Choice of Shape Function parameters for ICHEP-04

1 Reinterpretation of M_X analysis

In our previous publication [1] we were varying the Shape Function parameters [2] according to the results of a fit to $b \to c\ell\nu$ moments. There has been general consensus [3, 4] that this is not appropriate beyond the leading order to perturbation theory[1].

One of these publications [3] was also suggesting an alternative model which could be valid at higher orders and that would have allowed to use the parameters as extracted from the high statistics $b \to c\ell\nu$ measurements. Unfortunately this model is not usable over the whole phase space and it cannot be used as is to build an alternative MC [5]. We are, for the time being, not going to use this model for the simulation, but we are going to provide Branching Fractions in particular phase space regions so that theorists can turn the result into $|V_{ub}|$ themselves.

The other possible source of information on the Shape Function parameters are the $b \to s\gamma$ events, in particular their energy spectrum. CLEO [6] has provided a spectrum together with a constraint on the SF parameters (in the following called $\bar{\Lambda}^{SF}$and λ_1^{SF}). The $\Delta\chi^2 = 1$ points are in table 1 and can approximated by the empirical function

$$\chi^2 = (\frac{\lambda_1^{SF} + 2.4(\bar{\Lambda}^{SF})^2 - 0.35}{0.15})^2 + (\frac{(\bar{\Lambda}^{SF})^2 - 0.42}{0.22})^2 = 1 \tag{1}$$

(see yellow contours in Fig. 1a).

COULD WE GET OFFICIAL POINTS/CURVE FROM CLEO?

Considering also the new central value suggested by CLEO's data, λ_1^{SF}=-0.342 and $\bar{\Lambda}^{SF}$=0.545, the updated result is

$$R_{u/sl} = 0.0234 \pm 0.0027(\text{stat.}) \pm 0.0024(\text{sys.})^{+0.0064}_{-0.0038}(\text{theo.}) \tag{2}$$

which would translate into

$$|V_{ub}| = (4.93 \pm 0.29(stat) \pm (sys)^{+0.63}_{-0.40}(SF) \pm 0.25(pert + 1/mb^3))10^{-3} \tag{3}$$

[1] None of these papers have beenpublished to journal yet

m_b	λ_1^{SF}	$R_{u/sl}\pm$ (stat.) $\pm$ (MC stat.)
4.845	-0.16	0.019661 $\pm$ 0.00233284 $\pm$ 0.000834991
4.48	-1.22	0.029853 $\pm$ 0.00344077 $\pm$ 0.00127403
4.785	-0.34	0.0213843 $\pm$ 0.00252085 $\pm$ 0.000904662
4.785	-0.16	0.0218303 $\pm$ 0.00256934 $\pm$ 0.00092277
4.735	-0.47	0.0227531 $\pm$ 0.00267045 $\pm$ 0.000961654
4.69	-0.62	0.0238296 $\pm$ 0.0027886 $\pm$ 0.0010081
4.69	-0.342	0.0255003 $\pm$ 0.00296272 $\pm$ 0.00107196
4.58	-0.95	0.0267376 $\pm$ 0.00310526 $\pm$ 0.00113494
4.58	-0.69	0.0286639 $\pm$ 0.00330392 $\pm$ 0.0012105
4.53	-1.08	0.0282657 $\pm$ 0.00326975 $\pm$ 0.00120234

Table 1: Measurement of $R_{u/sl}$ for several values of the shape function parameters corresponding to a $\Delta\chi^2 = 1$ contour in the fit to CLEO data.

consistent within the Shape Function (SF) errors with the published $|V_{ub}| = (4.62 \pm 0.28(stat.) \pm 0.28(sys) \pm 0.40(SF) \pm 0.26(pert + 1/mb^3))10^{-3}$ but with a larger upper uncertainty. This error from SF parameters is universally acknowledged to be conservative, but comprising any possible effect. In particular we are neglecting the error from the shape of the parameterization.

2 Improved SF parameters

There are other possible approaches to the Shape Function parameters determination which are accepted often only by part of the community:

- exploiting the information from $b \to c\ell\nu$ only partially, namely taking $\bar{\Lambda}^{SF}$ from OPE fits. This seems quite natural because it is difficult to build a model where the m_b parameter in OPE fits and charmless decays are different, but there is no consensus on this.

  WHAT ARE THE CURRENT FEELINGS ON THIS?

- extracting the SF parameters from $b \to u\ell\nu$ events themselves. This would be the cleanest way to proceed but the study is still being finalized

- use all available information on $b \to s\gamma$ decays available on the market.

