Averages of b-hadron production fractions, lifetimes and mixing parameters (Spring 2014 update)

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Abstract

This is an updated version of Chapter 3 of the 2012 HFAG writeup `arXiv:1207.1158 [hep-ex]`. New results that became available between Summer 2012 and end of March 2014 have been incorporated in the averages. See also the corresponding sub-group web page at `http://www.slac.stanford.edu/xorg/hfag/osc`.

Averages or plots obtained from this document should be quoted as:

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Quantities such as b-hadron production fractions, b-hadron lifetimes, and neutral B-meson oscillation frequencies have been studied in the nineties at LEP and SLC (e^+e^- colliders at $\sqrt{s} = m_Z$) as well as at the first version of the Tevatron ($p\overline{p}$ collider at $\sqrt{s} = 1.8$ TeV). Since then precise measurements of the B^0 and B^+ mesons have also been performed at the asymmetric B factories, KEKB and PEPII (e^+e^- colliders at $\sqrt{s} = m_{\Upsilon(4S)}$) while measurements related to the other b-hadrons, in particular B^0_s, B^+_c and Λ^0_b, have been performed at the upgraded Tevatron ($\sqrt{s} = 1.96$ TeV) and are continuing at the LHC (pp collider at $\sqrt{s} = 7$ TeV). In most cases, these basic quantities, although interesting by themselves, became necessary ingredients for the more complicated and refined analyses at the asymmetric B factories, the Tevatron and the LHC, in particular the time-dependent CP asymmetry measurements. It is therefore important that the best experimental values of these quantities continue to be kept up-to-date and improved.

In several cases, the averages presented in this chapter are needed and used as input for the results given in the subsequent chapters. Within this chapter, some averages need the knowledge of other averages in a circular way. This coupling, which appears through the b-hadron fractions whenever inclusive or semi-exclusive measurements have to be considered, has reduced drastically in the past several years with increasingly precise exclusive measurements becoming available and dominating practically all averages.

In addition to b-hadron fractions, lifetimes and mixing parameters, this chapter also deals with the CP-violating phase $\phi_s^{c\overline{c}s} \simeq -2\beta_s$, which is the phase difference between the B^0_s mixing amplitude and the $b \to c\overline{c}s$ decay amplitude. The angle β, which is the equivalent of β_s for the B^0 system, is discussed in Chapter **??**.

3.1 b-hadron production fractions

We consider here the relative fractions of the different b-hadron species found in an unbiased sample of weakly-decaying b hadrons produced under some specific conditions. The knowledge of these fractions is useful to characterize the signal composition in inclusive b-hadron analyses, to predict the background composition in exclusive analyses, or to convert (relative) observe rates into (relative) branching fraction measurements. Many B-physics analyses need these fractions as input. We distinguish here the following three conditions: $\Upsilon(4S)$ decays, $\Upsilon(5S)$ decays, and high-energy collisions (including Z^0 decays).

3.1.1 b-hadron production fractions in $\Upsilon(4S)$ decays

Only pairs of the two lightest (charged and neutral) B mesons can be produced in $\Upsilon(4S)$ decays, and it is enough to determine the following branching fractions:

$$f^{+-} = \Gamma(\Upsilon(4S) \to B^+B^-)/\Gamma_{\rm tot}(\Upsilon(4S))\,, \quad (1)$$

$$f^{00} = \Gamma(\Upsilon(4S) \to B^0\overline{B}^0)/\Gamma_{\rm tot}(\Upsilon(4S))\,. \quad (2)$$

In practice, most analyses measure their ratio

$$R^{+-/00} = f^{+-}/f^{00} = \Gamma(\Upsilon(4S) \to B^+B^-)/\Gamma(\Upsilon(4S) \to B^0\overline{B}^0)\,, \quad (3)$$

Table 1: Published measurements of the B^+/B^0 production ratio in $\Upsilon(4S)$ decays, together with their average (see text). Systematic uncertainties due to the imperfect knowledge of $\tau(B^+)/\tau(B^0)$ are included. The latest *BABAR* result [1] supersedes the earlier *BABAR* measurements [2, 3].

Experiment and year	Ref.	Decay modes or method	Published value of $R^{+-/00} = f^{+-}/f^{00}$	Assumed value of $\tau(B^+)/\tau(B^0)$
CLEO, 2001	[4]	$J/\psi K^{(*)}$	$1.04 \pm 0.07 \pm 0.04$	1.066 ± 0.024
BABAR, 2002	[2]	$(c\bar{c})K^{(*)}$	$1.10 \pm 0.06 \pm 0.05$	1.062 ± 0.029
CLEO, 2002	[5]	$D^*\ell\nu$	$1.058 \pm 0.084 \pm 0.136$	1.074 ± 0.028
Belle, 2003	[6]	dilepton events	$1.01 \pm 0.03 \pm 0.09$	1.083 ± 0.017
BABAR, 2004	[3]	$J/\psi K$	$1.006 \pm 0.036 \pm 0.031$	1.083 ± 0.017
BABAR, 2005	[1]	$(c\bar{c})K^{(*)}$	$1.06 \pm 0.02 \pm 0.03$	1.086 ± 0.017
Average			1.059 ± 0.027 (tot)	1.076 ± 0.004

which is easier to access experimentally. Since an inclusive (but separate) reconstruction of B^+ and B^0 is difficult, specific exclusive decay modes, $B^+ \to x^+$ and $B^0 \to x^0$, are usually considered to perform a measurement of $R^{+-/00}$, whenever they can be related by isospin symmetry (for example $B^+ \to J/\psi K^+$ and $B^0 \to J/\psi K^0$). Under the assumption that $\Gamma(B^+ \to x^+) = \Gamma(B^0 \to x^0)$, *i.e.* that isospin invariance holds in these B decays, the ratio of the number of reconstructed $B^+ \to x^+$ and $B^0 \to x^0$ mesons is proportional to

$$\frac{f^{+-}\,\mathcal{B}(B^+ \to x^+)}{f^{00}\,\mathcal{B}(B^0 \to x^0)} = \frac{f^{+-}\,\Gamma(B^+ \to x^+)\,\tau(B^+)}{f^{00}\,\Gamma(B^0 \to x^0)\,\tau(B^0)} = \frac{f^{+-}}{f^{00}}\,\frac{\tau(B^+)}{\tau(B^0)}\,, \tag{4}$$

where $\tau(B^+)$ and $\tau(B^0)$ are the B^+ and B^0 lifetimes respectively. Hence the primary quantity measured in these analyses is $R^{+-/00}\,\tau(B^+)/\tau(B^0)$, and the extraction of $R^{+-/00}$ with this method therefore requires the knowledge of the $\tau(B^+)/\tau(B^0)$ lifetime ratio.

The published measurements of $R^{+-/00}$ are listed in Table 1 together with the corresponding assumed values of $\tau(B^+)/\tau(B^0)$. All measurements are based on the above-mentioned method, except the one from Belle, which is a by-product of the B^0 mixing frequency analysis using dilepton events (but note that it also assumes isospin invariance, namely $\Gamma(B^+ \to \ell^+\mathrm{X}) = \Gamma(B^0 \to \ell^+\mathrm{X})$). The latter is therefore treated in a slightly different manner in the following procedure used to combine these measurements:

- each published value of $R^{+-/00}$ from CLEO and *BABAR* is first converted back to the original measurement of $R^{+-/00}\,\tau(B^+)/\tau(B^0)$, using the value of the lifetime ratio assumed in the corresponding analysis;

- a simple weighted average of these original measurements of $R^{+-/00}\,\tau(B^+)/\tau(B^0)$ from CLEO and *BABAR* (which do not depend on the assumed value of the lifetime ratio) is then computed, assuming no statistical or systematic correlations between them;

- the weighted average of $R^{+-/00}\,\tau(B^+)/\tau(B^0)$ is converted into a value of $R^{+-/00}$, using the latest average of the lifetime ratios, $\tau(B^+)/\tau(B^0) = 1.076 \pm 0.004$ (see Sec. 3.2.3);

- the Belle measurement of $R^{+-/00}$ is adjusted to the current values of $\tau(B^0) = 1.519 \pm 0.005$ ps and $\tau(B^+)/\tau(B^0) = 1.076 \pm 0.004$ (see Sec. 3.2.3), using the quoted systematic uncertainties due to these parameters;
- the combined value of $R^{+-/00}$ from CLEO and *BABAR* is averaged with the adjusted value of $R^{+-/00}$ from Belle, assuming a 100% correlation of the systematic uncertainty due to the limited knowledge on $\tau(B^+)/\tau(B^0)$; no other correlation is considered.

The resulting global average,

$$R^{+-/00} = \frac{f^{+-}}{f^{00}} = 1.059 \pm 0.027\,, \tag{5}$$

is consistent with an equal production of charged and neutral B mesons, although only at the $2.2\,\sigma$ level.

On the other hand, the *BABAR* collaboration has performed a direct measurement of the f^{00} fraction using an original method, which does not rely on isospin symmetry nor requires the knowledge of $\tau(B^+)/\tau(B^0)$. Its analysis, based on a comparison between the number of events where a single $B^0 \to D^{*-}\ell^+\nu$ decay could be reconstructed and the number of events where two such decays could be reconstructed, yields [7]

$$f^{00} = 0.487 \pm 0.010\,(\text{stat}) \pm 0.008\,(\text{syst})\,. \tag{6}$$

The two results of Eqs. (5) and (6) are of very different natures and completely independent of each other. Their product is equal to $f^{+-} = 0.516 \pm 0.019$, while another combination of them gives $f^{+-} + f^{00} = 1.003 \pm 0.029$, compatible with unity. Assuming[1] $f^{+-} + f^{00} = 1$, also consistent with CLEO's observation that the fraction of $\Upsilon(4S)$ decays to $B\overline{B}$ pairs is larger than 0.96 at 95% CL [9], the results of Eqs. (5) and (6) can be averaged (first converting Eq. (5) into a value of $f^{00} = 1/(R^{+-/00} + 1)$) to yield the following more precise estimates:

$$f^{00} = 0.486 \pm 0.006\,, \quad f^{+-} = 1 - f^{00} = 0.514 \pm 0.006\,, \quad \frac{f^{+-}}{f^{00}} = 1.058 \pm 0.024\,. \tag{7}$$

The latter ratio differs from one by $2.4\,\sigma$.

3.1.2 b-hadron production fractions in $\Upsilon(5S)$ decays

Hadronic events produced in e^+e^- collisions at the $\Upsilon(5S)$ energy can be classified into three categories: light-quark (u, d, s, c) continuum events, $b\bar{b}$ continuum events, and $\Upsilon(5S)$ events. The latter two cannot be distinguished and will be called $b\bar{b}$ events in the following. These $b\bar{b}$ events, which also include $b\bar{b}\gamma$ events because of possible initial-state radiation, can hadronize in different final states. We define $f_{u,d}^{\Upsilon(5S)}$ as the fraction of $b\bar{b}$ events with a pair of non-strange bottom mesons ($B\overline{B}$, $B\overline{B}^*$, $B^*\overline{B}$, $B^*\overline{B}^*$, $B\overline{B}\pi$, $B\overline{B}^*\pi$, $B^*\overline{B}\pi$, $B^*\overline{B}^*\pi$, and $B\overline{B}\pi\pi$ final states, where B denotes a B^0 or B^+ meson and $\overline{B}$ denotes a $\overline{B}^0$ or B^- meson), $f_s^{\Upsilon(5S)}$ as the fraction

[1] A few non-$B\overline{B}$ decay modes of the $\Upsilon(4S)$ ($\Upsilon(1S)\pi^+\pi^-$, $\Upsilon(2S)\pi^+\pi^-$, $\Upsilon(1S)\eta$) have been observed with branching fractions of the order of 10^{-4} [8], corresponding to a partial width several times larger than that in the e^+e^- channel. However, this can still be neglected and the assumption $f^{+-} + f^{00} = 1$ remains valid in the present context of the determination of f^{+-} and f^{00}.

Table 2: Published measurements of $f_s^{\Upsilon(5S)}$. All values have been obtained assuming $f_{\not B}^{\Upsilon(5S)} = 0$. They are quoted as in the original publications, except for the most recent measurement which is quoted as $1 - f_{u,d}^{\Upsilon(5S)}$, with $f_{u,d}^{\Upsilon(5S)}$ from Ref. [10]. The last line gives our average of $f_s^{\Upsilon(5S)}$ assuming $f_{\not B}^{\Upsilon(5S)} = 0$.

