^0$	Belle	[109]	1554	7.09 ± 0.37	1.48 ± 0.35	0
$\mathcal{B}_{166} = e^-\omega$	*BABAR*	[110]	829	2.96 ± 0.13	0.35 ± 0.06	0
$\mathcal{B}_{166} = e^-\omega$	Belle	[109]	1554	2.92 ± 0.18	0.30 ± 0.14	0
$\mathcal{B}_{167} = \mu^-\omega$	*BABAR*	[110]	829	2.56 ± 0.16	0.73 ± 0.03	0
$\mathcal{B}_{167} = \mu^-\omega$	Belle	[109]	1554	2.38 ± 0.14	0.72 ± 0.18	0
$\mathcal{B}_{168} = e^-K^*(892)$	*BABAR*	[108]	829	8.00 ± 0.20	1.65 ± 0.23	2
$\mathcal{B}_{168} = e^-K^*(892)$	Belle	[109]	1554	4.37 ± 0.24	0.29 ± 0.14	0
$\mathcal{B}_{169} = \mu^-K^*(892)$	*BABAR*	[108]	829	4.60 ± 0.40	1.79 ± 0.21	4
$\mathcal{B}_{169} = \mu^-K^*(892)$	Belle	[109]	1554	3.39 ± 0.19	0.53 ± 0.20	1
$\mathcal{B}_{170} = e^-\overline{K}^*(892)$	*BABAR*	[108]	829	7.80 ± 0.20	2.76 ± 0.28	2
$\mathcal{B}_{170} = e^-\overline{K}^*(892)$	Belle	[109]	1554	4.41 ± 0.25	0.08 ± 0.08	0
$\mathcal{B}_{171} = \mu^-\overline{K}^*(892)$	*BABAR*	[108]	829	4.10 ± 0.30	1.72 ± 0.17	1
$\mathcal{B}_{171} = \mu^-\overline{K}^*(892)$	Belle	[109]	1554	3.60 ± 0.20	0.45 ± 0.17	1
$\mathcal{B}_{176} = e^-\phi$	*BABAR*	[108]	829	6.40 ± 0.20	0.68 ± 0.12	0
$\mathcal{B}_{176} = e^-\phi$	Belle	[109]	1554	4.18 ± 0.25	0.47 ± 0.19	0
$\mathcal{B}_{177} = \mu^-\phi$	*BABAR*	[108]	829	5.20 ± 0.30	2.76 ± 0.16	6
$\mathcal{B}_{177} = \mu^-\phi$	Belle	[109]	1554	3.21 ± 0.19	0.06 ± 0.06	1
$\mathcal{B}_{178} = e^-e^+e^-$	*BABAR*	[112]	868	8.60 ± 0.20	0.12 ± 0.02	0

Table 17 – continued from previous page

Decay mode	Exp.	Ref.	N_τ (millions)	efficiency (%)	N_{bkg}	N_{obs}
$\mathcal{B}_{178} = e^- e^+ e^-$	Belle	[113]	1437	6.00 ± 0.59	0.21 ± 0.15	0
$\mathcal{B}_{179} = e^- \mu^+ \mu^-$	*BABAR*	[112]	868	6.40 ± 0.40	0.54 ± 0.14	0
$\mathcal{B}_{179} = e^- \mu^+ \mu^-$	Belle	[113]	1437	6.10 ± 0.58	0.10 ± 0.04	0
$\mathcal{B}_{180} = \mu^- e^+ \mu^-$	*BABAR*	[112]	868	10.20 ± 0.60	0.03 ± 0.02	0
$\mathcal{B}_{180} = \mu^- e^+ \mu^-$	Belle	[113]	1437	10.10 ± 0.77	0.02 ± 0.02	0
$\mathcal{B}_{181} = \mu^- e^+ e^-$	*BABAR*	[112]	868	8.80 ± 0.50	0.64 ± 0.19	0
$\mathcal{B}_{181} = \mu^- e^+ e^-$	Belle	[113]	1437	9.30 ± 0.73	0.04 ± 0.04	0
$\mathcal{B}_{182} = e^- \mu^+ e^-$	*BABAR*	[112]	868	12.70 ± 0.70	0.34 ± 0.12	0
$\mathcal{B}_{182} = e^- \mu^+ e^-$	Belle	[113]	1437	11.50 ± 0.89	0.01 ± 0.01	0
$\mathcal{B}_{183} = \mu^- \mu^+ \mu^-$	*BABAR*	[112]	868	6.60 ± 0.60	0.44 ± 0.17	0
$\mathcal{B}_{183} = \mu^- \mu^+ \mu^-$	Belle	[113]	1437	7.60 ± 0.56	0.13 ± 0.20	0