We are currently concentrating on the last item. In particular we are interested in utilizing the latest moments of the photon energy as measured from Belle.

To interpret the measurement of the photon energy moments we compute at generator level the first two moments ($M1_{th}$ and $M2_{th}$) and the branching fractions (BR) for the inclusive $b \to s\gamma$ decays using the Kagan-Neubert model with the exponential Shape Function which corresponds to the deFazio-Neubert one. We redo the calculation for a grid in $\bar{\Lambda}^{SF}$and λ_1^{SF}. We consider only events where E_γ (in the B meson frame) is greater than the cut applied in the analysis. This will give the prediction of the moments in the inclusive spectrum. Fig. 2 shows the dependence of these moments on $m_b = m_B - \bar{\Lambda}^{SF}$ and $\lambda_1^{SF}$for a cutoff of 1.815 GeV.

Since there is some ongoing discussion on whether the exclusive $B \to K^*\gamma$ component should be considered separately or not we also perform the moments calculation with a lower cut on m_X such that the

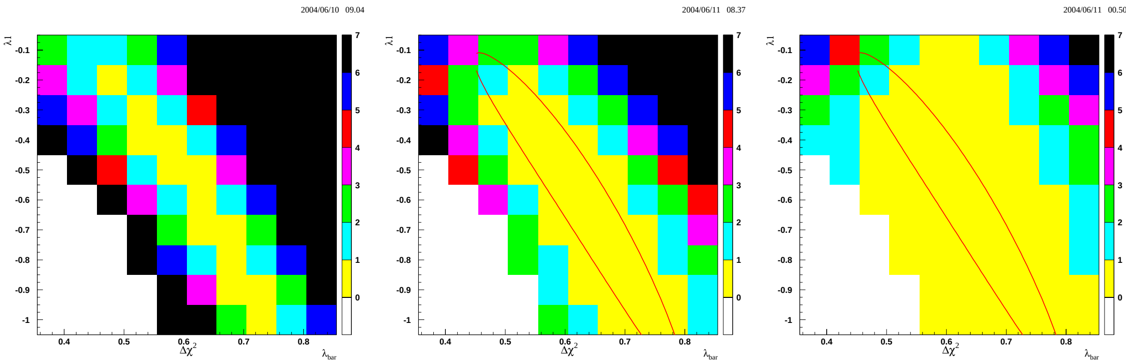

Figure 1: $\Delta\chi^2 = 1$ regions (in yellow) in the $\bar{\Lambda}^{SF}$-λ_1^{SF}plane for the CLEO analysis of their photon energy spectrum (left) and for the moments analysis discussed here (center). The right plot shows what happens replacing the exclusive contributions with their value(see text).The red ellipse is the analytic contour of the left plot, drawn for comparison.

removed BF is equal to the measured $B \to K^*\gamma$ BF (0.000039). The moments and BF computed in this way are called $< E_\gamma >_{in}$, $V[E_\gamma]_{in}$ and BR_{in}. We then compute at generator level the moments in the exclusive sample ($< E_\gamma >_{ex} = 2.561\,\mathrm{GeV}$ and $V[E_\gamma]_{ex} = 0.009\,\mathrm{GeV}^2$) and assume a fraction $f = 1/(1 + \frac{BR_{in}}{0.000039})$ of exclusive decays in the sample. We can then compute the expected moments for each value of $\bar{\Lambda}^{SF}$and λ_1^{SF}as:

$$M1_{th} = f < E_\gamma >_{ex} + (1-f) < E_\gamma >_{in} \tag{4}$$
$$M2_{th} = fV[E_\gamma]_{ex} + (1-f) * V[E_\gamma]_{in} + f * (1-f) * (< E_\gamma >_{ex} - < E_\gamma >_{in})^2$$

In both cases, given the theoretical expectations ($M1_{th}$ and $M2_{th}$) we build, in each $\bar{\Lambda}^{SF}$-λ_1^{SF}point a

$$\chi^2 = (\frac{< E_\gamma > - M1_{th}}{\sigma_{<>}})^2 + (\frac{V[E_\gamma] - M2_{th}}{\sigma_V})^2 \tag{5}$$

and consider as error ellipses the corresponding $\Delta\chi^2 = 1$ regions.

I AM A BIT CONFUSED WHY DO WE PICK UP $\Delta\chi^2 = 1$ REGIONS? THEY DON'T CORRESPOND TO 68% C.L. !