Experiment, year, dataset	Decay mode or method	Value of $f_s^{\Upsilon(5S)}$
CLEO, 2006, $0.42\,\mathrm{fb}^{-1}$ [11]	$\Upsilon(5S) \to D_s X$	$0.168 \pm 0.026^{+0.067}_{-0.034}$
	$\Upsilon(5S) \to \phi X$	$0.246 \pm 0.029^{+0.110}_{-0.053}$
	$\Upsilon(5S) \to B\overline{B}X$	$0.411 \pm 0.100 \pm 0.092$
	CLEO average of above 3	$0.21^{+0.06}_{-0.03}$
Belle, 2006, $1.86\,\mathrm{fb}^{-1}$ [12]	$\Upsilon(5S) \to D_s X$	$0.179 \pm 0.014 \pm 0.041$
	$\Upsilon(5S) \to D^0 X$	$0.181 \pm 0.036 \pm 0.075$
	Belle average of above 2	$0.180 \pm 0.013 \pm 0.032$
Belle, 2010, $23.6\,\mathrm{fb}^{-1}$ [10]	$\Upsilon(5S) \to B\overline{B}X$	$0.263 \pm 0.032 \pm 0.051$
Average of all above after adjustments to inputs of Table 3		0.215 ± 0.031

Table 3: External inputs on which the $f_s^{\Upsilon(5S)}$ averages are based.

Branching fraction	Value	Explanation and reference
$\mathcal{B}(B \to D_s X) \times \mathcal{B}(D_s \to \phi\pi)$	0.00374 ± 0.00014	derived from [13]
$\mathcal{B}(B_s^0 \to D_s X)$	0.92 ± 0.11	model-dependent estimate [14]
$\mathcal{B}(D_s \to \phi\pi)$	0.045 ± 0.004	[13]
$\mathcal{B}(B \to D^0 X) \times \mathcal{B}(D^0 \to K\pi)$	0.0243 ± 0.0011	derived from [13]
$\mathcal{B}(B_s^0 \to D^0 X)$	0.08 ± 0.07	model-dependent estimate [12, 14]
$\mathcal{B}(D^0 \to K\pi)$	0.0387 ± 0.0005	[13]
$\mathcal{B}(B \to \phi X)$	0.0343 ± 0.0012	world average [11, 13]
$\mathcal{B}(B_s^0 \to \phi X)$	0.161 ± 0.024	model-dependent estimate [11]

of $b\bar{b}$ events with a pair of strange bottom mesons ($B_s^0\overline{B}_s^0$, $B_s^0\overline{B}_s^{0*}$, $B_s^{0*}\overline{B}_s^0$, and $B_s^{0*}\overline{B}_s^{0*}$ final states), and $f_{\not B}^{\Upsilon(5S)}$ as the fraction of $b\bar{b}$ events without bottom meson in the final state. Note that the excited bottom-meson states decay via $B^* \to B\gamma$ and $B_s^{0*} \to B_s^0\gamma$. These fractions satisfy

$$f_{u,d}^{\Upsilon(5S)} + f_s^{\Upsilon(5S)} + f_{\not B}^{\Upsilon(5S)} = 1\,. \tag{8}$$

The CLEO and Belle collaborations have published in 2006 measurements of several inclusive $\Upsilon(5S)$ branching fractions, $\mathcal{B}(\Upsilon(5S) \to D_s X)$, $\mathcal{B}(\Upsilon(5S) \to \phi X)$ and $\mathcal{B}(\Upsilon(5S) \to D^0 X)$, from which they extracted the model-dependent estimates of $f_s^{\Upsilon(5S)}$ reported in Table 2. This extraction was performed under the implicit assumption $f_{\not B}^{\Upsilon(5S)} = 0$, using the relation

$$\frac{1}{2}\mathcal{B}(\Upsilon(5S) \to D_s X) = f_s^{\Upsilon(5S)} \times \mathcal{B}(B_s^0 \to D_s X) + \left(1 - f_s^{\Upsilon(5S)} - f_{\not B}^{\Upsilon(5S)}\right) \times \mathcal{B}(B \to D_s X)\,, \tag{9}$$

and similar relations for $\mathcal{B}(\Upsilon(5S) \to D^0 X)$ and $\mathcal{B}(\Upsilon(5S) \to \phi X)$. We list also in Table 2 the values of $f_s^{\Upsilon(5S)}$ derived from measurements of $f_{u,d}^{\Upsilon(5S)} = \mathcal{B}(\Upsilon(5S) \to B\overline{B}X)$ [10, 11], as well as our average value of $f_s^{\Upsilon(5S)}$, all obtained under the assumption $f_{\not B}^{\Upsilon(5S)} = 0$.

However, the assumption $f_{\not B}^{\Upsilon(5S)} = 0$ is no longer valid since the observation of the following final states in e^+e^- collisions at the $\Upsilon(5S)$ energy: $\Upsilon(1S)\pi^+\pi^-$, $\Upsilon(2S)\pi^+\pi^-$, $\Upsilon(3S)\pi^+\pi^-$ and $\Upsilon(1S)K^+K^-$ [15, 16], $h_b(1P)\pi^+\pi^-$ and $h_b(2P)\pi^+\pi^-$ [17], and more recently $\Upsilon(1S)\pi^0\pi^0$, $\Upsilon(2S)\pi^0\pi^0$ and $\Upsilon(3S)\pi^0\pi^0$ [18]. The sum of the mesaurements of the corresponding visible cross-sections, adding also the contributions of the unmeasured $\Upsilon(1S)K^0\overline{K}^0$, $h_b(1P)\pi^0\pi^0$ and $h_b(2P)\pi^0\pi^0$ final states assuming isospin conservation, amounts to

$$\sigma^{\rm vis}(e^+e^- \to (b\bar{b})hh) = 13.2 \pm 1.4 \text{ pb}\,, \quad \text{for } (b\bar{b}) = \Upsilon(1S, 2S, 3S), h_b(1P, 2P) \text{ and } hh = \pi\pi, KK\,.$$

We divide this by the $b\bar{b}$ production cross section, $\sigma(e^+e^- \to b\bar{b}X) = 337 \pm 15$ pb, obtained as the average of the CLEO [14] and Belle [19] measurements, to obtain

$$\mathcal{B}(\Upsilon(5S) \to (b\bar{b})hh) = 0.039 \pm 0.004\,, \quad \text{for } (b\bar{b}) = \Upsilon(1S, 2S, 3S), h_b(1P, 2P) \text{ and } hh = \pi\pi, KK\,,$$

which is to be considered as a lower bound for $f_{\not B}^{\Upsilon(5S)}$.

Following the method described in Ref. [20], we perform a χ^2 fit of the original measurements of the $\Upsilon(5S)$ branching fractions of Refs. [10–12], using the inputs of Table 3, the relations of Eqs. (8) and (9) and the one-sided Gaussian constraint $f_{\not B}^{\Upsilon(5S)} \geq \mathcal{B}(\Upsilon(5S) \to (b\bar{b})hh)$, to simultaneously extract $f_{u,d}^{\Upsilon(5S)}$, $f_s^{\Upsilon(5S)}$ and $f_{\not B}^{\Upsilon(5S)}$. Taking all known correlations into account, the best fit values are

$$f_{u,d}^{\Upsilon(5S)} = 0.761^{+0.027}_{-0.042}\,, \tag{10}$$
$$f_s^{\Upsilon(5S)} = 0.200^{+0.030}_{-0.031}\,, \tag{11}$$
$$f_{\not B}^{\Upsilon(5S)} = 0.039^{+0.050}_{-0.004}\,, \tag{12}$$

where the strongly asymmetric uncertainty on $f_{\not B}^{\Upsilon(5S)}$ is due to the one-sided constraint from the observed $(b\bar{b})hh$ decays. These results, together with their correlation, imply

$$f_s^{\Upsilon(5S)}/f_{u,d}^{\Upsilon(5S)} = 0.263^{+0.052}_{-0.044}\,, \tag{13}$$

in fair agreement with the results of a *BABAR* analysis [21] performed as a function of centre-of-mass energy[2].

The production of B_s^0 mesons at the $\Upsilon(5S)$ is observed to be dominated by the $B_s^{0*}\overline{B}_s^{0*}$ channel, with $\sigma(e^+e^- \to B_s^{0*}\overline{B}_s^{0*})/\sigma(e^+e^- \to B_s^{0(*)}\overline{B}_s^{0(*)}) = (87.0 \pm 1.7)\%$ [22,23]. The proportion of the various production channels for non-strange B mesons have also been measured [10].

3.1.3 *b*-hadron production fractions at high energy

At high energy, all species of weakly-decaying b hadrons may be produced, either directly or in strong and electromagnetic decays of excited b hadrons. It is often assumed that the fractions of these different species are the same in unbiased samples of high-$p_{\rm T}$ b jets originating from Z^0 decays, from $p\overline{p}$ collisions at the Tevatron, or from pp collisions at the LHC. This hypothesis is plausible under the condition that the square of the momentum transfer to the produced b

[2] This has not been included in the average, since no numerical value is given for $f_s^{\Upsilon(5S)}/f_{u,d}^{\Upsilon(5S)}$ in Ref. [21].

quarks, Q^2, is large compared with the square of the hadronization energy scale, $Q^2 \gg \Lambda_{\rm QCD}^2$. On the other hand, there is no strong argument to claim that the fractions at different machines should be strictly equal, so this assumption should be checked experimentally. Although the available data is not sufficient at this time to perform a definitive check, it is expected that more refined analyses of the Tevatron Run II data and new analyses from LHC experiments may improve this situation and allow one to confirm or disprove this assumption with reasonable confidence. Meanwhile, the attitude adopted here is that these fractions are assumed to be equal at all high-energy colliders until demonstrated otherwise by experiment. However, both CDF and LHCb report a $p_{\rm T}$ dependence for Λ_b production relative to B^+ and B^0; the number of Λ_b baryons observed at low $p_{\rm T}$ is enhanced with respect to that seen at LEP at higher $p_{\rm T}$. Therefore we present three sets of complete averages: one set including only measurements performed at LEP, a second set including only measurements performed at the Tevatron, a third set including measurements performed at LEP, Tevatron and LHCb. The LHCb production fractions results, by themselves, are still incomplete, lacking measurements on the production of other weakly decaying heavy flavour baryons, Ξ_b and Ω_b, and a measurement of $\overline{\chi}$ giving an extra constraint between f_d and f_s.

Contrary to what happens in the charm sector where the fractions of D^+ and D^0 are different, the relative amount of B^+ and B^0 is not affected by the electromagnetic decays of excited B^{+*} and B^{0*} states and strong decays of excited B^{+**} and B^{0**} states. Decays of the type $B_s^{0**} \to B^{(*)}K$ also contribute to the B^+ and B^0 rates, but with the same magnitude if mass effects can be neglected. We therefore assume equal production of B^+ and B^0. We also neglect the production of weakly-decaying states made of several heavy quarks (like B_c^+ and other heavy baryons) which is known to be very small. Hence, for the purpose of determining the b-hadron fractions, we use the constraints

$$f_u = f_d \quad \text{and} \quad f_u + f_d + f_s + f_{\rm baryon} = 1\,, \tag{14}$$

where f_u, f_d, f_s and $f_{\rm baryon}$ are the unbiased fractions of B^+, B^0, B_s^0 and b baryons, respectively.

The LEP experiments have measured $f_s \times \mathcal{B}(B_s^0 \to D_s^- \ell^+ \nu_\ell X)$ [24], $\mathcal{B}(b \to \Lambda_b^0) \times \mathcal{B}(\Lambda_b^0 \to \Lambda_c^+ \ell^- \overline{\nu}_\ell X)$ [25,26] and $\mathcal{B}(b \to \Xi_b^-) \times \mathcal{B}(\Xi_b^- \to \Xi^- \ell^- \overline{\nu}_\ell X)$ [27,28][3] from partially reconstructed final states including a lepton, $f_{\rm baryon}$ from protons identified in b events [30], and the production rate of charged b hadrons [31]. Ratios of b-hadron fractions have been measured at CDF using lepton+charm final states [32–34][4], double semileptonic decays with $K^*\mu\mu$ and $\phi\mu\mu$ final states [35], and fully reconstructed $B_s^0 \to J/\psi\,\phi$ decays [36]. Measurements of the production of other heavy flavour baryons at the Tevatron are included in the determination of $f_{\rm baryon}$ [37–39][5] using the constraint

$$\begin{aligned} f_{\rm baryon} &= f_{\Lambda_b} + f_{\Xi_b^0} + f_{\Xi_b^-} + f_{\Omega_b^-} \\ &= f_{\Lambda_b}\left(1 + 2\frac{f_{\Xi_b^-}}{f_{\Lambda_b}} + \frac{f_{\Omega_b^-}}{f_{\Lambda_b}}\right), \end{aligned} \tag{15}$$

where isospin invariance is assumed in the production of Ξ_b^0 and Ξ_b^-. Other b-baryons are expected to decay strongly or electromagnetically to those baryons listed. For the production

[3]The DELPHI result of Ref. [28] is considered to supersede an older one [29].