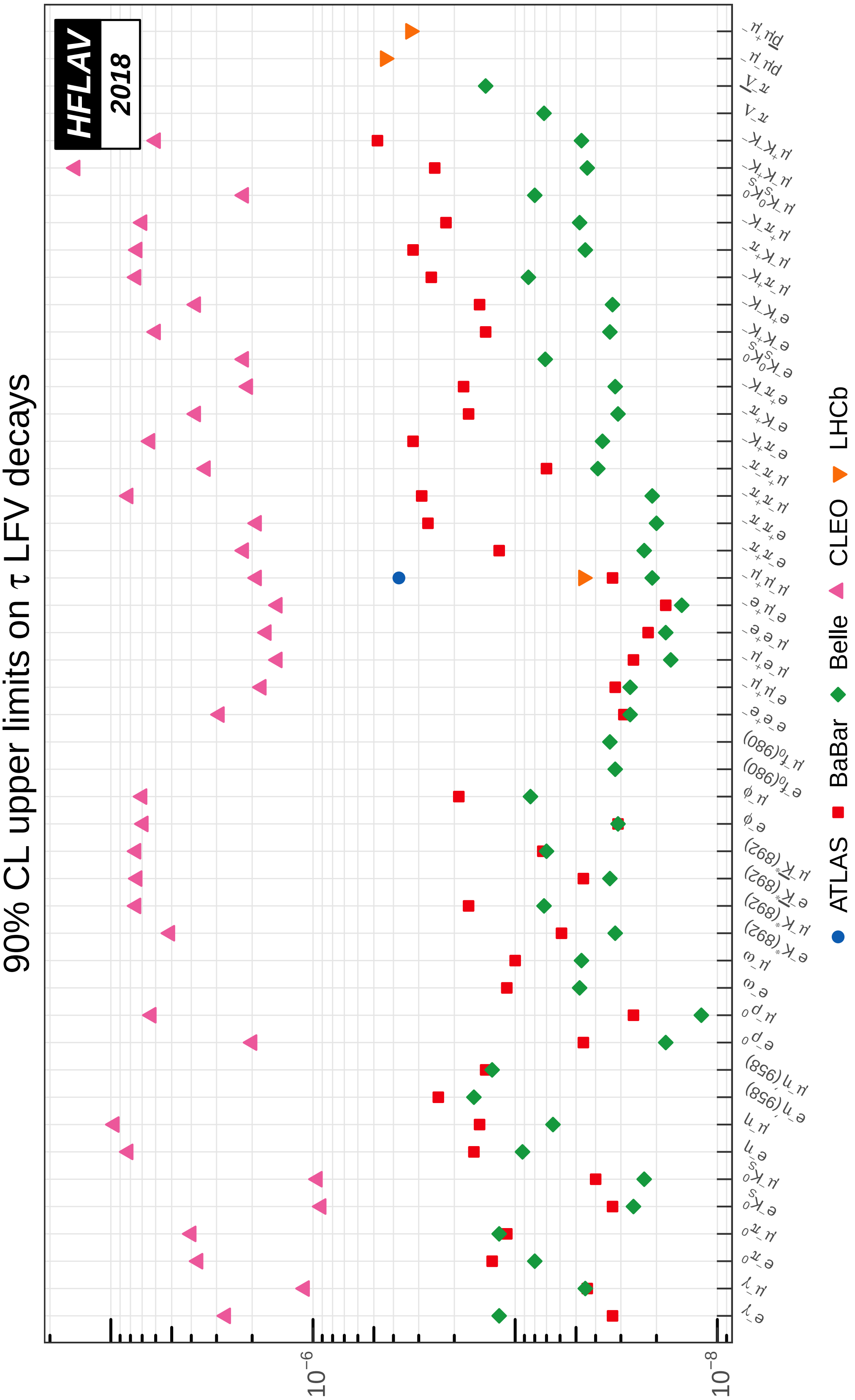

Figure 2: Tau lepton-flavour-violating branching fraction upper limits summary plot. In order to appreciate the physics reach improvement over time, the plot includes also the CLEO upper limits reported by PDG 2018 [6].

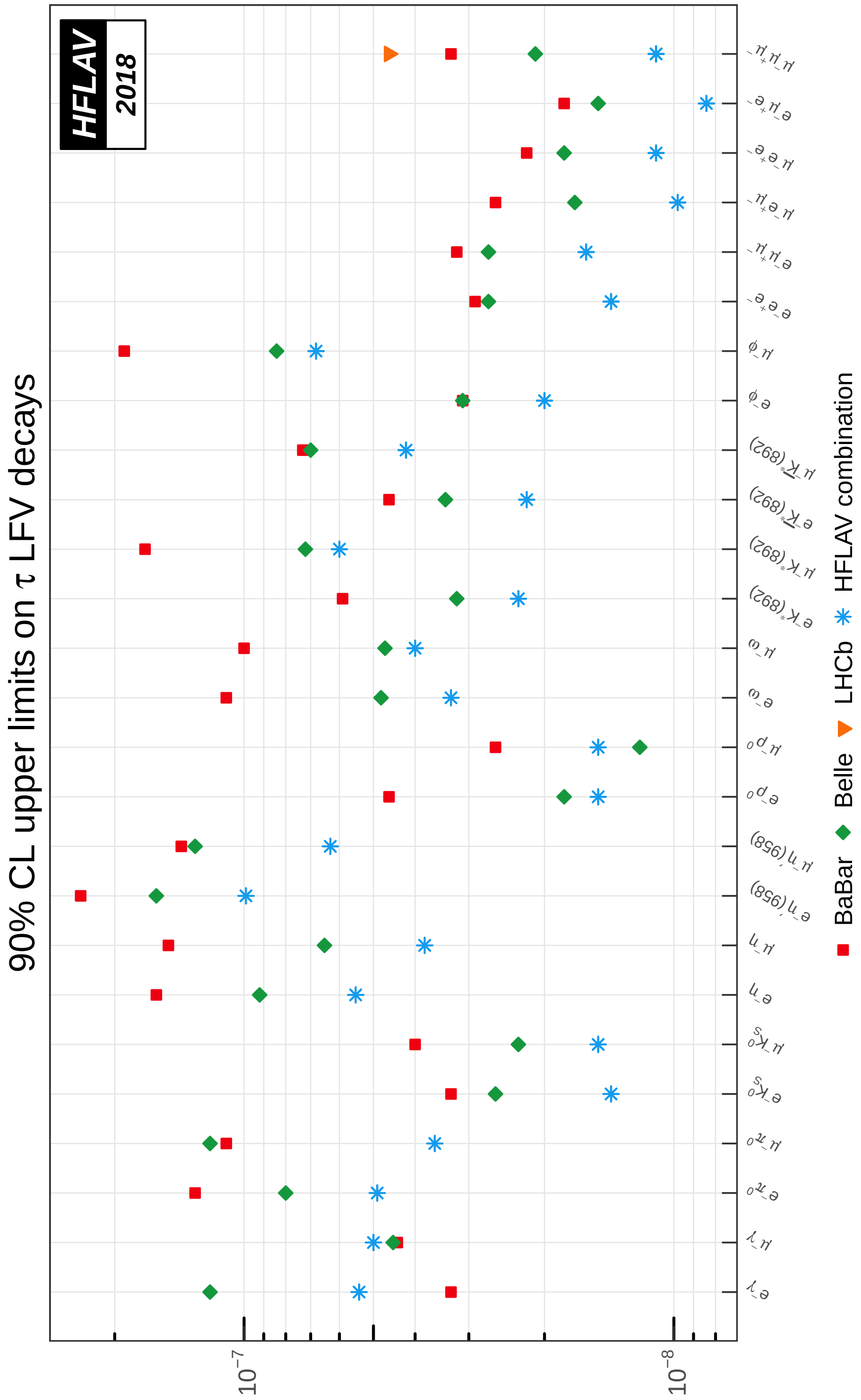

Figure 3: Tau lepton-flavour-violating branching fraction upper limits combinations summary plot. For each channel we report the HFLAV combined limit, and the experimental published limits. In some cases, the combined limit is weaker than the limit published by a single experiment. This arises since the CL$_s$ method used in the combination can be more conservative compared to other legitimate methods, especially when the number of observed events fluctuates below the expected background.