To see what is the sensitivity of the moments analysis compared to a study of the overall shape we applied this method to the results for the photon energy moments published by CLEO ($< E_\gamma > = 2.346 \pm 0.034\,\mathrm{GeV}$ and $V[E_\gamma] = 0.0226 \pm 0.0069\,\mathrm{GeV}^2$ [9]) and compared them to the $\Delta\chi^2 = 1$ region fitted by CLEO with the full shape (see fig. 1b). The moments analysis is a bit less sensitive, but not inconsistent, and we will therefore utilize the contour obtained from the full shape. Fig. 1c shows what happens if one considers the exclusive modes separately. There a clear loss in sensitivity. Also it looks like it compares less with the CLEO constraint, which seems to indicate that CLEO's method is closer to the one ignoring exclusive contributions.

IS THERE AGREEMENT ON THE FACT THAT THE EXCLUSIVE COMPONENT SHOULD NOT BE SEPARATED OUT?

We want to apply this technique to the moments as measured by Belle: $< E_\gamma > = 2.292 \pm 0.042\,\mathrm{GeV}$ and $V[E_\gamma] = 0.0305 \pm 0.0097\,\mathrm{GeV}^2$. They are measured with a cut on the photon energy reconstructed

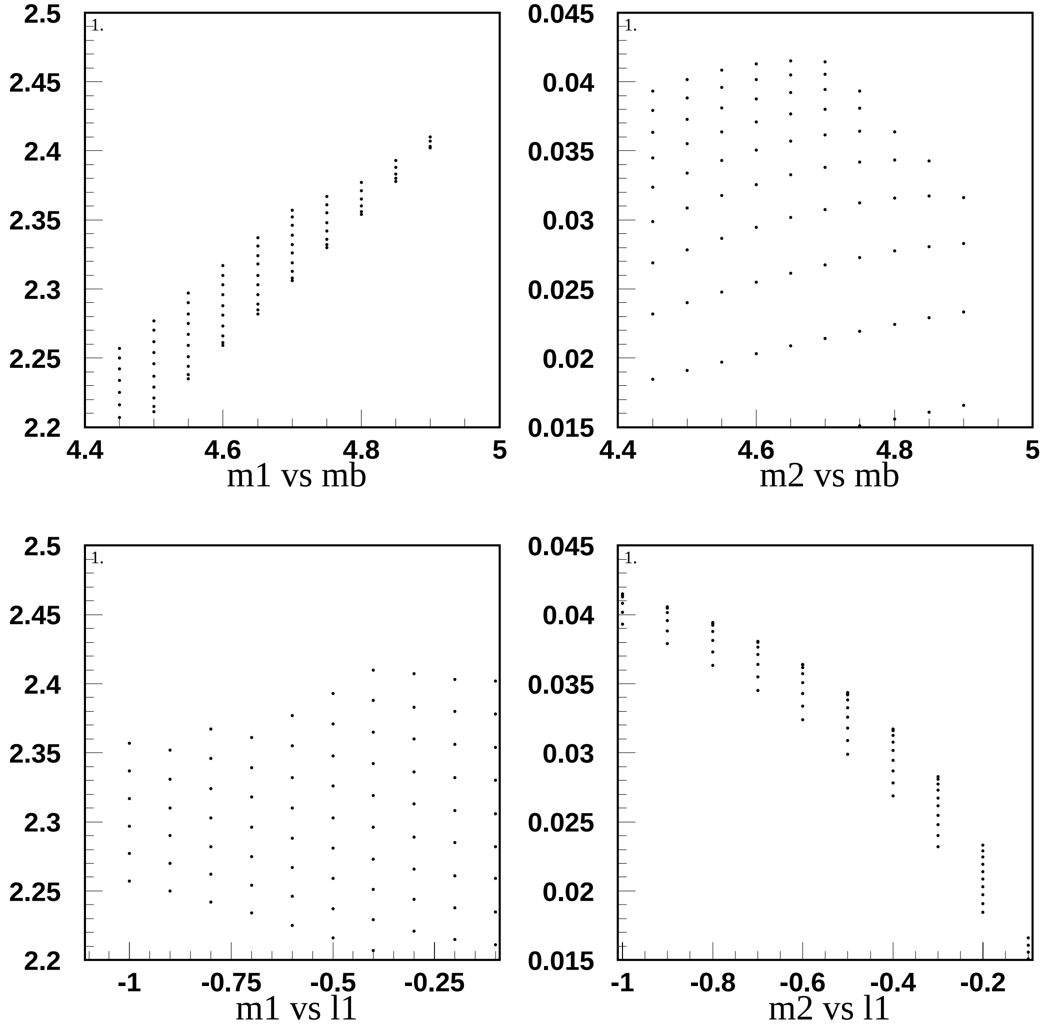

Figure 2: Dependence of the first and second moment on $m_b = m_B - \bar{\Lambda}^{SF}$ and λ_1^{SF} for $E_\gamma < 1.815\,\mathrm{GeV}$