[4]CDF updated their measurement of f_{Λ_b}/f_d [32] to account for a measured $p_{\rm T}$ dependence between exclusively reconstructed Λ_b and B^0 [34].

[5]D0 reports $f_{\Omega_b^-}/f_{\Xi_b^-}$. We use the CDF+D0 average of $f_{\Xi_b^-}/f_{\Lambda_b}$ to obtain $f_{\Omega_b^-}/f_{\Lambda_b}$ and then combine with the CDF result.

measurements, both CDF and D0 reconstruct their b-baryons exclusively to final states which include a J/ψ and a hyperon ($\Lambda_b \to J/\psi\,\Lambda$, $\Xi_b^- \to J/\psi\,\Xi^-$ and $\Omega_b^- \to J/\psi\,\Omega^-$). We assume that the partial decay width of a b-baryon to a J/ψ and the corresponding hyperon is equal to the partial width of any other b-baryon to a J/ψ and the corresponding hyperon. LHCb has also measured ratios of b-hadron fractions, $f_s/(f_u+f_d)$ and $f_{\Lambda_b}/(f_u+f_d)$, in charm+lepton final states [40] and f_s/f_d in fully reconstructed hadronic final states using theoretical values for the branching fractions of two-body B_s^0 and B^0 decays [41].

Both CDF and LHCb observe that the ratio f_{Λ_b}/f_d depends on the p_T of the charm+lepton system [34,40][6]. CDF chose to correct an older result to account for the p_T dependence whereas LHCb chose to report a linear dependence of $f_{\Lambda_b}/(f_u+f_d)$, which yields unphysical results for $p_\mathrm{T} > 32$ GeV/c. In a second result, CDF binned their data in p_T of the charm+electron system. Figure 1 shows the ratio $R_{\Lambda_b} = f_{\Lambda_b}/(f_u+f_d)$ as a function of this p_T, as measured by both CDF and LHCb. Two fits are performed. The first fit using the LHCb parameterization yields $R_{\Lambda_b} = (0.386{\pm}0.21)\,[1-(0.0270\pm0.0056)\times p_\mathrm{T}]$. A second fit using a simple exponential yields $R_{\Lambda_b} = \exp\{(-0.928\pm0.066)-(0.0344\pm0.0086)\times p_\mathrm{T}\}$. A common systematic uncertainty of 26% on the scale of both results arises from the $\Lambda_c^+ \to pK^-\pi^+$ branching fraction. The quality of the two fits are similar, but the second parameterization gives a physical result for all p_T. A value of R_{Λ_b} is also calculated for LEP and placed at the approximate p_T for the charm+lepton system, but this value does not participate in any fit. Note that the p_T dependence of R_{Λ_b} combined with the constraint in Eq. (14) implies a compensating p_T dependence in one or more of the production fractions, f_u, f_d, or f_s.

Both CDF and LHCb have investigated the p_T dependence of f_s/f_d using fully reconstructed B_s^0 and B^0 decays. The CDF analysis reconstructed decays that include a J/ψ in the final state [36] and reports no significant p_T dependence on the ratio. However, their result is dominated by an 18% scale uncertainty from preliminary measurements of the branching ratios of the $\mathcal{B}(B_s^0 \to J/\psi\phi)$ and $\mathcal{B}(B^0 \to J\psi K^*(892))$. LHCb reported 3σ evidence that the ratio f_s/f_d decreases with p_T using fully reconstructed B_s^0 and B^0 decays and theoretical predictions for branching ratios [41]. Figure 2 shows the ratio $R_s = f_s/f_u$ as a function of p_T measured by CDF and LHCb. Two fits are performed similar to the result for R_{Λ_b} above. The first fit using a linear parameterization yields $R_s = (0.2760\pm0.0068)-(0.00191\pm0.00059)\times p_\mathrm{T}$. A second fit using a simple exponential yields $R_s = \exp\{(-1.293\pm0.028)-(-0.0077\pm0.0025)\times p_\mathrm{T}\}$. The two fits are nearly indistinguisible over the p_T range of the results but the second gives a physical value for all p_T. R_s is also calculated for LEP and placed at the approximate p_T for the b hadron, though the LEP result doesn't participate in the fit. The PDG world average for R_s is also included in the figure for reference.

In order to combine or compare LHCb results with other experiments, the p_T-dependent $f_{\Lambda_b}/(f_u+f_d)$ is weighted by the p_T spectrum[7]. Table 4 compares the p_T-weighted LHCb data with comparable averages from CDF. The average CDF and LHCb data are in good agreement despite the b hadrons being produced in different kinematic regimes.

All these published results have been combined following the procedure and assumptions

[6]CDF compares the p_T distribution of fully reconstructed $\Lambda_b \to \Lambda_c^+\pi^-$ with $\overline{B^0} \to \mathcal{D}^+\pi^-$ which compares f_{Λ_b}/f_d up to a scale factor. LHCb compares the p_T in the charm+lepton system between Λ_b and B^0 and B^+ comparing $R_{\Lambda_b} = f_{\Lambda_b}/(f_u+f_d) = f_{\Lambda_b}/2f_d$.

[7]In practice the LHCb data are given in 14 bins in p_T and η with a full covariance matrix [40]. The weighted average is calculated as $D^TC^{-1}M/\sigma$, where $\sigma = D^TC^{-1}D$, M is a vector of measurements, C^{-1} is the inverse covariance matrix and D^T is the transpose of the design matrix (vector of 1's)

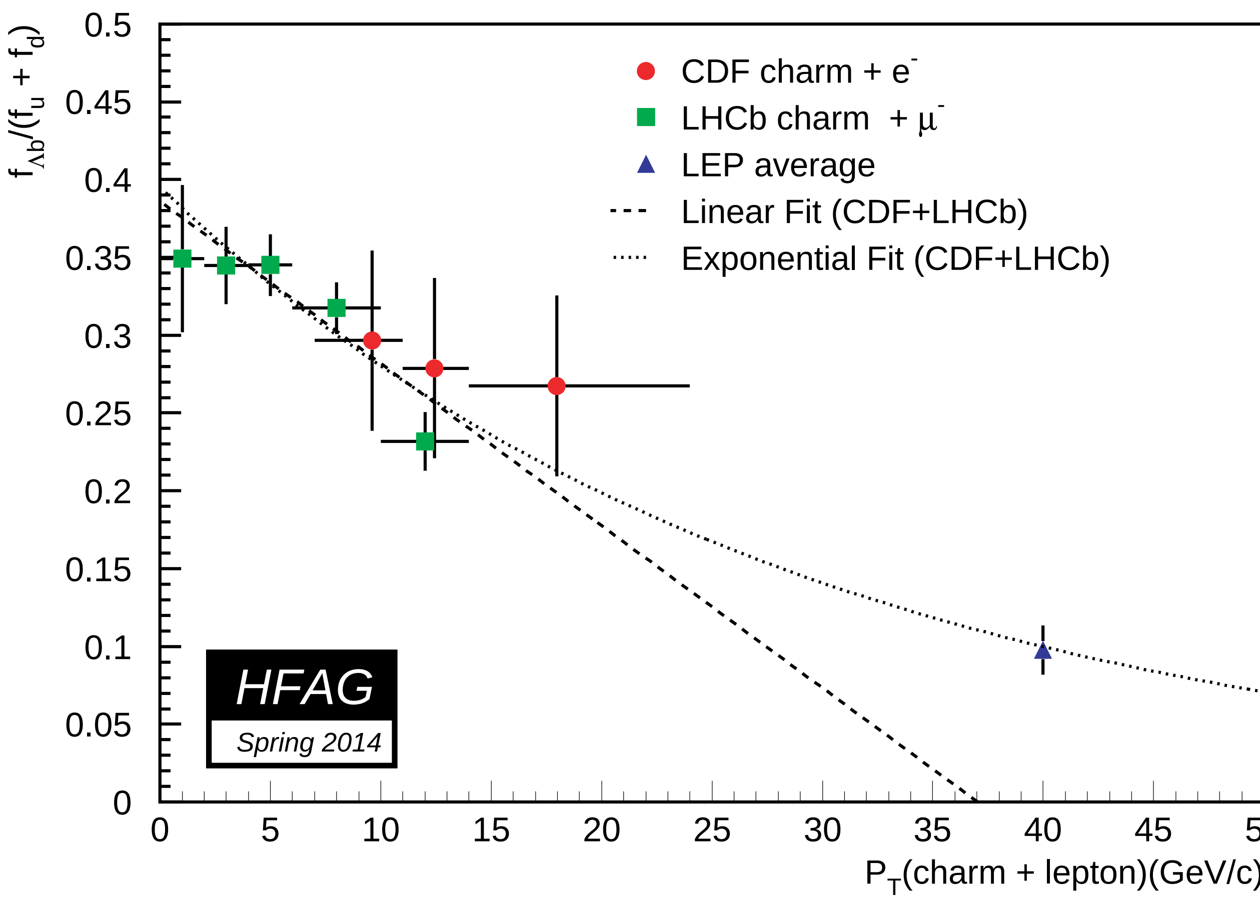

Figure 1: Ratio of production fractions $f_{\Lambda_b}/(f_u + f_d)$ as a function of p_{T} of the charm+lepton system for CDF [34] and LHCb [40] data. A scale uncertainty due to the common systematic uncertainty from the $\Lambda_c^+ \to pK^-\pi^+$ branching fraction is omitted. The curves represent fits to the data: a linear fit using the LHCb parameterization (dashed), and an exponential fit described in the text (dotted). The computed LEP ratio is included at an approximate p_{T} in Z decays, but does not participate in any fit.

Table 4: Comparison of average production fraction ratios from CDF and LHCb. The kinematic regime of the charm+lepton system reconstructed in each experiment is also shown.

Quantity	CDF	LHCb
$f_s/(f_u + f_d)$	0.211 ± 0.054	0.131 ± 0.009
$f_{\Lambda_b}/(f_u + f_d)$	0.290 ± 0.109	0.305 ± 0.022
Average charm+lepton p_{T}	~ 13 GeV/c	~ 7 GeV/c
Pseudo-rapidity range	$-1 < \eta < 1$	$2 < \eta < 5$

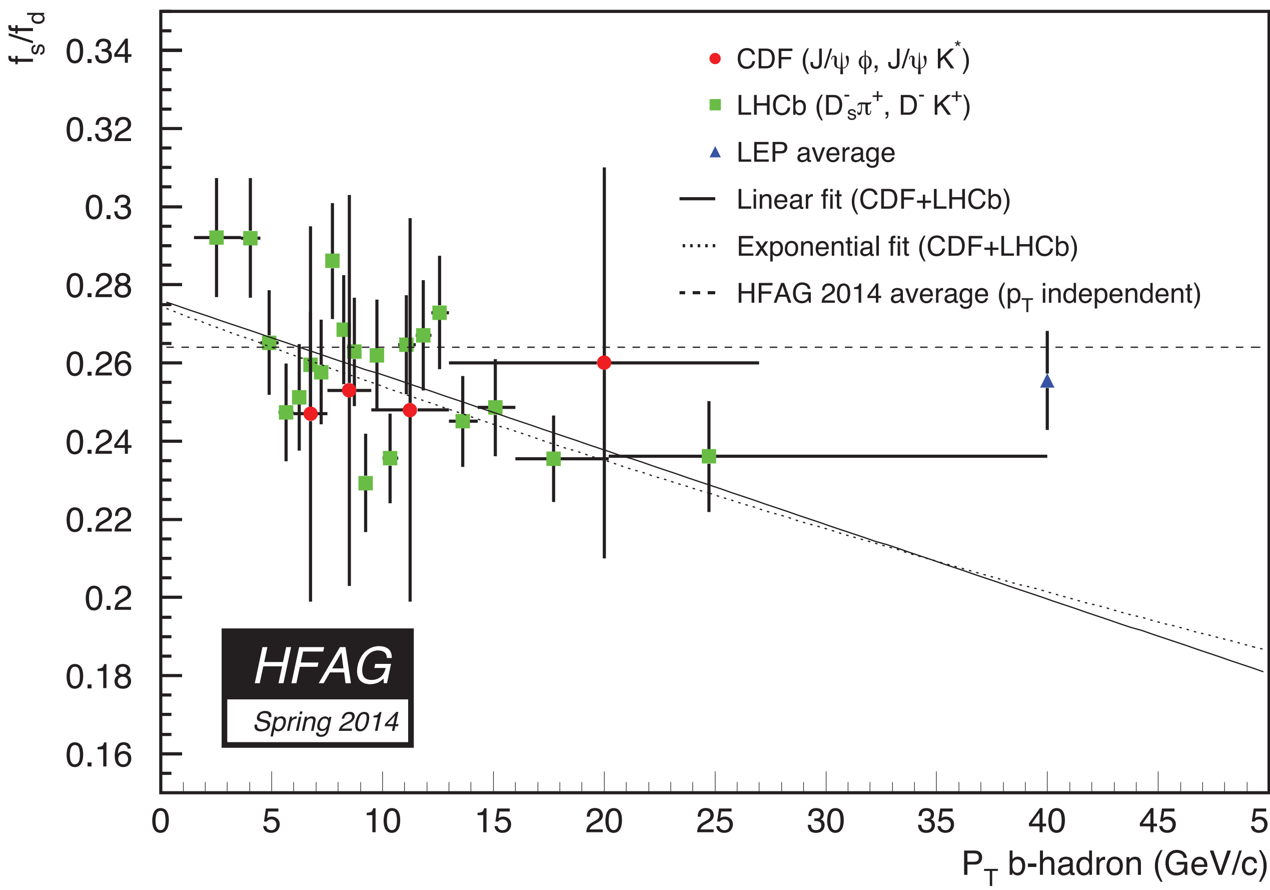

Figure 2: Ratio of production fractions f_s/f_u as a function of p_{T} of the reconstructed b-hadrons for the CDF [36] and LHCb [41] data. Note the suppressed zero for the vertical axis. The curves represent fits to the data: a linear fit (solid), and an exponential fit described in the text (dotted). The p_{T} independent value average of R_s (dashed) is shown for comparison. The computed LEP ratio is included at an approximate p_{T} in Z decays, but does not participate in any fit.