A Branching fractions fit measurement list by reference

Table 18 reports the measurements used for the HFLAV-Tau branching fraction fit grouped by their bibliographic reference.

Table 18: By-reference measurements list.

Reference / Branching Fraction	**Value**
BARATE 98 (ALEPH) [50]	
$\mathcal{B}_{85} = K^-\pi^+\pi^-\nu_\tau$ (ex. K^0)	$0.00214 \pm 0.00037 \pm 0.00029$
$\mathcal{B}_{88} = K^-\pi^-\pi^+\pi^0\nu_\tau$ (ex. K^0)	$0.00061 \pm 0.00039 \pm 0.00018$
$\mathcal{B}_{93} = \pi^-K^-K^+\nu_\tau$	$0.00163 \pm 0.00021 \pm 0.00017$
$\mathcal{B}_{94} = \pi^-K^-K^+\pi^0\nu_\tau$	$0.00075 \pm 0.00029 \pm 0.00015$
BARATE 98E (ALEPH) [28]	
$\mathcal{B}_{33} = K^0_S(\text{particles})^-\nu_\tau$	$0.0097 \pm 0.00058 \pm 0.00062$
$\mathcal{B}_{37} = K^-K^0\nu_\tau$	$0.00158 \pm 0.00042 \pm 0.00017$
$\mathcal{B}_{40} = \pi^-\overline{K}^0\pi^0\nu_\tau$	$0.00294 \pm 0.00073 \pm 0.00037$
$\mathcal{B}_{42} = K^-\pi^0K^0\nu_\tau$	$0.00152 \pm 0.00076 \pm 0.00021$
$\mathcal{B}_{47} = \pi^-K^0_SK^0_S\nu_\tau$	$0.00026 \pm 0.0001 \pm 5\cdot10^{-5}$
$\mathcal{B}_{48} = \pi^-K^0_SK^0_L\nu_\tau$	$0.00101 \pm 0.00023 \pm 0.00013$
$\mathcal{B}_{51} = \pi^-\pi^0K^0_SK^0_L\nu_\tau$	$(3.1 \pm 1.1 \pm 0.5)\cdot10^{-4}$
$\mathcal{B}_{53} = \overline{K}^0h^-h^-h^+\nu_\tau$	$0.00023 \pm 0.00019 \pm 0.00007$
BARATE 99K (ALEPH) [20]	
$\mathcal{B}_{10} = K^-\nu_\tau$	$0.00696 \pm 0.00025 \pm 0.00014$
$\mathcal{B}_{16} = K^-\pi^0\nu_\tau$	$0.00444 \pm 0.00026 \pm 0.00024$
$\mathcal{B}_{23} = K^-2\pi^0\nu_\tau$ (ex. K^0)	$0.00056 \pm 0.0002 \pm 0.00015$
$\mathcal{B}_{28} = K^-3\pi^0\nu_\tau$ (ex. K^0, η)	$0.00037 \pm 0.00021 \pm 0.00011$
$\mathcal{B}_{35} = \pi^-\overline{K}^0\nu_\tau$	$0.00928 \pm 0.00045 \pm 0.00034$
$\mathcal{B}_{37} = K^-K^0\nu_\tau$	$0.00162 \pm 0.00021 \pm 0.00011$
$\mathcal{B}_{40} = \pi^-\overline{K}^0\pi^0\nu_\tau$	$0.00347 \pm 0.00053 \pm 0.00037$
$\mathcal{B}_{42} = K^-\pi^0K^0\nu_\tau$	$0.00143 \pm 0.00025 \pm 0.00015$
BARATE 99R (ALEPH) [34]	
$\mathcal{B}_{44} = \pi^-\overline{K}^02\pi^0\nu_\tau$ (ex. K^0)	0.00026 ± 0.00024
BUSKULIC 96 (ALEPH) [64]	
$\dfrac{\mathcal{B}_{150}}{\mathcal{B}_{66}} = \dfrac{h^-\omega\nu_\tau}{h^-h^-h^+\pi^0\nu_\tau\ (\text{ex. } K^0)}$	0.431 ± 0.033
BUSKULIC 97C (ALEPH) [57]	
$\mathcal{B}_{126} = \pi^-\pi^0\eta\nu_\tau$	$0.0018 \pm 0.0004 \pm 0.0002$
$\mathcal{B}_{128} = K^-\eta\nu_\tau$	$(2.9^{+1.3\cdot10^{-4}}_{-1.2} \pm 0.7)\cdot10^{-4}$
$\mathcal{B}_{150} = h^-\omega\nu_\tau$	$0.0191 \pm 0.0007 \pm 0.0006$
$\mathcal{B}_{152} = h^-\pi^0\omega\nu_\tau$	$0.0043 \pm 0.0006 \pm 0.0005$
SCHAEL 05C (ALEPH) [7]	
$\mathcal{B}_{3} = \mu^-\overline{\nu}_\mu\nu_\tau$	$0.17319 \pm 0.0007 \pm 0.00032$
$\mathcal{B}_{5} = e^-\overline{\nu}_e\nu_\tau$	$0.17837 \pm 0.00072 \pm 0.00036$
$\mathcal{B}_{8} = h^-\nu_\tau$	$(11.524 \pm 0.070 \pm 0.078)\cdot10^{-2}$
$\mathcal{B}_{13} = h^-\pi^0\nu_\tau$	$(25.924 \pm 0.097 \pm 0.085)\cdot10^{-2}$
$\mathcal{B}_{19} = h^-2\pi^0\nu_\tau$ (ex. K^0)	$(9.295 \pm 0.084 \pm 0.088)\cdot10^{-2}$
$\mathcal{B}_{26} = h^-3\pi^0\nu_\tau$	$(1.08200 \pm 0.0709295 \pm 0.0594643)\cdot10^{-2}$
$\mathcal{B}_{30} = h^-4\pi^0\nu_\tau$ (ex. K^0, η)	$0.00112 \pm 0.00037 \pm 0.00035$
$\mathcal{B}_{58} = h^-h^-h^+\nu_\tau$ (ex. K^0, ω)	$0.09469 \pm 0.00062 \pm 0.00073$
$\mathcal{B}_{66} = h^-h^-h^+\pi^0\nu_\tau$ (ex. K^0)	$0.04734 \pm 0.00059 \pm 0.00049$