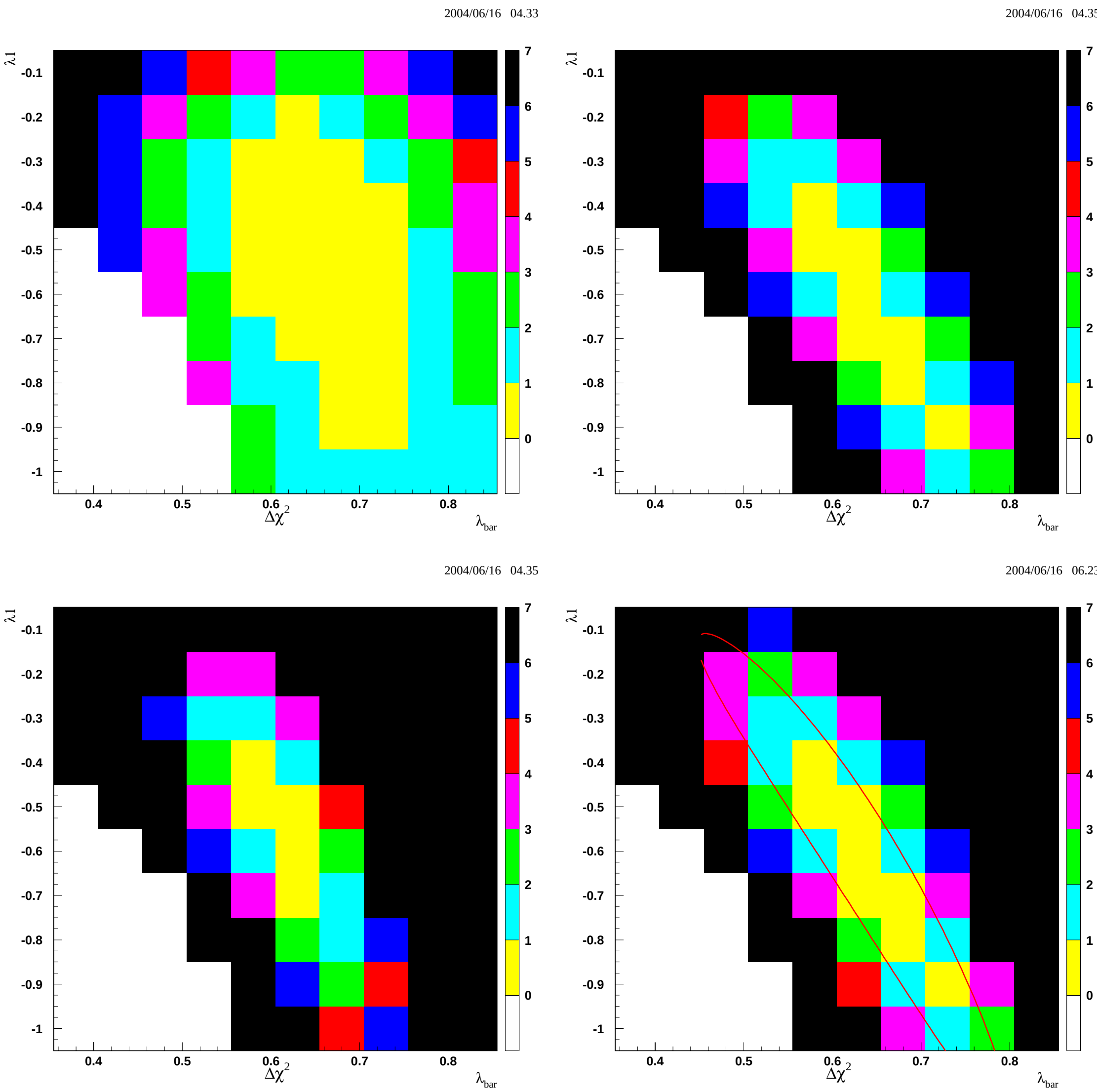

Figure 3: $\Delta\chi^2 = 1$ regions (in yellow) in the $\bar{\Lambda}^{SF}$-λ_1^{SF}plane for the (top left) Belle moments , (top right) Belle moments combined with CLEO analysis, (bottom left) Belle, Cleo and the external constraint from the OPE fits. The bottom right plots shows the result utilizing the CLEO constraint and the belle moments treating the exclusive decays separately. The red ellipse represents the CLEO constraint.

in the $\Upsilon(4S)$ frame $E_\gamma^Y > 1.8\,\mathrm{GeV}$ and this corresponds effectively to a cut in the B meson rest frame $E_\gamma > 1.815\,\mathrm{GeV}$. The resulting contour is in Fig. 3a.