described in Ref. [42], to yield $f_u = f_d = 0.400 \pm 0.007$, $f_s = 0.103 \pm 0.006$ and $f_{\rm baryon} = 0.097 \pm 0.016$ under the constraints of Eq. (14). Repeating the combinations, for LEP and the Tevatron, we obtain $f_u = f_d = 0.407 \pm 0.009$, $f_s = 0.087 \pm 0.014$ and $f_{\rm baryon} = 0.099 \pm 0.016$ when using the LEP data only, $f_u = f_d = 0.324 \pm 0.032$, $f_s = 0.090 \pm 0.013$ $f_{\rm baryon} = 0.263 \pm 0.073$ when using the Tevatron data only. As noted previously, the LHCb data are insufficient to determine a complete set of b-hadron production fractions. The world averages (LEP, Tevatron and LHCb) for the various fractions are presented here for comparison with previous averages. Significant differences exist between the LEP and Tevatron fractions, therefore use of the world averages should be taken with some care. For these combinations other external inputs are used, *e.g.* the branching ratios of B mesons to final states with a D, D^* or D^{**} in semileptonic decays, which are needed to evaluate the fraction of semileptonic B_s^0 decays with a D_s^- in the final state.

Time-integrated mixing analyses performed with lepton pairs from $b\bar{b}$ events produced at

Table 5: Time-integrated mixing probability $\overline{\chi}$ (defined in Eq. (16)), and fractions of the different b-hadron species in an unbiased sample of weakly-decaying b hadrons, obtained from both direct and mixing measurements. The correlation coefficients between the fractions are also given. The last column includes measurements performed at LEP, Tevatron and LHCb.

Quantity		Z decays	Tevatron	LHCb [41]	all
Mixing probability $\overline{\chi}$		0.1259 ± 0.0042	0.127 ± 0.008		0.1260 ± 0.0037
B^+ or B^0 fraction	$f_u = f_d$	0.403 ± 0.009	0.336 ± 0.030		0.402 ± 0.007
B^0_s fraction	f_s	0.103 ± 0.009	0.098 ± 0.011		0.106 ± 0.005
b-baryon fraction	$f_{\rm baryon}$	0.090 ± 0.015	0.231 ± 0.067		0.090 ± 0.014
B^0_s/B^0 ratio	f_s/f_d	0.255 ± 0.025	0.291 ± 0.029	0.256 ± 0.020	0.263 ± 0.014
$\rho(f_s, f_u) = \rho(f_s, f_d)$		-0.539	$+0.530$		-0.170
$\rho(f_{\rm baryon}, f_u) = \rho(f_{\rm baryon}, f_d)$		-0.874	-0.991		-0.944
$\rho(f_{\rm baryon}, f_s)$		$+0.062$	-0.640		-0.164

high-energy colliders measure the quantity

$$\overline{\chi} = f'_d\,\chi_d + f'_s\,\chi_s\,, \tag{16}$$

where f'_d and f'_s are the fractions of B^0 and B^0_s hadrons in a sample of semileptonic b-hadron decays, and where χ_d and χ_s are the B^0 and B^0_s time-integrated mixing probabilities. Assuming that all b hadrons have the same semileptonic decay width implies $f'_i = f_i R_i$, where $R_i = \tau_i/\tau_b$ is the ratio of the lifetime τ_i of species i to the average b-hadron lifetime $\tau_b = \sum_i f_i \tau_i$. Hence measurements of the mixing probabilities $\overline{\chi}$, χ_d and χ_s can be used to improve our knowledge of f_u, f_d, f_s and $f_{\rm baryon}$. In practice, the above relations yield another determination of f_s obtained from $f_{\rm baryon}$ and mixing information,

$$f_s = \frac{1}{R_s}\frac{(1+r)\overline{\chi} - (1 - f_{\rm baryon}R_{\rm baryon})\chi_d}{(1+r)\chi_s - \chi_d}\,, \tag{17}$$

where $r = R_u/R_d = \tau(B^+)/\tau(B^0)$.

The published measurements of $\overline{\chi}$ performed by the LEP experiments have been combined by the LEP Electroweak Working Group to yield $\overline{\chi} = 0.1259 \pm 0.0042$ [43]. This can be compared with the Tevatron average, $\overline{\chi} = 0.127 \pm 0.008$, obtained from D0 [44] and CDF [45] measurements with Run II data.[8] The two averages agree, showing no evidence that the production fractions of B^0 and B^0_s mesons at the Z peak or at the Tevatron are different. We combine these two results in a simple weighted average, assuming no correlations, and obtain $\overline{\chi} = 0.1260 \pm 0.0037$.

Introducing the $\overline{\chi}$ average in Eq. (17), together with our world average $\chi_d = 0.1874 \pm 0.0018$ (see Eq. (46) of Sec. 3.3.1), the assumption $\chi_s = 1/2$ (justified by Eq. (55) in Sec. 3.3.2), the best knowledge of the lifetimes (see Sec. 3.2) and the estimate of $f_{\rm baryon}$ given above, yields $f_s = 0.114 \pm 0.011$ (or $f_s = 0.114 \pm 0.012$ using only LEP data, or $f_s = 0.116 \pm 0.020$ using only Tevatron data), an estimate dominated by the mixing information. Taking into account all known correlations (including the one introduced by $f_{\rm baryon}$), this result is then combined with

[8] As explained in Ref. [45], a previous CDF analysis [46] performed with Run I data overlooked a background component, so the corresponding result is not included in the average.

the set of fractions obtained from direct measurements (given above), to yield the improved estimates of Table 5, still under the constraints of Eq. (14). As can be seen, our knowledge on the mixing parameters substantially reduces the uncertainty on f_s. It should be noted that the results are correlated, as indicated in Table 5.

3.2 *b*-hadron lifetimes

In the spectator model the decay of b-flavoured hadrons H_b is governed entirely by the flavour changing $b \to Wq$ transition ($q = c, u$). For this very reason, lifetimes of all b-flavoured hadrons are the same in the spectator approximation regardless of the (spectator) quark content of the H_b. In the early 1990's experiments became sophisticated enough to start seeing the differences of the lifetimes among various H_b species. The first theoretical calculations of the spectator quark effects on H_b lifetime emerged only few years earlier.

Currently, most of such calculations are performed in the framework of the Heavy Quark Expansion, HQE. In the HQE, under certain assumptions (most important of which is that of quark-hadron duality), the decay rate of an H_b to an inclusive final state f is expressed as the sum of a series of expectation values of operators of increasing dimension, multiplied by the correspondingly higher powers of $\Lambda_{\rm QCD}/m_b$:

$$\Gamma_{H_b \to f} = |CKM|^2 \sum_n c_n^{(f)} \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^n \langle H_b|O_n|H_b\rangle, \tag{18}$$

where $|CKM|^2$ is the relevant combination of the CKM matrix elements. Coefficients $c_n^{(f)}$ of this expansion, known as Operator Product Expansion [47], can be calculated perturbatively. Hence, the HQE predicts $\Gamma_{H_b \to f}$ in the form of an expansion in both $\Lambda_{\rm QCD}/m_b$ and $\alpha_s(m_b)$. The precision of current experiments makes it mandatory to go to the next-to-leading order in QCD, *i.e.* to include correction of the order of $\alpha_s(m_b)$ to the $c_n^{(f)}$'s. All non-perturbative physics is shifted into the expectation values $\langle H_b|O_n|H_b\rangle$ of operators O_n. These can be calculated using lattice QCD or QCD sum rules, or can be related to other observables via the HQE [48]. One may reasonably expect that powers of $\Lambda_{\rm QCD}/m_b$ provide enough suppression that only the first few terms of the sum in Eq. (18) matter.

Theoretical predictions are usually made for the ratios of the lifetimes (with $\tau(B^0)$ chosen as the common denominator) rather than for the individual lifetimes, for this allows several uncertainties to cancel. The precision of the current HQE calculations (see Refs. [49–51] for the latest updates) is in some instances already surpassed by the measurements, *e.g.* in the case of $\tau(B^+)/\tau(B^0)$. Also, HQE calculations are not assumption-free. More accurate predictions are a matter of progress in the evaluation of the non-perturbative hadronic matrix elements and verifying the assumptions that the calculations are based upon. However, the HQE, even in its present shape, draws a number of important conclusions, which are in agreement with experimental observations:

- The heavier the mass of the heavy quark the smaller is the variation in the lifetimes among different hadrons containing this quark, which is to say that as $m_b \to \infty$ we retrieve the spectator picture in which the lifetimes of all H_b's are the same. This is well illustrated by the fact that lifetimes are rather similar in the b sector, while they differ by large factors in the c sector ($m_c < m_b$).

- The non-perturbative corrections arise only at the order of $\Lambda_{\rm QCD}^2/m_b^2$, which translates into differences among H_b lifetimes of only a few percent.
- It is only the difference between meson and baryon lifetimes that appears at the $\Lambda_{\rm QCD}^2/m_b^2$ level. The splitting of the meson lifetimes occurs at the $\Lambda_{\rm QCD}^3/m_b^3$ level, yet it is enhanced by a phase space factor $16\pi^2$ with respect to the leading free b decay.

To ensure that certain sources of systematic uncertainty cancel, lifetime analyses are sometimes designed to measure a ratio of lifetimes. However, because of the differences in decay topologies, abundance (or lack thereof) of decays of a certain kind, *etc.*, measurements of the individual lifetimes are more common. In the following section we review the most common types of the lifetime measurements. This discussion is followed by the presentation of the averaging of the various lifetime measurements, each with a brief description of its particularities.

3.2.1 Lifetime measurements, uncertainties and correlations

In most cases lifetime of an H_b is estimated from a flight distance and a $\beta\gamma$ factor which is used to convert the geometrical distance into the proper decay time. Methods of accessing lifetime information can roughly be divided in the following five categories:

1. ***Inclusive (flavour-blind) measurements***. These measurements are aimed at extracting the lifetime from a mixture of b-hadron decays, without distinguishing the decaying species. Often the knowledge of the mixture composition is limited, which makes these measurements experiment-specific. Also, these measurements have to rely on Monte Carlo for estimating the $\beta\gamma$ factor, because the decaying hadrons are not fully reconstructed. On the bright side, these usually are the largest statistics b-hadron lifetime measurements that are accessible to a given experiment, and can, therefore, serve as an important performance benchmark.

2. ***Measurements in semileptonic decays of a specific H_b***. W from $b \to Wc$ produces $\ell\nu_l$ pair ($\ell = e, \mu$) in about 21% of the cases. Electron or muon from such decays is usually a well-detected signature, which provides for clean and efficient trigger. c quark from $b \to Wc$ transition and the other quark(s) making up the decaying H_b combine into a charm hadron, which is reconstructed in one or more exclusive decay channels. Knowing what this charmed hadron is allows one to separate, at least statistically, different H_b species. The advantage of these measurements is in statistics, which usually is superior to that of the exclusively reconstructed H_b decays. Some of the main disadvantages are related to the difficulty of estimating lepton+charm sample composition and Monte Carlo reliance for the $\beta\gamma$ factor estimate.