Table 18 – continued from previous page

Reference / Branching Fraction	Value
$\mathcal{B}_{76} = h^- h^- h^+ 2\pi^0 \nu_\tau$ (ex. K^0)	$0.00435 \pm 0.0003 \pm 0.00035$
$\mathcal{B}_{103} = 3h^- 2h^+ \nu_\tau$ (ex. K^0)	$0.00072 \pm 0.00009 \pm 0.00012$
$\mathcal{B}_{104} = 3h^- 2h^+ \pi^0 \nu_\tau$ (ex. K^0)	$(0.021 \pm 0.007 \pm 0.009) \cdot 10^{-2}$
$\mathcal{B}_{805} = a_1^- (\to \pi^- \gamma) \nu_\tau$	$(4 \pm 2) \cdot 10^{-4}$
ALBRECHT 88B (ARGUS) [55]	
$\mathcal{B}_{103} = 3h^- 2h^+ \nu_\tau$ (ex. K^0)	$0.00064 \pm 0.00023 \pm 0.0001$
ALBRECHT 92D (ARGUS) [11]	
$\frac{\mathcal{B}_3}{\mathcal{B}_5} = \frac{\mu^- \overline{\nu}_\mu \nu_\tau}{e^- \overline{\nu}_e \nu_\tau}$	$0.997 \pm 0.035 \pm 0.04$
AUBERT 08 (*BABAR*) [42]	
$\mathcal{B}_{60} = \pi^- \pi^+ \pi^- \nu_\tau$ (ex. K^0)	$0.0883 \pm 0.0001 \pm 0.0013$
$\mathcal{B}_{85} = K^- \pi^+ \pi^- \nu_\tau$ (ex. K^0)	$0.00273 \pm 2 \cdot 10^{-5} \pm 9 \cdot 10^{-5}$
$\mathcal{B}_{93} = \pi^- K^- K^+ \nu_\tau$	$0.001346 \pm 1 \cdot 10^{-5} \pm 3.6 \cdot 10^{-5}$
$\mathcal{B}_{96} = K^- K^- K^+ \nu_\tau$	$1.5777 \cdot 10^{-5} \pm 1.3 \cdot 10^{-6} \pm 1.2308 \cdot 10^{-6}$
AUBERT 10F (*BABAR*) [12]	
$\frac{\mathcal{B}_3}{\mathcal{B}_5} = \frac{\mu^- \overline{\nu}_\mu \nu_\tau}{e^- \overline{\nu}_e \nu_\tau}$	$0.9796 \pm 0.0016 \pm 0.0036$
$\frac{\mathcal{B}_9}{\mathcal{B}_5} = \frac{\pi^- \nu_\tau}{e^- \overline{\nu}_e \nu_\tau}$	$0.5945 \pm 0.0014 \pm 0.0061$
$\frac{\mathcal{B}_{10}}{\mathcal{B}_5} = \frac{K^- \nu_\tau}{e^- \overline{\nu}_e \nu_\tau}$	$0.03882 \pm 0.00032 \pm 0.00057$
DEL-AMO-SANCHEZ 11E (*BABAR*) [60]	
$\mathcal{B}_{128} = K^- \eta \nu_\tau$	$0.000142 \pm 1.1 \cdot 10^{-5} \pm 7 \cdot 10^{-6}$
LEES 12X (*BABAR*) [65]	
$\mathcal{B}_{811} = \pi^- 2\pi^0 \omega \nu_\tau$ (ex. K^0)	$(7.3 \pm 1.2 \pm 1.2) \cdot 10^{-5}$
$\mathcal{B}_{812} = 2\pi^- \pi^+ 3\pi^0 \nu_\tau$ (ex. K^0, η, ω, f_1)	$(0.1 \pm 0.08 \pm 0.30) \cdot 10^{-4}$
$\mathcal{B}_{821} = 3\pi^- 2\pi^+ \nu_\tau$ (ex. K^0, ω, f_1)	$(7.68 \pm 0.04 \pm 0.40) \cdot 10^{-4}$
$\mathcal{B}_{822} = K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	$(0.6 \pm 0.5 \pm 1.1) \cdot 10^{-6}$
$\mathcal{B}_{831} = 2\pi^- \pi^+ \omega \nu_\tau$ (ex. K^0)	$(8.4 \pm 0.4 \pm 0.6) \cdot 10^{-5}$
$\mathcal{B}_{832} = 3\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0, η, ω, f_1)	$(0.36 \pm 0.03 \pm 0.09) \cdot 10^{-4}$
$\mathcal{B}_{833} = K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	$(1.1 \pm 0.4 \pm 0.4) \cdot 10^{-6}$
$\mathcal{B}_{910} = 2\pi^- \pi^+ \eta \nu_\tau$ ($\eta \to 3\pi^0$) (ex. K^0)	$(8.27 \pm 0.88 \pm 0.81) \cdot 10^{-5}$
$\mathcal{B}_{911} = \pi^- 2\pi^0 \eta \nu_\tau$ ($\eta \to \pi^+ \pi^- \pi^0$) (ex. K^0)	$(4.57 \pm 0.77 \pm 0.50) \cdot 10^{-5}$
$\mathcal{B}_{920} = \pi^- f_1 \nu_\tau$ ($f_1 \to 2\pi^- 2\pi^+$)	$(5.20 \pm 0.31 \pm 0.37) \cdot 10^{-5}$
$\mathcal{B}_{930} = 2\pi^- \pi^+ \eta \nu_\tau$ ($\eta \to \pi^+ \pi^- \pi^0$) (ex. K^0)	$(5.39 \pm 0.27 \pm 0.41) \cdot 10^{-5}$
$\mathcal{B}_{944} = 2\pi^- \pi^+ \eta \nu_\tau$ ($\eta \to \gamma\gamma$) (ex. K^0)	$(8.26 \pm 0.35 \pm 0.51) \cdot 10^{-5}$
LEES 12Y (*BABAR*) [35]	
$\mathcal{B}_{47} = \pi^- K_S^0 K_S^0 \nu_\tau$	$(2.31 \pm 0.04 \pm 0.08) \cdot 10^{-4}$
$\mathcal{B}_{50} = \pi^- \pi^0 K_S^0 K_S^0 \nu_\tau$	$(1.60 \pm 0.20 \pm 0.22) \cdot 10^{-5}$
LEES 18B (*BABAR*) [3]	
$\mathcal{B}_{37} = K^- K^0 \nu_\tau$	$(14.78 \pm 0.22 \pm 0.40) \cdot 10^{-4}$
BABAR prelim. ICHEP2018 [4]	
$\mathcal{B}_{10} = K^- \nu_\tau$	$(7.17 \pm 0.031 \pm 0.21) \cdot 10^{-3}$
$\mathcal{B}_{16} = K^- \pi^0 \nu_\tau$	$(5.05 \pm 0.02 \pm 0.15) \cdot 10^{-3}$
$\mathcal{B}_{23} = K^- 2\pi^0 \nu_\tau$ (ex. K^0)	$(6.15 \pm 0.12 \pm 0.34) \cdot 10^{-4}$
$\mathcal{B}_{27} = \pi^- 3\pi^0 \nu_\tau$ (ex. K^0)	$(1.168 \pm 0.006 \pm 0.038) \cdot 10^{-2}$
$\mathcal{B}_{28} = K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	$(1.25 \pm 0.16 \pm 0.24) \cdot 10^{-4}$
$\mathcal{B}_{809} = \pi^- 4\pi^0 \nu_\tau$ (ex. K^0, η)	$(9.02 \pm 0.40 \pm 0.65) \cdot 10^{-4}$
FUJIKAWA 08 (Belle) [24]	