Fig. 3b shows the contours for the combination of the Belle moments analysis and the CLEO fit to the spectrum, while Fig. 3c shows the arguable result where these two measurements are combined with m_b as fitted in $b \to c\ell\nu$ events [10], $\bar{\Lambda}^{SF} = 0.588 \pm 0.068\,\mathrm{GeV}$. Finally, Fig. 3d shows the analysis combining Belle and CLEO (with no external constraint) separating out the exclusive component. The inclusion of the Belle result with such a low cut-off is a very strong contraint and it also makes the result relatively insensitive to the treatement of the exclusive modes.

3 Summary

The best solution, from both the accuracy and reliability point of view, is to utilize the shape analysis from CLEO and the moments from Belle (assuming a full shape analysis from them is not expected on summer time scale). This would lead to the contour in Fig 4, whose best fit expression is (see blue line)

$$\chi^2 = (\frac{\lambda_1^{SF} + 2.4(\bar{\Lambda}^{SF})^2 - 0.37}{0.145})^2 + (\frac{(\bar{\Lambda}^{SF})^2 - 0.42}{0.11})^2 = 1 \tag{6}$$

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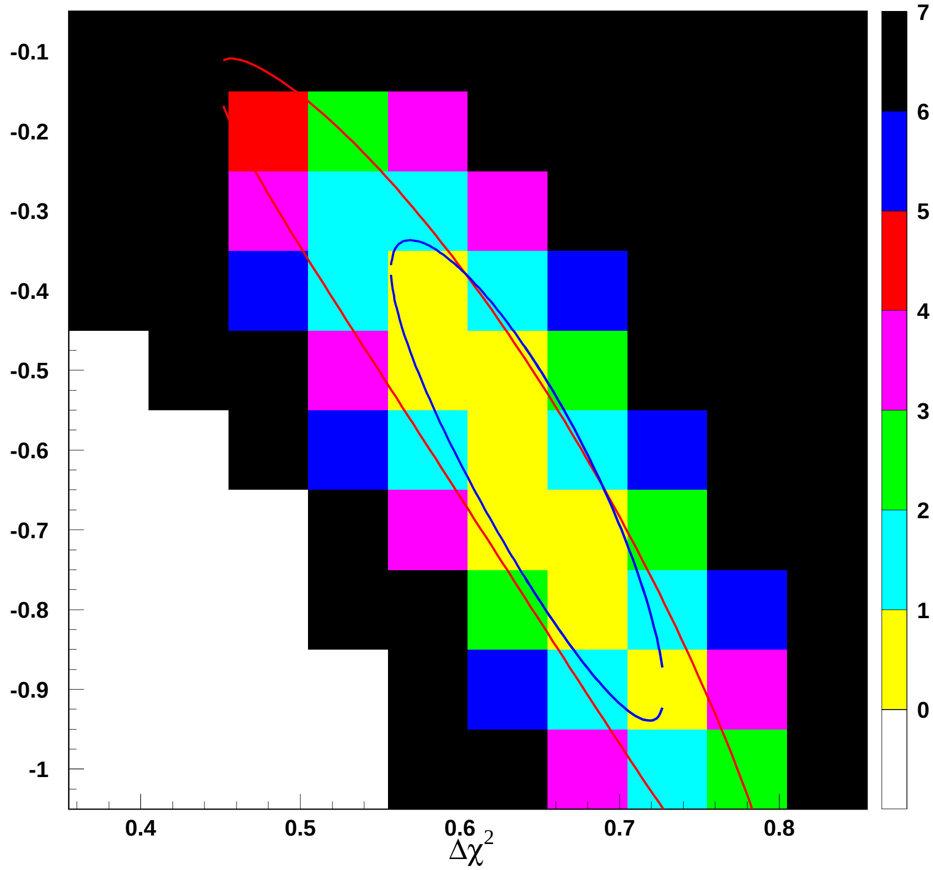

Figure 4: $\Delta\chi^2 = 1$ regions (in yellow) in the $\bar{\Lambda}^{SF}$-λ_1^{SF}plane for the Belle moments combined with CLEO analysis. The red ellipse represents the CLEO constraint while the blue curve represents the best fit to this contour.