3. ***Measurements in exclusively reconstructed hadronic decays***. These have the advantage of complete reconstruction of decaying H_b, which allows one to infer the decaying species as well as to perform precise measurement of the $\beta\gamma$ factor. Both lead to generally smaller systematic uncertainties than in the above two categories. The downsides are smaller branching ratios, larger combinatoric backgrounds, especially in $H_b \to H_c\pi(\pi\pi)$ and multi-body H_c decays, or in a hadron collider environment with non-trivial underlying event. $H_b \to J/\psi\, H_s$ are relatively clean and easy to trigger on $J/\psi \to \ell^+\ell^-$, but their branching fraction is only about 1%.

4. ***Measurements at asymmetric B factories***.

   In the $\Upsilon(4S) \to B\overline{B}$ decay, the B mesons (B^+ or B^0) are essentially at rest in the $\Upsilon(4S)$ frame. This makes direct lifetime measurements impossible in experiments at symmetric colliders producing $\Upsilon(4S)$ at rest. At asymmetric B factories the $\Upsilon(4S)$ meson is boosted resulting in B and $\overline{B}$ moving nearly parallel to each other with the same boost. The lifetime is inferred from the distance Δz separating the B and $\overline{B}$ decay vertices along the beam axis and from the $\Upsilon(4S)$ boost known from the beam energies. This boost is equal to $\beta\gamma \approx 0.55$ (0.43) in the *BABAR* (Belle) experiment, resulting in an average B decay length of approximately 250 (190) μm.

   In order to determine the charge of the B mesons in each event, one of the them is fully reconstructed in a semileptonic or hadronic decay mode. The other B is typically not fully reconstructed, only the position of its decay vertex is determined from the remaining tracks in the event. These measurements benefit from large statistics, but suffer from poor proper time resolution, comparable to the B lifetime itself. This resolution is dominated by the uncertainty on the decay vertices, which is typically 50 (100) μm for a fully (partially) reconstructed B meson. With very large future statistics, the resolution and purity could be improved (and hence the systematics reduced) by fully reconstructing both B mesons in the event.

5. ***Direct measurement of lifetime ratios***. This method has so far been only applied in the measurement of $\tau(B^+)/\tau(B^0)$. The ratio of the lifetimes is extracted from the dependence of the observed relative number of B^+ and B^0 candidates (both reconstructed in semileptonic decays) on the proper decay time.

In some of the latest analyses, measurements of two (*e.g.* $\tau(B^+)$ and $\tau(B^+)/\tau(B^0)$) or three (*e.g.* $\tau(B^+)$, $\tau(B^+)/\tau(B^0)$, and Δm_d) quantities are combined. This introduces correlations among measurements. Another source of correlations among the measurements are the systematic effects, which could be common to an experiment or to an analysis technique across the experiments. When calculating the averages, such correlations are taken into account per general procedure, described in Ref. [52].

3.2.2 Inclusive *b*-hadron lifetimes

The inclusive b hadron lifetime is defined as $\tau_b = \sum_i f_i \tau_i$ where τ_i are the individual species lifetimes and f_i are the fractions of the various species present in an unbiased sample of weakly-decaying b hadrons produced at a high-energy collider.[9] This quantity is certainly less fundamental than the lifetimes of the individual species, the latter being much more useful in comparisons of the measurements with the theoretical predictions. Nonetheless, we perform the averaging of the inclusive lifetime measurements for completeness as well as for the reason that they might be of interest as "technical numbers."

In practice, an unbiased measurement of the inclusive lifetime is difficult to achieve, because it would imply an efficiency which is guaranteed to be the same across species. So most of the measurements are biased. In an attempt to group analyses which are expected to select the

[9] In principle such a quantity could be slightly different in Z decays and at the Tevatron, in case the fractions of b-hadron species are not exactly the same; see the discussion in Sec. 3.1.3.

Table 6: Measurements of average b-hadron lifetimes.

Experiment	Method	Data set	τ_b (ps)	Ref.
ALEPH	Dipole	91	$1.511 \pm 0.022 \pm 0.078$	[53]
DELPHI	All track i.p. (2D)	91–92	$1.542 \pm 0.021 \pm 0.045$	[54][a]
DELPHI	Sec. vtx	91–93	$1.582 \pm 0.011 \pm 0.027$	[55][a]
DELPHI	Sec. vtx	94–95	$1.570 \pm 0.005 \pm 0.008$	[56]
L3	Sec. vtx + i.p.	91–94	$1.556 \pm 0.010 \pm 0.017$	[57][b]
OPAL	Sec. vtx	91–94	$1.611 \pm 0.010 \pm 0.027$	[58]
SLD	Sec. vtx	93	$1.564 \pm 0.030 \pm 0.036$	[59]
Average set 1 (b vertex)			1.572 ± 0.009	
ALEPH	Lepton i.p. (3D)	91–93	$1.533 \pm 0.013 \pm 0.022$	[60]
L3	Lepton i.p. (2D)	91–94	$1.544 \pm 0.016 \pm 0.021$	[57][b]
OPAL	Lepton i.p. (2D)	90–91	$1.523 \pm 0.034 \pm 0.038$	[61]
Average set 2 ($b \to \ell$)			1.537 ± 0.020	
CDF1	J/ψ vtx	92–95	$1.533 \pm 0.015^{+0.035}_{-0.031}$	[62]
ATLAS	J/ψ vtx	2010	$1.489 \pm 0.016 \pm 0.043$	[63][p]
Average set 3 ($b \to J/\psi$)			1.516 ± 0.028	
Average of all above			1.566 ± 0.009	

[a] The combined DELPHI result quoted in [55] is $1.575 \pm 0.010 \pm 0.026$ ps.
[b] The combined L3 result quoted in [57] is $1.549 \pm 0.009 \pm 0.015$ ps.
[p] Preliminary.

same mixture of b hadrons, the available results (given in Table 6) are divided into the following three sets:

1. measurements at LEP and SLD that accept any b-hadron decay, based on topological reconstruction (secondary vertex or track impact parameters);

2. measurements at LEP based on the identification of a lepton from a b decay; and

3. measurements at the Tevatron based on inclusive $H_b \to J/\psi X$ reconstruction, where the J/ψ is fully reconstructed.

The measurements of the first set are generally considered as estimates of τ_b, although the efficiency to reconstruct a secondary vertex most probably depends, in an analysis-specific way, on the number of tracks coming from the vertex, thereby depending on the type of the H_b. Even though these efficiency variations can in principle be accounted for using Monte Carlo simulations (which inevitably contain assumptions on branching fractions), the H_b mixture in that case can remain somewhat ill-defined and could be slightly different among analyses in this set.

On the contrary, the mixtures corresponding to the other two sets of measurements are better defined in the limit where the reconstruction and selection efficiency of a lepton or a J/ψ from an H_b does not depend on the decaying hadron type. These mixtures are given by the production fractions and the inclusive branching fractions for each H_b species to give a lepton or a J/ψ. In particular, under the assumption that all b hadrons have the same semileptonic decay width, the analyses of the second set should measure $\tau(b \to \ell) = (\sum_i f_i \tau_i^2)/(\sum_i f_i \tau_i)$ which is necessarily larger than τ_b if lifetime differences exist. Given the present knowledge on τ_i and f_i, $\tau(b \to \ell) - \tau_b$ is expected to be of the order of 0.01 ps.

Measurements by SLC and LEP experiments are subject to a number of common systematic uncertainties, such as those due to (lack of knowledge of) b and c fragmentation, b and c decay models, $\mathcal{B}(B \to \ell)$, $\mathcal{B}(B \to c \to \ell)$, $\mathcal{B}(c \to \ell)$, τ_c, and H_b decay multiplicity. In the averaging, these systematic uncertainties are assumed to be 100% correlated. The averages for the sets defined above (also given in Table 6) are

$$\tau(b \text{ vertex}) = 1.572 \pm 0.009 \text{ ps}, \quad (19)$$

$$\tau(b \to \ell) = 1.537 \pm 0.020 \text{ ps}, \quad (20)$$

$$\tau(b \to J/\psi) = 1.516 \pm 0.028 \text{ ps}, \quad (21)$$

whereas an average of all measurements, ignoring mixture differences, yields 1.566 ± 0.009 ps.

3.2.3 B^0 and B^+ lifetimes and their ratio

After a number of years of dominating these averages the LEP experiments yielded the scene to the asymmetric B factories and the Tevatron experiments. The B factories have been very successful in utilizing their potential – in only a few years of running, BABAR and, to a greater extent, Belle, have struck a balance between the statistical and the systematic uncertainties, with both being close to (or even better than) the impressive 1%. In the meanwhile, CDF and D0 have emerged as significant contributors to the field as the Tevatron Run II data flowed in, with CDF eventually providing the most precise results.

At present time we are in an interesting position of having three sets of measurements (from LEP/SLC, B factories and the Tevatron) that originate from different environments, obtained using substantially different techniques and are precise enough for incisive comparison.

The averaging of $\tau(B^+)$, $\tau(B^0)$ and $\tau(B^+)/\tau(B^0)$ measurements is summarized[10] in Tables 7, 8, and 9. For $\tau(B^+)/\tau(B^0)$ we averaged only the measurements of this quantity provided by experiments rather than using all available knowledge, which would have included, for example, $\tau(B^+)$ and $\tau(B^0)$ measurements which did not contribute to any of the ratio measurements.

The following sources of correlated (within experiment/machine) systematic uncertainties have been considered:

- for SLC/LEP measurements – D^{**} branching ratio uncertainties [42], momentum estimation of b mesons from Z^0 decays (b-quark fragmentation parameter $\langle X_E \rangle = 0.702 \pm 0.008$ [42]), B_s^0 and b baryon lifetimes (see Secs. 3.2.4 and 3.2.6), and b-hadron fractions at high energy (see Table 5);

- for BABAR measurements – alignment, z scale, PEP-II boost, sample composition (where applicable);

- for D0 and CDF Run II measurements – alignment (separately within each experiment).

[10] We do not include the old unpublished measurements of Refs. [89, 90].

Table 7: Measurements of the B^0 lifetime.

Experiment	Method	Data set	$\tau(B^0)$ (ps)	Ref.
ALEPH	$D^{(*)}\ell$	91–95	$1.518 \pm 0.053 \pm 0.034$	[64]
ALEPH	Exclusive	91–94	$1.25^{+0.15}_{-0.13} \pm 0.05$	[65]
ALEPH	Partial rec. $\pi^+\pi^-$	91–94	$1.49^{+0.17+0.08}_{-0.15-0.06}$	[65]
DELPHI	$D^{(*)}\ell$	91–93	$1.61^{+0.14}_{-0.13} \pm 0.08$	[66]
DELPHI	Charge sec. vtx	91–93	$1.63 \pm 0.14 \pm 0.13$	[67]
DELPHI	Inclusive $D^*\ell$	91–93	$1.532 \pm 0.041 \pm 0.040$	[68]
DELPHI	Charge sec. vtx	94–95	$1.531 \pm 0.021 \pm 0.031$	[56]
L3	Charge sec. vtx	94–95	$1.52 \pm 0.06 \pm 0.04$	[69]
OPAL	$D^{(*)}\ell$	91–93	$1.53 \pm 0.12 \pm 0.08$	[70]
OPAL	Charge sec. vtx	93–95	$1.523 \pm 0.057 \pm 0.053$	[71]
OPAL	Inclusive $D^*\ell$	91–00	$1.541 \pm 0.028 \pm 0.023$	[72]
SLD	Charge sec. vtx ℓ	93–95	$1.56^{+0.14}_{-0.13} \pm 0.10$	[73][a]
SLD	Charge sec. vtx	93–95	$1.66 \pm 0.08 \pm 0.08$	[73][a]
CDF1	$D^{(*)}\ell$	92–95	$1.474 \pm 0.039^{+0.052}_{-0.051}$	[74]
CDF1	Excl. $J/\psi K^{*0}$	92–95	$1.497 \pm 0.073 \pm 0.032$	[75]
CDF2	Excl. $J/\psi K_S$, $J/\psi K^{*0}$	02–09	$1.507 \pm 0.010 \pm 0.008$	[76]
D0	Excl. $J/\psi K^{*0}$	03–07	$1.414 \pm 0.018 \pm 0.034$	[77]
D0	Excl. $J/\psi K_S$	02–11	$1.508 \pm 0.025 \pm 0.043$	[78]
BABAR	Exclusive	99–00	$1.546 \pm 0.032 \pm 0.022$	[79]
BABAR	Inclusive $D^*\ell$	99–01	$1.529 \pm 0.012 \pm 0.029$	[80]
BABAR	Exclusive $D^*\ell$	99–02	$1.523^{+0.024}_{-0.023} \pm 0.022$	[81]
BABAR	Incl. $D^*\pi$, $D^*\rho$	99–01	$1.533 \pm 0.034 \pm 0.038$	[82]
BABAR	Inclusive $D^*\ell$	99–04	$1.504 \pm 0.013^{+0.018}_{-0.013}$	[83]
Belle	Exclusive	00–03	$1.534 \pm 0.008 \pm 0.010$	[84]
ATLAS	Excl. $J/\psi K^{*0}$	2010	$1.51 \pm 0.04 \pm 0.04$	[85][p]
LHCb	Excl. $J/\psi K^{*0}$	2011	$1.524 \pm 0.006 \pm 0.004$	[86]
LHCb	Excl. $J/\psi K_S$	2011	$1.499 \pm 0.013 \pm 0.005$	[86]
Average			1.519 ± 0.005	

[a] The combined SLD result quoted in [73] is 1.64 ± 0.08 ± 0.08 ps.
[p] Preliminary.