Table 18 – continued from previous page

Reference / Branching Fraction	Value
$\mathcal{B}_{13} = h^- \pi^0 \nu_\tau$	$0.2567 \pm 1 \cdot 10^{-4} \pm 0.0039$
INAMI 09 (Belle) [58]	
$\mathcal{B}_{126} = \pi^- \pi^0 \eta \nu_\tau$	$0.00135 \pm 3 \cdot 10^{-5} \pm 7 \cdot 10^{-5}$
$\mathcal{B}_{128} = K^- \eta \nu_\tau$	$0.000158 \pm 5 \cdot 10^{-6} \pm 9 \cdot 10^{-6}$
$\mathcal{B}_{130} = K^- \pi^0 \eta \nu_\tau$	$4.6 \cdot 10^{-5} \pm 1.1 \cdot 10^{-5} \pm 4 \cdot 10^{-6}$
$\mathcal{B}_{132} = \pi^- \overline{K}^0 \eta \nu_\tau$	$8.8 \cdot 10^{-5} \pm 1.4 \cdot 10^{-5} \pm 6 \cdot 10^{-6}$
LEE 10 (Belle) [43]	
$\mathcal{B}_{60} = \pi^- \pi^+ \pi^- \nu_\tau$ (ex. K^0)	$0.0842 \pm 0^{+0.0026}_{-0.0025}$
$\mathcal{B}_{85} = K^- \pi^+ \pi^- \nu_\tau$ (ex. K^0)	$0.0033 \pm 1 \cdot 10^{-5}{}^{+0.00016}_{-0.00017}$
$\mathcal{B}_{93} = \pi^- K^- K^+ \nu_\tau$	$0.00155 \pm 1 \cdot 10^{-5}{}^{+6 \cdot 10^{-5}}_{-5 \cdot 10^{-5}}$
$\mathcal{B}_{96} = K^- K^- K^+ \nu_\tau$	$3.29 \cdot 10^{-5} \pm 1.7 \cdot 10^{-6}{}^{+1.9 \cdot 10^{-6}}_{-2.0 \cdot 10^{-6}}$
RYU 14vpc (Belle) [31]	
$\mathcal{B}_{35} = \pi^- \overline{K}^0 \nu_\tau$	$8.32 \cdot 10^{-3} \pm 0.3\% \pm 1.8\%$
$\mathcal{B}_{37} = K^- K^0 \nu_\tau$	$14.8 \cdot 10^{-4} \pm 0.9\% \pm 3.7\%$
$\mathcal{B}_{40} = \pi^- \overline{K}^0 \pi^0 \nu_\tau$	$3.86 \cdot 10^{-3} \pm 0.8\% \pm 3.5\%$
$\mathcal{B}_{42} = K^- \pi^0 K^0 \nu_\tau$	$14.96 \cdot 10^{-4} \pm 1.3\% \pm 4.9\%$
$\mathcal{B}_{47} = \pi^- K_S^0 K_S^0 \nu_\tau$	$2.33 \cdot 10^{-4} \pm 1.4\% \pm 4.0\%$
$\mathcal{B}_{50} = \pi^- \pi^0 K_S^0 K_S^0 \nu_\tau$	$2.00 \cdot 10^{-5} \pm 10.8\% \pm 10.1\%$
BEHREND 89B (CELLO) [36]	
$\mathcal{B}_{54} = h^- h^- h^+ \geq 0\,\text{neutrals} \geq 0\, K_L^0\, \nu_\tau$	$0.15 \pm 0.004 \pm 0.003$
ANASTASSOV 01 (CLEO) [47]	
$\mathcal{B}_{78} = h^- h^- h^+ 3\pi^0 \nu_\tau$	$0.00022 \pm 3 \cdot 10^{-5} \pm 4 \cdot 10^{-5}$
$\mathcal{B}_{104} = 3h^- 2h^+ \pi^0 \nu_\tau$ (ex. K^0)	$0.00017 \pm 2 \cdot 10^{-5} \pm 2 \cdot 10^{-5}$
ANASTASSOV 97 (CLEO) [13]	
$\frac{\mathcal{B}_3}{\mathcal{B}_5} = \frac{\mu^- \overline{\nu}_\mu \nu_\tau}{e^- \overline{\nu}_e \nu_\tau}$	$0.9777 \pm 0.0063 \pm 0.0087$
$\mathcal{B}_5 = e^- \overline{\nu}_e \nu_\tau$	$0.1776 \pm 0.0006 \pm 0.0017$
$\mathcal{B}_8 = h^- \nu_\tau$	$0.1152 \pm 0.0005 \pm 0.0012$
ARTUSO 92 (CLEO) [59]	
$\mathcal{B}_{126} = \pi^- \pi^0 \eta \nu_\tau$	$0.0017 \pm 0.0002 \pm 0.0002$
ARTUSO 94 (CLEO) [25]	
$\mathcal{B}_{13} = h^- \pi^0 \nu_\tau$	$0.2587 \pm 0.0012 \pm 0.0042$
BALEST 95C (CLEO) [41]	
$\mathcal{B}_{57} = h^- h^- h^+ \nu_\tau$ (ex. K^0)	$0.0951 \pm 0.0007 \pm 0.002$
$\mathcal{B}_{66} = h^- h^- h^+ \pi^0 \nu_\tau$ (ex. K^0)	$0.0423 \pm 0.0006 \pm 0.0022$
$\frac{\mathcal{B}_{150}}{\mathcal{B}_{66}} = \frac{h^- \omega \nu_\tau}{h^- h^- h^+ \pi^0 \nu_\tau \text{ (ex. } K^0\text{)}}$	$0.464 \pm 0.016 \pm 0.017$
BARINGER 87 (CLEO) [63]	
$\mathcal{B}_{150} = h^- \omega \nu_\tau$	$0.016 \pm 0.0027 \pm 0.0041$
BARTELT 96 (CLEO) [61]	
$\mathcal{B}_{128} = K^- \eta \nu_\tau$	$(2.6 \pm 0.5 \pm 0.5) \cdot 10^{-4}$
BATTLE 94 (CLEO) [21]	
$\mathcal{B}_{10} = K^- \nu_\tau$	$0.0066 \pm 0.0007 \pm 0.0009$
$\mathcal{B}_{16} = K^- \pi^0 \nu_\tau$	$0.0051 \pm 0.001 \pm 0.0007$
$\mathcal{B}_{23} = K^- 2\pi^0 \nu_\tau$ (ex. K^0)	$0.0009 \pm 0.001 \pm 0.0003$
$\mathcal{B}_{31} = K^- \geq 0\,\pi^0 \geq 0\, K^0 \geq 0\,\gamma \nu_\tau$	$0.017 \pm 0.0012 \pm 0.0019$
BISHAI 99 (CLEO) [62]	
$\mathcal{B}_{130} = K^- \pi^0 \eta \nu_\tau$	$(1.77 \pm 0.56 \pm 0.71) \cdot 10^{-4}$