The resultant averages are:

$$\tau(B^0) = 1.519 \pm 0.005 \text{ ps}\,, \quad (22)$$
$$\tau(B^+) = 1.638 \pm 0.004 \text{ ps}\,, \quad (23)$$
$$\tau(B^+)/\tau(B^0) = 1.076 \pm 0.004\,. \quad (24)$$

3.2.4 B^0_s lifetimes

Like neutral kaons, neutral B mesons contain short- and long-lived components, since the light (L) and heavy (H) eigenstates, B_L and B_H, differ not only in their masses, but also in their total decay widths, with a decay width difference defined as $\Delta\Gamma = \Gamma_\mathrm{L} - \Gamma_\mathrm{H}$. Neglecting *CP*

Table 8: Measurements of the B^+ lifetime.

Experiment	Method	Data set	$\tau(B^+)$ (ps)	Ref.
ALEPH	$D^{(*)}\ell$	91–95	$1.648 \pm 0.049 \pm 0.035$	[64]
ALEPH	Exclusive	91–94	$1.58^{+0.21+0.04}_{-0.18-0.03}$	[65]
DELPHI	$D^{(*)}\ell$	91–93	$1.61 \pm 0.16 \pm 0.12$	[66][a]
DELPHI	Charge sec. vtx	91–93	$1.72 \pm 0.08 \pm 0.06$	[67][a]
DELPHI	Charge sec. vtx	94–95	$1.624 \pm 0.014 \pm 0.018$	[56]
L3	Charge sec. vtx	94–95	$1.66 \pm 0.06 \pm 0.03$	[69]
OPAL	$D^{(*)}\ell$	91–93	$1.52 \pm 0.14 \pm 0.09$	[70]
OPAL	Charge sec. vtx	93–95	$1.643 \pm 0.037 \pm 0.025$	[71]
SLD	Charge sec. vtx ℓ	93–95	$1.61^{+0.13}_{-0.12} \pm 0.07$	[73][b]
SLD	Charge sec. vtx	93–95	$1.67 \pm 0.07 \pm 0.06$	[73][b]
CDF1	$D^{(*)}\ell$	92–95	$1.637 \pm 0.058^{+0.045}_{-0.043}$	[74]
CDF1	Excl. $J/\psi K$	92–95	$1.636 \pm 0.058 \pm 0.025$	[75]
CDF2	Excl. $J/\psi K$	02–09	$1.639 \pm 0.009 \pm 0.009$	[76]
CDF2	Excl. $D^0\pi$	02–06	$1.663 \pm 0.023 \pm 0.015$	[87]
BABAR	Exclusive	99–00	$1.673 \pm 0.032 \pm 0.023$	[79]
Belle	Exclusive	00–03	$1.635 \pm 0.011 \pm 0.011$	[84]
LHCb	Excl. $J/\psi K$	2011	$1.637 \pm 0.004 \pm 0.003$	[86]
Average			1.638 ± 0.004	

[a] The combined DELPHI result quoted in [67] is 1.70 ± 0.09 ps.
[b] The combined SLD result quoted in [73] is $1.66 \pm 0.06 \pm 0.05$ ps.

violation in $B - \overline{B}$ mixing, which is expected to be very small [91, 92], the mass eigenstates are also *CP* eigenstates, with the light $B_{\rm L}$ state being *CP*-even and the heavy $B_{\rm H}$ state being *CP*-odd. While the decay width difference $\Delta\Gamma_d$ can be neglected in the B^0 system, the B^0_s system exhibits a significant value of $\Delta\Gamma_s$: the sign of $\Delta\Gamma_s$ is known to be positive [93], *i.e.* the heavy eigenstates lives longer than the light eigenstate. Specific measurements of $\Delta\Gamma_s$ and $\Gamma_s = (\Gamma_{\rm L} + \Gamma_{\rm H})/2$ are explained and averaged in Sec. 3.3.2, but the results for $1/\Gamma_{\rm L}$, $1/\Gamma_{\rm H}$ and the mean B^0_s lifetime, defined as $\tau(B^0_s) = 1/\Gamma_s$, are also quoted at the end of this section.

Many B^0_s lifetime analyses, in particular the early ones performed before the non-zero value of $\Delta\Gamma_s$ was firmly established, ignore $\Delta\Gamma_s$ and fit the proper time distribution of a sample of B^0_s candidates reconstructed in a certain final state f with a model assuming a single exponential function for the signal. We denote such effective lifetime measurements as $\tau_{\rm single}(B^0_s \to f)$; their true values may lie *a priori* anywhere between $1/\Gamma_{\rm L} = 1/(\Gamma_s + \Delta\Gamma_s/2)$ and $1/\Gamma_{\rm H} = 1/(\Gamma_s - \Delta\Gamma_s/2)$, depending on the proportion of $B_{\rm L}$ and $B_{\rm H}$ in the final state f. Table 10 summarizes the effective lifetime measurements.

Averaging measurements of $\tau_{\rm single}(B^0_s \to f)$ over several final states f will yield a result corresponding to an ill-defined observable when the proportions of $B_{\rm L}$ and $B_{\rm H}$ differ. Therefore, the effective B^0_s lifetime measurements are broken down into several categories and averaged separately.

- ***Flavour-specific decays***, such as semileptonic $B^0_s \to D^-_s \ell^+ \nu$ or $B^0_s \to D^-_s \pi^+$, have equal fractions of $B_{\rm L}$ and $B_{\rm H}$ at time zero. If the resulting superposition of two exponential

Table 9: Measurements of the ratio $\tau(B^+)/\tau(B^0)$.

Experiment	Method	Data set	Ratio $\tau(B^+)/\tau(B^0)$	Ref.
ALEPH	$D^{(*)}\ell$	91–95	$1.085 \pm 0.059 \pm 0.018$	[64]
ALEPH	Exclusive	91–94	$1.27^{+0.23+0.03}_{-0.19-0.02}$	[65]
DELPHI	$D^{(*)}\ell$	91–93	$1.00^{+0.17}_{-0.15} \pm 0.10$	[66]
DELPHI	Charge sec. vtx	91–93	$1.06^{+0.13}_{-0.11} \pm 0.10$	[67]
DELPHI	Charge sec. vtx	94–95	$1.060 \pm 0.021 \pm 0.024$	[56]
L3	Charge sec. vtx	94–95	$1.09 \pm 0.07 \pm 0.03$	[69]
OPAL	$D^{(*)}\ell$	91–93	$0.99 \pm 0.14^{+0.05}_{-0.04}$	[70]
OPAL	Charge sec. vtx	93–95	$1.079 \pm 0.064 \pm 0.041$	[71]
SLD	Charge sec. vtx ℓ	93–95	$1.03^{+0.16}_{-0.14} \pm 0.09$	[73][a]
SLD	Charge sec. vtx	93–95	$1.01^{+0.09}_{-0.08} \pm 0.05$	[73][a]
CDF1	$D^{(*)}\ell$	92–95	$1.110 \pm 0.056^{+0.033}_{-0.030}$	[74]
CDF1	Excl. $J/\psi K$	92–95	$1.093 \pm 0.066 \pm 0.028$	[75]
CDF2	Excl. $J/\psi K^{(*)}$	02–09	$1.088 \pm 0.009 \pm 0.004$	[76]
D0	$D^{*+}\mu$ $D^0\mu$ ratio	02–04	$1.080 \pm 0.016 \pm 0.014$	[88]
BABAR	Exclusive	99–00	$1.082 \pm 0.026 \pm 0.012$	[79]
Belle	Exclusive	00–03	$1.066 \pm 0.008 \pm 0.008$	[84]
LHCb	Excl. $J/\psi K^{(*)}$	2011	$1.074 \pm 0.005 \pm 0.003$	[86]
Average			1.076 ± 0.004	

[a] The combined SLD result quoted in [73] is $1.01 \pm 0.07 \pm 0.06$.

distributions is fitted with a single exponential function, one obtains a measure of the so-called *flavour-specific lifetime* [111]:

$$\tau_{\rm single}(B_s^0 \to \text{flavour specific}) = \frac{1}{\Gamma_s} \frac{1 + \left(\frac{\Delta\Gamma_s}{2\Gamma_s}\right)^2}{1 - \left(\frac{\Delta\Gamma_s}{2\Gamma_s}\right)^2}. \tag{25}$$

The average of all flavour-specific B_s^0 lifetime measurements[11] is

$$\tau_{\rm single}(B_s^0 \to \text{flavour specific}) = 1.465 \pm 0.031 \text{ ps}\,. \tag{26}$$

- $\boldsymbol{B_s^0 \to D_s^{\mp} X}$ ***decays*** include flavour-specific decays but also decays with a less known mixture of light and heavy components. The corresponding effective lifetime average,

$$\tau_{\rm single}(B_s^0 \to D_s^{\mp} X) = 1.469 \pm 0.031 \text{ ps}\,, \tag{27}$$

can still be a useful input for analyses examining an inclusive D_s sample. The following correlated systematic errors were considered: average B lifetime used in backgrounds, B_s^0 decay multiplicity, and branching ratios used to determine backgrounds (*e.g.* $\mathcal{B}(B \to D_s D)$). A knowledge of the multiplicity of B_s^0 decays is important for measurements

[11] An old unpublished measurement [112] is not included.

Table 10: Measurements of the effective B_s^0 lifetimes obtained from single exponential fits, without attempting to separate the short and long components.

Experiment	Final state f		Data set		$\tau_{\text{single}}(B_s^0 \to f)$ (ps)	Ref.
ALEPH	$D_s^- \ell^+$	flavour-specific	91–95		$1.54^{+0.14}_{-0.13} \pm 0.04$	[94]
CDF1	$D_s^- \ell^+$	flavour-specific	92–96		$1.36 \pm 0.09^{+0.06}_{-0.05}$	[95]
DELPHI	$D_s^- \ell^+$	flavour-specific	91–95		$1.42^{+0.14}_{-0.13} \pm 0.03$	[96]
OPAL	$D_s^- \ell^+$	flavour-specific	90–95		$1.50^{+0.16}_{-0.15} \pm 0.04$	[97]
D0	$D_s^- \mu^+$	flavour-specific	02–04		$1.398 \pm 0.044^{+0.028}_{-0.025}$	[98]
CDF2	$D_s^- \pi^+(X)$	flavour-specific	02–06	1.3 fb^{-1}	$1.518 \pm 0.041 \pm 0.027$	[99]
LHCb	$D_s^- D^+$	flavour-specific	11–12	3 fb^{-1}	$1.52 \pm 0.15 \pm 0.01$	[100]
Average of above 7 flavour-specific measurements					1.465 ± 0.031	
ALEPH	$D_s h$	mixture	91–95		$1.47 \pm 0.14 \pm 0.08$	[101]
DELPHI	$D_s h$	mixture	91–95		$1.53^{+0.16}_{-0.15} \pm 0.07$	[102]
OPAL	D_s incl.	mixture	90–95		$1.72^{+0.20+0.18}_{-0.19-0.17}$	[103]
Average of above 10 D_s measurements					1.469 ± 0.031	
CDF1	$J/\psi\,\phi$	CP even+odd	92–95		$1.34^{+0.23}_{-0.19} \pm 0.05$	[62]
D0	$J/\psi\,\phi$	CP even+odd	02–04		$1.444^{+0.098}_{-0.090} \pm 0.02$	[104]
ATLAS	$J/\psi\,\phi$	CP even+odd	10	40 pb^{-1}	$1.41 \pm 0.08 \pm 0.05$	[85][p]
LHCb	$J/\psi\,\phi$	CP even+odd	11	1 fb^{-1}	$1.480 \pm 0.011 \pm 0.005$	[86]
Average of above 4 $J/\psi\,\phi$ measurements					1.478 ± 0.012	
ALEPH	$D_s^{(*)+} D_s^{(*)-}$	CP even+odd	91–95		$1.27 \pm 0.33 \pm 0.08$	[105]
LHCb	K^+K^-	CP-even	10	0.037 fb^{-1}	$1.440 \pm 0.096 \pm 0.009$	[106]
LHCb	K^+K^-	CP-even	11	1.0 fb^{-1}	$1.455 \pm 0.046 \pm 0.006$	[107]
Average of above 2 K^+K^- measurements					1.452 ± 0.042	
LHCb	$D_s^+ D_s^-$	CP-even	11–12	3 fb^{-1}	$1.379 \pm 0.026 \pm 0.017$	[100]
Average of above 3 CP-even measurements					1.405 ± 0.025	
CDF2	$J/\psi\, f_0(980)$	CP-odd	02–08	3.8 fb^{-1}	$1.70^{+0.12}_{-0.11} \pm 0.03$	[108]
LHCb	$J/\psi\,\pi^+\pi^-$	CP-odd	11	1.0 fb^{-1}	$1.652 \pm 0.024 \pm 0.024$	[109]
Average of above 2 $J/\psi\, f_0(980)$, $J/\psi\,\pi^+\pi^-$ measurements					1.656 ± 0.033	
LHCb	$J/\psi\, K_S^0$	CP-odd	11	1.0 fb^{-1}	$1.75 \pm 0.12 \pm 0.07$	[110]
Average of above 3 CP-odd measurements					1.661 ± 0.032	

[p] Preliminary.

that partially reconstruct the final state such as $B \to D_s X$ (where X is not a lepton). The boost deduced from Monte Carlo simulation depends on the multiplicity used. Since this is not well known, the multiplicity in the simulation is varied and this range of values observed is taken to be a systematic. Similarly not all the branching ratios for the potential background processes are measured. Where they are available, the PDG values are used for the error estimate. Where no measurements are available estimates can usually be made by using measured branching ratios of related processes and using some reasonable extrapolation.

- ***$B_s^0 \to J/\psi\,\phi$ decays*** contain a well-defined mixture of CP-even and CP-odd states

There are no known correlations between the existing $B_s^0 \to J/\psi\,\phi$ effective lifetime measurements; these are combined into the average[12]

$$\tau_{\text{single}}(B_s^0 \to J/\psi\,\phi) = 1.478 \pm 0.012 \text{ ps}\,. \tag{28}$$

A caveat is that different experimental acceptances may lead to different admixtures of the CP-even and CP-odd states, and simple fits to a single exponential may result in inherently different values of $\tau_{\text{single}}(B_s^0 \to J/\psi\,\phi)$. Analyses that separate the CP-even and CP-odd components in this decay through a full angular study, outlined in Sec. 3.3.2, provide directly measurements of $1/\Gamma_s$ and $\Delta\Gamma_s$ (see Table 21).

- ***Decays to pure CP eigenstates*** have also been measured, in the CP-even modes $B_s^0 \to K^+K^-$ by LHCb [106, 107][13] and $B_s^0 \to D_s^+D_s^-$ by LHCb [100], as well as in the CP-odd modes $B_s^0 \to J/\psi\,f_0(980)$ by CDF [108], $B_s^0 \to J/\psi\,\pi^+\pi^-$ by LHCb [109] and $B_s^0 \to J/\psi\,K_S^0$ by LHCb [110]. If these decays are dominated by a single weak phase and if CP violation can be neglected, then $\tau_{\text{single}}(B_s^0 \to CP\text{-even}) \sim 1/\Gamma_L$ and $\tau_{\text{single}}(B_s^0 \to CP\text{-odd}) \sim 1/\Gamma_H$ (see Eqs. (50) and (51) for approximate relations in presence of CP violation in the mixing). The averages for the effective lifetimes obtained with CP-even and CP-odd final states are

$$\tau_{\text{single}}(B_s^0 \to CP\text{-even}) = 1.405 \pm 0.025 \text{ ps}\,, \tag{29}$$
$$\tau_{\text{single}}(B_s^0 \to CP\text{-odd}) = 1.661 \pm 0.032 \text{ ps}\,. \tag{30}$$

A measurement of the effective lifetime of $B_s^0 \to D_s^{(*)+}D_s^{(*)-}$ decays by ALEPH [105] is not included in the above CP-even average, since a small CP-odd component is most probably present.

As described in Sec. 3.3.2, the effective lifetime averages of Eqs. (26), (29), and (30) are used as ingredients to improve the determination of $1/\Gamma_s$ and $\Delta\Gamma_s$ obtained from the full angular analyses of $B_s^0 \to J/\psi\,\phi$ decays. The resulting world averages for the B_s^0 lifetimes are

$$\frac{1}{\Gamma_L} = \frac{1}{\Gamma_s + \Delta\Gamma_s/2} = 1.427 \pm 0.009 \text{ ps}\,, \tag{31}$$
$$\frac{1}{\Gamma_H} = \frac{1}{\Gamma_s - \Delta\Gamma_s/2} = 1.620 \pm 0.014 \text{ ps}\,, \tag{32}$$
$$\tau(B_s^0) = \frac{1}{\Gamma_s} = \frac{2}{\Gamma_L + \Gamma_H} = 1.517 \pm 0.007 \text{ ps}\,. \tag{33}$$

3.2.5 B_c^+ lifetime

Early measurements of the B_c^+ meson lifetime, from CDF [115–117] and D0 [118], use the semileptonic decay mode $B_c^+ \to J/\psi\,\ell$ and are based on a simultaneous fit to the mass and lifetime using the vertex formed with the leptons from the decay of the J/ψ and the third lepton. Correction factors to estimate the boost due to the missing neutrino are used. In the analysis of the CDF Run I data [115], a mass value of $6.40 \pm 0.39 \pm 0.13$ GeV/c^2 is found by

[12] An old unpublished measurement [113] is not included.
[13] An old unpublished measurement of the $B_s^0 \to K^+K^-$ effective lifetime by CDF [114] is no longer considered.

Table 11: Measurements of the B_c^+ lifetime.

Experiment	Method	Data set		$\tau(B_c^+)$ (ps)	Ref.
CDF1	$J/\psi\,\ell$	92–95	0.11 fb^{-1}	$0.46^{+0.18}_{-0.16} \pm 0.03$	[115]
CDF2	$J/\psi\,e$	02–04	0.36 fb^{-1}	$0.463^{+0.073}_{-0.065} \pm 0.036$	[117]
D0	$J/\psi\,\mu$	02–06	1.3 fb^{-1}	$0.448^{+0.038}_{-0.036} \pm 0.032$	[118]
CDF2	$J/\psi\,\pi$		6.7 fb^{-1}	$0.452 \pm 0.048 \pm 0.027$	[121]
LHCb	$J/\psi\,\mu$	12	2 fb^{-1}	$0.509 \pm 0.008 \pm 0.012$	[122]
Average				0.500 ± 0.013	

fitting to the tri-lepton invariant mass spectrum. In the CDF Run II result [117], the mass is fixed to 6.271 GeV/c^2, but then varied between 6.2 and 6.4 GeV/c^2 to assess the systematic error on the lifetime due to the B_c^+ mass value. Finally, in the D0 Run II result [118], the B_c^+ mass is assumed to be $6285.7 \pm 5.3 \pm 1.2$ MeV/c^2, taken from a CDF result [119]. These mass measurements are consistent within uncertainties, and also consistent with the most recent precision determination from CDF of $6275.6 \pm 2.9 \pm 2.5$ MeV/c^2 [120]. Correlated systematic errors include the impact of the uncertainty of the B_c^+ p_T spectrum on the correction factors, the level of feed-down from $\psi(2S)$, Monte-Carlo modeling of the decay model varying from phase space to the ISGW model, and mass variations.

The latest determination of the B_c^+ lifetime from CDF2 [121] is based on fully reconstructed $B_c^+ \to J/\psi\pi$ decays and does not suffer from a missing neutrino. All the measurements[14] are summarized in Table 11 and the world average, dominated by a recent LHCb measurement [122], is determined to be

$$\tau(B_c^+) = 0.500 \pm 0.013 \text{ ps}\,. \tag{34}$$

3.2.6 Λ_b^0 and b-baryon lifetimes

The first measurements of b-baryon lifetimes originate from two classes of partially reconstructed decays. In the first class, decays with an exclusively reconstructed Λ_c^+ baryon and a lepton of opposite charge are used. These products are more likely to occur in the decay of Λ_b^0 baryons. In the second class, more inclusive final states with a baryon (p, $\overline{p}$, Λ, or $\overline{\Lambda}$) and a lepton have been used, and these final states can generally arise from any b baryon. With the large b-hadron samples available at the Tevatron, the most precise measurements of b-baryons now come from fully reconstructed exclusive decays.

The following sources of correlated systematic uncertainties have been considered: experimental time resolution within a given experiment, b-quark fragmentation distribution into weakly decaying b baryons, Λ_b^0 polarization, decay model, and evaluation of the b-baryon purity in the selected event samples. In computing the averages the central values of the masses are scaled to $M(\Lambda_b^0) = 5620 \pm 2$ MeV/c^2 [123] and $M(b\text{-baryon}) = 5670 \pm 100$ MeV/c^2.

For the semi-inclusive lifetime measurements, the meaning of decay model systematic uncertainties and the correlation of these uncertainties between measurements are not always clear. Uncertainties related to the decay model are dominated by assumptions on the fraction of n-body semileptonic decays. To be conservative it is assumed that these are 100% correlated

[14] We do not list (nor include in the average) an unpublished result from CDF2 [116].

Table 12: Measurements of the b-baryon lifetimes.

Experiment	Method	Data set	Lifetime (ps)	Ref.
ALEPH	$\Lambda_c^+\ell$	91–95	$1.18^{+0.13}_{-0.12} \pm 0.03$	[26][a]
ALEPH	$\Lambda\ell^-\ell^+$	91–95	$1.30^{+0.26}_{-0.21} \pm 0.04$	[26][a]
DELPHI	$\Lambda_c^+\ell$	91–94	$1.11^{+0.19}_{-0.18} \pm 0.05$	[125][b]
OPAL	$\Lambda_c^+\ell$, $\Lambda\ell^-\ell^+$	90–95	$1.29^{+0.24}_{-0.22} \pm 0.06$	[97]
CDF1	$\Lambda_c^+\ell$	91–95	$1.32 \pm 0.15 \pm 0.07$	[126]
CDF2	$\Lambda_c^+\pi$	02–06	$1.401 \pm 0.046 \pm 0.035$	[127]
CDF2	$J/\psi\,\Lambda$	02–11	$1.565 \pm 0.035 \pm 0.020$	[128]
D0	$\Lambda_c^+\mu$	02–06	$1.290^{+0.119+0.087}_{-0.110-0.091}$	[129]
D0	$J/\psi\,\Lambda$	02–11	$1.303 \pm 0.075 \pm 0.035$	[78]
ATLAS	$J/\psi\,\Lambda$	2011	$1.449 \pm 0.036 \pm 0.017$	[130]
CMS	$J/\psi\,\Lambda$	2011	$1.503 \pm 0.052 \pm 0.031$	[131]
LHCb	$J/\psi\,\Lambda$	2011	$1.415 \pm 0.027 \pm 0.006$	[86]
LHCb	$J/\psi\,pK$	11–12	$1.479 \pm 0.009 \pm 0.010$	[132]
Average of above 13:		Λ_b^0 lifetime =	1.466 ± 0.010	
ALEPH	$\Lambda\ell$	91–95	$1.20 \pm 0.08 \pm 0.06$	[26]
DELPHI	$\Lambda\ell\pi$ vtx	91–94	$1.16 \pm 0.20 \pm 0.08$	[125][b]
DELPHI	$\Lambda\mu$ i.p.	91–94	$1.10^{+0.19}_{-0.17} \pm 0.09$	[133][b]
DELPHI	$p\ell$	91–94	$1.19 \pm 0.14 \pm 0.07$	[125][b]
OPAL	$\Lambda\ell$ i.p.	90–94	$1.21^{+0.15}_{-0.13} \pm 0.10$	[134][c]
OPAL	$\Lambda\ell$ vtx	90–94	$1.15 \pm 0.12 \pm 0.06$	[134][c]
Average of above 19:	mean b-baryon lifetime =		1.466 ± 0.010	
CDF2	$J/\psi\,\Xi^-$	02–11	$1.32 \pm 0.14 \pm 0.02$	[128]
Average of above 1:		Ξ_b^- lifetime =	1.32 ± 0.14	
ALEPH	$\Xi\ell$	90–95	$1.35^{+0.37+0.15}_{-0.28-0.17}$	[27]
DELPHI	$\Xi\ell$	91–93	$1.5^{+0.7}_{-0.4} \pm 0.3$	[29][d]
DELPHI	$\Xi\ell$	92–95	$1.45^{+0.55}_{-0.43} \pm 0.13$	[28][d]
Average of above 4:		mean Ξ_b lifetime =	1.34 ± 0.12	
CDF2	$J/\psi\,\Omega^-$	02–11	$1.66^{+0.53}_{-0.40} \pm 0.02$	[128]
Average of above 1:		Ω_b^- lifetime =	$1.66^{+0.53}_{-0.40}$	

[a] The combined ALEPH result quoted in [26] is 1.21 ± 0.11 ps.
[b] The combined DELPHI result quoted in [125] is $1.14 \pm 0.08 \pm 0.04$ ps.
[c] The combined OPAL result quoted in [134] is $1.16 \pm 0.11 \pm 0.06$ ps.
[d] The combined DELPHI result quoted in [28] is $1.48^{+0.40}_{-0.31} \pm 0.12$ ps.
[p] Preliminary.

whenever given as an error. DELPHI varies the fraction of 4-body decays from 0.0 to 0.3. In computing the average, the DELPHI result is corrected to a value of 0.2 ± 0.2 for this fraction.