Table 18 – continued from previous page

Reference / Branching Fraction	Value
$\mathcal{B}_{132} = \pi^- \overline{K}^0 \eta \nu_\tau$	$(2.2 \pm 0.70 \pm 0.22) \cdot 10^{-4}$
BORTOLETTO 93 (CLEO) [46]	
$\frac{\mathcal{B}_{76}}{\mathcal{B}_{54}} = \frac{h^- h^- h^+ 2\pi^0 \nu_\tau \text{ (ex. } K^0)}{h^- h^- h^+ \geq 0 \text{ neutrals} \geq 0\, K_L^0\, \nu_\tau}$	$0.034 \pm 0.002 \pm 0.003$
$\frac{\mathcal{B}_{152}}{\mathcal{B}_{76}} = \frac{h^- \omega \pi^0 \nu_\tau}{h^- h^- h^+ 2\pi^0 \nu_\tau \text{ (ex. } K^0)}$	$0.81 \pm 0.06 \pm 0.06$
COAN 96 (CLEO) [30]	
$\mathcal{B}_{34} = h^- \overline{K}^0 \nu_\tau$	$0.00855 \pm 0.00036 \pm 0.00073$
$\mathcal{B}_{37} = K^- K^0 \nu_\tau$	$0.00151 \pm 0.00021 \pm 0.00022$
$\mathcal{B}_{39} = h^- \overline{K}^0 \pi^0 \nu_\tau$	$0.00562 \pm 0.0005 \pm 0.00048$
$\mathcal{B}_{42} = K^- \pi^0 K^0 \nu_\tau$	$0.00145 \pm 0.00036 \pm 0.0002$
$\mathcal{B}_{47} = \pi^- K_S^0 K_S^0 \nu_\tau$	$0.00023 \pm 5 \cdot 10^{-5} \pm 3 \cdot 10^{-5}$
EDWARDS 00A (CLEO) [45]	
$\mathcal{B}_{69} = \pi^- \pi^+ \pi^- \pi^0 \nu_\tau \text{ (ex. } K^0)$	$0.0419 \pm 0.001 \pm 0.0021$
GIBAUT 94B (CLEO) [53]	
$\mathcal{B}_{102} = 3h^- 2h^+ \geq 0 \text{ neutrals}\, \nu_\tau \text{ (ex. } K^0)$	$0.00097 \pm 5 \cdot 10^{-5} \pm 0.00011$
$\mathcal{B}_{103} = 3h^- 2h^+ \nu_\tau \text{ (ex. } K^0)$	$0.00077 \pm 5 \cdot 10^{-5} \pm 9 \cdot 10^{-5}$
PROCARIO 93 (CLEO) [27]	
$\frac{\mathcal{B}_{19}}{\mathcal{B}_{13}} = \frac{h^- 2\pi^0 \nu_\tau \text{ (ex. } K^0)}{h^- \pi^0 \nu_\tau}$	$0.342 \pm 0.006 \pm 0.016$
$\frac{\mathcal{B}_{26}}{\mathcal{B}_{13}} = \frac{h^- 3\pi^0 \nu_\tau}{h^- \pi^0 \nu_\tau}$	$0.044 \pm 0.003 \pm 0.005$
$\mathcal{B}_{29} = h^- 4\pi^0 \nu_\tau \text{ (ex. } K^0)$	$0.0016 \pm 0.0005 \pm 0.0005$
RICHICHI 99 (CLEO) [48]	
$\frac{\mathcal{B}_{80}}{\mathcal{B}_{60}} = \frac{K^- \pi^- h^+ \nu_\tau \text{ (ex. } K^0)}{\pi^- \pi^+ \pi^- \nu_\tau \text{ (ex. } K^0)}$	$0.0544 \pm 0.0021 \pm 0.0053$
$\frac{\mathcal{B}_{81}}{\mathcal{B}_{69}} = \frac{K^- \pi^- h^+ \pi^0 \nu_\tau \text{ (ex. } K^0)}{\pi^- \pi^+ \pi^- \pi^0 \nu_\tau \text{ (ex. } K^0)}$	$0.0261 \pm 0.0045 \pm 0.0042$
$\frac{\mathcal{B}_{93}}{\mathcal{B}_{60}} = \frac{\pi^- K^- K^+ \nu_\tau}{\pi^- \pi^+ \pi^- \nu_\tau \text{ (ex. } K^0)}$	$0.016 \pm 0.0015 \pm 0.003$
$\frac{\mathcal{B}_{94}}{\mathcal{B}_{69}} = \frac{\pi^- K^- K^+ \pi^0 \nu_\tau}{\pi^- \pi^+ \pi^- \pi^0 \nu_\tau \text{ (ex. } K^0)}$	$0.0079 \pm 0.0044 \pm 0.0016$
ARMS 05 (CLEO3) [51]	
$\mathcal{B}_{88} = K^- \pi^- \pi^+ \pi^0 \nu_\tau \text{ (ex. } K^0)$	$0.00074 \pm 8 \cdot 10^{-5} \pm 0.00011$
$\mathcal{B}_{94} = \pi^- K^- K^+ \pi^0 \nu_\tau$	$(5.5 \pm 1.4 \pm 1.2) \cdot 10^{-5}$
$\mathcal{B}_{151} = K^- \omega \nu_\tau$	$(4.1 \pm 0.6 \pm 0.7) \cdot 10^{-4}$
BRIERE 03 (CLEO3) [44]	
$\mathcal{B}_{60} = \pi^- \pi^+ \pi^- \nu_\tau \text{ (ex. } K^0)$	$0.0913 \pm 0.0005 \pm 0.0046$
$\mathcal{B}_{85} = K^- \pi^+ \pi^- \nu_\tau \text{ (ex. } K^0)$	$0.00384 \pm 0.00014 \pm 0.00038$
$\mathcal{B}_{93} = \pi^- K^- K^+ \nu_\tau$	$0.00155 \pm 6 \cdot 10^{-5} \pm 9 \cdot 10^{-5}$
ABDALLAH 06A (DELPHI) [18]	
$\mathcal{B}_{8} = h^- \nu_\tau$	$0.11571 \pm 0.0012 \pm 0.00114$
$\mathcal{B}_{13} = h^- \pi^0 \nu_\tau$	$0.2574 \pm 0.00201 \pm 0.00138$
$\mathcal{B}_{19} = h^- 2\pi^0 \nu_\tau \text{ (ex. } K^0)$	$0.09498 \pm 0.0032 \pm 0.00275$
$\mathcal{B}_{25} = h^- \geq 3\pi^0\, \nu_\tau \text{ (ex. } K^0)$	$0.01403 \pm 0.00214 \pm 0.00224$
$\mathcal{B}_{57} = h^- h^- h^+ \nu_\tau \text{ (ex. } K^0)$	$0.09317 \pm 0.0009 \pm 0.00082$
$\mathcal{B}_{66} = h^- h^- h^+ \pi^0 \nu_\tau \text{ (ex. } K^0)$	$0.04545 \pm 0.00106 \pm 0.00103$
$\mathcal{B}_{74} = h^- h^- h^+ \geq 2\pi^0\, \nu_\tau \text{ (ex. } K^0)$	$0.00561 \pm 0.00068 \pm 0.00095$
$\mathcal{B}_{103} = 3h^- 2h^+ \nu_\tau \text{ (ex. } K^0)$	$0.00097 \pm 0.00015 \pm 5 \cdot 10^{-5}$
$\mathcal{B}_{104} = 3h^- 2h^+ \pi^0 \nu_\tau \text{ (ex. } K^0)$	$0.00016 \pm 0.00012 \pm 6 \cdot 10^{-5}$
ABREU 92N (DELPHI) [15]	

Table 18 – continued from previous page

Reference / Branching Fraction	**Value**
$\mathcal{B}_7 = h^- \geq 0\, K_L^0\, \nu_\tau$	$0.124 \pm 0.007 \pm 0.007$
ABREU 94K (DELPHI) [22]	
$\mathcal{B}_{10} = K^- \nu_\tau$	0.0085 ± 0.0018
$\mathcal{B}_{31} = K^- \geq 0\, \pi^0 \geq 0\, K^0 \geq 0\, \gamma \nu_\tau$	0.0154 ± 0.0024
ABREU 99X (DELPHI) [8]	
$\mathcal{B}_3 = \mu^- \overline{\nu}_\mu \nu_\tau$	$0.17325 \pm 0.00095 \pm 0.00077$
$\mathcal{B}_5 = e^- \overline{\nu}_e \nu_\tau$	$0.17877 \pm 0.00109 \pm 0.0011$
BYLSMA 87 (HRS) [54]	
$\mathcal{B}_{102} = 3h^- 2h^+ \geq 0\,\text{neutrals}\, \nu_\tau$ (ex. K^0)	0.00102 ± 0.00029
$\mathcal{B}_{103} = 3h^- 2h^+ \nu_\tau$ (ex. K^0)	0.00051 ± 0.0002
ACCIARRI 01F (L3) [9]	
$\mathcal{B}_3 = \mu^- \overline{\nu}_\mu \nu_\tau$	$0.17342 \pm 0.0011 \pm 0.00067$
$\mathcal{B}_5 = e^- \overline{\nu}_e \nu_\tau$	$0.17806 \pm 0.00104 \pm 0.00076$
ACCIARRI 95 (L3) [16]	
$\mathcal{B}_7 = h^- \geq 0\, K_L^0\, \nu_\tau$	$0.1247 \pm 0.0026 \pm 0.0043$
$\mathcal{B}_{13} = h^- \pi^0 \nu_\tau$	$0.2505 \pm 0.0035 \pm 0.005$
$\mathcal{B}_{19} = h^- 2\pi^0 \nu_\tau$ (ex. K^0)	$0.0888 \pm 0.0037 \pm 0.0042$
$\mathcal{B}_{26} = h^- 3\pi^0 \nu_\tau$	$0.017 \pm 0.0024 \pm 0.0038$
ACCIARRI 95F (L3) [32]	
$\mathcal{B}_{35} = \pi^- \overline{K}^0 \nu_\tau$	$0.0095 \pm 0.0015 \pm 0.0006$
$\mathcal{B}_{40} = \pi^- \overline{K}^0 \pi^0 \nu_\tau$	$0.0041 \pm 0.0012 \pm 0.0003$
ACHARD 01D (L3) [39]	
$\mathcal{B}_{55} = h^- h^- h^+ \geq 0\,\text{neutrals}\, \nu_\tau$ (ex. K^0)	$0.14556 \pm 0.00105 \pm 0.00076$
$\mathcal{B}_{102} = 3h^- 2h^+ \geq 0\,\text{neutrals}\, \nu_\tau$ (ex. K^0)	$0.0017 \pm 0.00022 \pm 0.00026$
ADEVA 91F (L3) [37]	
$\mathcal{B}_{54} = h^- h^- h^+ \geq 0\,\text{neutrals} \geq 0\, K_L^0\, \nu_\tau$	$0.144 \pm 0.006 \pm 0.003$
ABBIENDI 00C (OPAL) [33]	
$\mathcal{B}_{35} = \pi^- \overline{K}^0 \nu_\tau$	$0.00933 \pm 0.00068 \pm 0.00049$
$\mathcal{B}_{38} = K^- K^0 \geq 0\, \pi^0\, \nu_\tau$	$0.0033 \pm 0.00055 \pm 0.00039$
$\mathcal{B}_{43} = \pi^- \overline{K}^0 \geq 1\, \pi^0\, \nu_\tau$	$0.00324 \pm 0.00074 \pm 0.00066$
ABBIENDI 00D (OPAL) [52]	
$\mathcal{B}_{92} = \pi^- K^- K^+ \geq 0\,\text{neutrals}\, \nu_\tau$	$0.00159 \pm 0.00053 \pm 0.0002$
ABBIENDI 01J (OPAL) [23]	
$\mathcal{B}_{10} = K^- \nu_\tau$	$0.00658 \pm 0.00027 \pm 0.00029$
$\mathcal{B}_{31} = K^- \geq 0\, \pi^0 \geq 0\, K^0 \geq 0\, \gamma \nu_\tau$	$0.01528 \pm 0.00039 \pm 0.0004$
ABBIENDI 03 (OPAL) [10]	
$\mathcal{B}_3 = \mu^- \overline{\nu}_\mu \nu_\tau$	$0.1734 \pm 0.0009 \pm 0.0006$
ABBIENDI 04J (OPAL) [26]	
$\mathcal{B}_{16} = K^- \pi^0 \nu_\tau$	$0.00471 \pm 0.00059 \pm 0.00023$
$\mathcal{B}_{85} = K^- \pi^+ \pi^- \nu_\tau$ (ex. K^0)	$0.00415 \pm 0.00053 \pm 0.0004$
ABBIENDI 99H (OPAL) [14]	
$\mathcal{B}_5 = e^- \overline{\nu}_e \nu_\tau$	$0.1781 \pm 0.0009 \pm 0.0006$
ACKERSTAFF 98M (OPAL) [19]	
$\mathcal{B}_8 = h^- \nu_\tau$	$0.1198 \pm 0.0013 \pm 0.0016$
$\mathcal{B}_{13} = h^- \pi^0 \nu_\tau$	$0.2589 \pm 0.0017 \pm 0.0029$
$\mathcal{B}_{17} = h^- \geq 2\, \pi^0\, \nu_\tau$	$0.0991 \pm 0.0031 \pm 0.0027$

Table 18 – continued from previous page

Reference / Branching Fraction	Value
ACKERSTAFF 99E (OPAL) [56]	
$\mathcal{B}_{103} = 3h^- 2h^+ \nu_\tau$ (ex. K^0)	$0.00091 \pm 0.00014 \pm 6 \cdot 10^{-5}$
$\mathcal{B}_{104} = 3h^- 2h^+ \pi^0 \nu_\tau$ (ex. K^0)	$0.00027 \pm 0.00018 \pm 9 \cdot 10^{-5}$
AKERS 94G (OPAL) [29]	
$\mathcal{B}_{33} = K^0_S(\text{particles})^- \nu_\tau$	$0.0097 \pm 0.0009 \pm 0.0006$
AKERS 95Y (OPAL) [40]	
$\mathcal{B}_{55} = h^- h^- h^+ \geq 0\,\text{neutrals}\,\nu_\tau$ (ex. K^0)	$0.1496 \pm 0.0009 \pm 0.0022$
$\frac{\mathcal{B}_{57}}{\mathcal{B}_{55}} = \frac{h^- h^- h^+ \nu_\tau \text{ (ex. } K^0)}{h^- h^- h^+ \geq 0\,\text{neutrals}\,\nu_\tau \text{ (ex. } K^0)}$	$0.66 \pm 0.004 \pm 0.014$
ALEXANDER 91D (OPAL) [17]	
$\mathcal{B}_7 = h^- \geq 0\, K^0_L\, \nu_\tau$	$0.121 \pm 0.007 \pm 0.005$
AIHARA 87B (TPC) [38]	
$\mathcal{B}_{54} = h^- h^- h^+ \geq 0\,\text{neutrals} \geq 0\, K^0_L\, \nu_\tau$	$0.151 \pm 0.008 \pm 0.006$
BAUER 94 (TPC) [49]	
$\mathcal{B}_{82} = K^- \pi^- \pi^+ \geq 0\,\text{neutrals}\,\nu_\tau$	$0.0058^{+0.0015}_{-0.0013} \pm 0.0012$
$\mathcal{B}_{92} = \pi^- K^- K^+ \geq 0\,\text{neutrals}\,\nu_\tau$	$0.0015^{+0.0009}_{-0.0007} \pm 0.0003$

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