Furthermore, in computing the average, the semileptonic decay results from LEP are corrected for a polarization of $-0.45^{+0.19}_{-0.17}$ [42] and a Λ_b^0 fragmentation parameter $\langle X_E \rangle = 0.70 \pm 0.03$ [124].

Inputs to the averages are given in Table 12. The CDF $\Lambda_b \to J/\psi\,\Lambda$ lifetime result [128] is

Table 13: Summary of lifetimes of different b-hadron species.

b-hadron species	Measured lifetime
B^+	1.638 ± 0.004 ps
B^0	1.519 ± 0.005 ps
B_s^0 $(1/\Gamma_s)$	1.517 ± 0.007 ps
B_c^+	0.500 ± 0.013 ps
Λ_b^0	1.466 ± 0.010 ps
Ξ_b^-	1.32 ± 0.14 ps
Ω_b^-	$1.66^{+0.53}_{-0.40}$ ps
b-hadron mixture	1.566 ± 0.009 ps
b-baryon mixture	1.466 ± 0.010 ps
Ξ_b mixture	1.34 ± 0.12 ps

Table 14: Measured ratios of b-hadron lifetimes relative to the B^0 lifetime and ranges predicted by theory [50, 51].

Lifetime ratio	Measured value	Predicted range
$\tau(B^+)/\tau(B^0)$	1.076 ± 0.004	1.04 – 1.08
$\tau(B_s^0)/\tau(B^0)$	0.999 ± 0.006	0.99 – 1.01
$\tau(\Lambda_b^0)/\tau(B^0)$	0.965 ± 0.007	0.86 – 0.95

$2.5\,\sigma$ larger than the world average computed excluding this result. It is nonetheless combined with the rest without adjustment of input errors. The world average lifetime of b baryons is then

$$\langle \tau(b\text{-baryon}) \rangle = 1.466 \pm 0.010 \text{ ps}. \tag{35}$$

Keeping only $\Lambda_c^{\pm}\ell^{\mp}$, $\Lambda\ell^-\ell^+$, and fully exclusive final states, as representative of the Λ_b^0 baryon, the following lifetime is obtained:

$$\tau(\Lambda_b^0) = 1.466 \pm 0.010 \text{ ps}. \tag{36}$$

Averaging the measurements based on the $\Xi^{\mp}\ell^{\mp}$ [27–29] and $J/\psi\,\Xi^{\mp}$ [128] final states gives a lifetime value for a sample of events containing Ξ_b^0 and Ξ_b^- baryons:

$$\langle \tau(\Xi_b) \rangle = 1.34 \pm 0.12 \text{ ps}. \tag{37}$$

First measurements of fully reconstructed $\Xi_b^- \to J/\psi\,\Xi^-$ and $\Omega_b^- \to J/\psi\,\Omega^-$ baryons yield [128]

$$\tau(\Xi_b^-) = 1.32 \pm 0.14 \text{ ps}, \tag{38}$$

$$\tau(\Omega_b^-) = 1.66^{+0.53}_{-0.40} \text{ ps}. \tag{39}$$

3.2.7 Summary and comparison with theoretical predictions

Averages of lifetimes of specific b-hadron species are collected in Table 13. As described in Sec. 3.2, Heavy Quark Effective Theory can be employed to explain the hierarchy of $\tau(B_c^+) \ll$

$\tau(\Lambda_b^0) < \tau(B_s^0) \approx \tau(B^0) < \tau(B^+)$, and used to predict the ratios between lifetimes. Typical predictions are compared to the measured lifetime ratios in Table 14. The prediction of the ratio between the B^+ and B^0 lifetimes, 1.06 ± 0.02 [50], is in good agreement with experiment.

The total widths of the B_s^0 and B^0 mesons are expected to be very close and differ by at most 1% [51, 135]. This prediction is consistent with the experimental ratio $\tau(B_s^0)/\tau(B^0) = \Gamma_d/\Gamma_s$, which is smaller than 1 by (0.1 ± 0.6)%.

The ratio $\tau(\Lambda_b^0)/\tau(B^0)$ has particularly been the source of theoretical scrutiny since earlier calculations using Heavy Quark Effective Theory [47, 136] predicted a value larger than 0.90, almost $2\,\sigma$ above the world average at the time. Many predictions cluster around a most likely central value of 0.94 [137]. More recent calculations of this ratio that include higher-order effects predict a lower ratio between the Λ_b^0 and B^0 lifetimes [50, 51] and reduce this difference. References [50, 51] present probability density functions of their predictions with variation of theoretical inputs, and the indicated ranges in Table 14 are the RMS of the distributions from the most probable values, and for $\tau(\Lambda_b^0)/\tau(B^0)$, also encompass the earlier theoretical predictions [47,136,137]. Note that in contrast to the B mesons, complete NLO QCD corrections and fully reliable lattice determinations of the matrix elements for Λ_b^0 are not yet available. As already mentioned, the CDF measurement of the Λ_b lifetime in the exclusive decay mode $J/\psi\,\Lambda$ [128] is significantly higher than the world average before inclusion, with a ratio to the $\tau(B^0)$ world average of $\tau(\Lambda_b^0)/\tau(B^0) = 1.012 \pm 0.031$, resulting in continued interest in lifetimes of b baryons.

3.3 Neutral *B*-meson mixing

The $B^0-\overline{B}^0$ and $B_s^0-\overline{B}_s^0$ systems both exhibit the phenomenon of particle-antiparticle mixing. For each of them, there are two mass eigenstates which are linear combinations of the two flavour states, B and $\overline{B}$. The heaviest (lightest) of the these mass states is denoted $B_{\rm H}$ ($B_{\rm L}$), with mass $m_{\rm H}$ ($m_{\rm L}$) and total decay width $\Gamma_{\rm H}$ ($\Gamma_{\rm L}$). We define

$$\Delta m = m_{\rm H} - m_{\rm L}\,, \qquad x = \Delta m/\Gamma\,, \tag{40}$$

$$\Delta\Gamma = \Gamma_{\rm L} - \Gamma_{\rm H}\,, \qquad y = \Delta\Gamma/(2\Gamma)\,, \tag{41}$$

where $\Gamma = (\Gamma_{\rm H} + \Gamma_{\rm L})/2 = 1/\overline{\tau}(B)$ is the average decay width. Δm is positive by definition, and $\Delta\Gamma$ is expected to be positive within the Standard Model.[15]

There are four different time-dependent probabilities describing the case of a neutral B meson produced as a flavour state and decaying to a flavour-specific final state. If *CPT* is conserved (which will be assumed throughout), they can be written as

$$\left\{ \begin{array}{rcl} \mathcal{P}(B \to B) & = & \frac{e^{-\Gamma t}}{2} \left[\cosh\!\left(\frac{\Delta\Gamma}{2}t\right) + \cos(\Delta m t)\right] \\ \mathcal{P}(B \to \overline{B}) & = & \frac{e^{-\Gamma t}}{2} \left[\cosh\!\left(\frac{\Delta\Gamma}{2}t\right) - \cos(\Delta m t)\right] \left|\frac{q}{p}\right|^2 \\ \mathcal{P}(\overline{B} \to B) & = & \frac{e^{-\Gamma t}}{2} \left[\cosh\!\left(\frac{\Delta\Gamma}{2}t\right) - \cos(\Delta m t)\right] \left|\frac{p}{q}\right|^2 \\ \mathcal{P}(\overline{B} \to \overline{B}) & = & \frac{e^{-\Gamma t}}{2} \left[\cosh\!\left(\frac{\Delta\Gamma}{2}t\right) + \cos(\Delta m t)\right] \end{array} \right. , \tag{42}$$

[15] For reason of symmetry in Eqs. (40) and (41), $\Delta\Gamma$ is sometimes defined with the opposite sign. The definition adopted here, *i.e.* Eq. (41), is the one used by most experimentalists and many phenomenologists in B physics.

where t is the proper time of the system (*i.e.* the time interval between the production and the decay in the rest frame of the B meson). At the B factories, only the proper-time difference Δt between the decays of the two neutral B mesons from the $\Upsilon(4S)$ can be determined, but, because the two B mesons evolve coherently (keeping opposite flavours as long as none of them has decayed), the above formulae remain valid if t is replaced with Δt and the production flavour is replaced by the flavour at the time of the decay of the accompanying B meson in a flavour-specific state. As can be seen in the above expressions, the mixing probabilities depend on three mixing observables: Δm, $\Delta\Gamma$, and $|q/p|^2$ which signals *CP* violation in the mixing if $|q/p|^2 \neq 1$.

In the next sections we review in turn the experimental knowledge on the B^0 decay-width and mass differences, the B_s^0 decay-width and mass differences, *CP* violation in B^0 and B_s^0 mixing, and mixing-induced *CP* violation in B_s^0 decays.

3.3.1 B^0 mixing parameters $\Delta\Gamma_d$ and Δm_d

Many time-dependent B^0–$\overline{B}^0$ oscillation analyses have been performed by the ALEPH, *BABAR*, Belle, CDF, D0, DELPHI, LHCb, L3 and OPAL collaborations. The corresponding measurements of Δm_d are summarized in Table 15, where only the most recent results are listed (*i.e.* measurements superseded by more recent ones are omitted)[16]. Although a variety of different techniques have been used, the individual Δm_d results obtained at the LEP and Tevatron energy colliders have remarkably similar precision. The systematic uncertainties are not negligible; they are often dominated by sample composition, mistag probability, or b-hadron lifetime contributions. Their average is compatible with the recent and more precise measurements from the asymmetric B factories and the LHCb experiment. Before being combined, the measurements are adjusted on the basis of a common set of input values, including the averages of the b-hadron fractions and lifetimes given in this report (see Secs. 3.1 and 3.2). Some measurements are statistically correlated. Systematic correlations arise both from common physics sources (fractions, lifetimes, branching ratios of b hadrons), and from purely experimental or algorithmic effects (efficiency, resolution, flavour tagging, background description). Combining all published measurements listed in Table 15 and accounting for all identified correlations as described in Ref. [42] yields $\Delta m_d = 0.510 \pm 0.003 \pm 0.002\ \text{ps}^{-1}$.

On the other hand, ARGUS and CLEO have published measurements of the time-integrated mixing probability χ_d [158–160], which average to $\chi_d = 0.182 \pm 0.015$. Following Ref. [160], the width difference $\Delta\Gamma_d$ could in principle be extracted from the measured value of $\Gamma_d = 1/\tau(B^0)$ and the above averages for Δm_d and χ_d (provided that $\Delta\Gamma_d$ has a negligible impact on the $\Delta m_d\ \tau(B^0)$ analyses that have assumed $\Delta\Gamma_d = 0$), using the relation

$$\chi_d = \frac{x_d^2 + y_d^2}{2(x_d^2+1)} \quad \text{with} \quad x_d = \frac{\Delta m_d}{\Gamma_d} \quad \text{and} \quad y_d = \frac{\Delta\Gamma_d}{2\Gamma_d}\,. \tag{43}$$

However, direct time-dependent studies provide much stronger constraints: $|\Delta\Gamma_d|/\Gamma_d < 18\%$ at 95% CL from DELPHI [140], $-6.8\% < \text{sign}(\text{Re}\lambda_{CP})\Delta\Gamma_d/\Gamma_d < 8.4\%$ at 90% CL from *BABAR* [161], and $\text{sign}(\text{Re}\lambda_{CP})\Delta\Gamma_d/\Gamma_d = (1.7 \pm 1.8 \pm 1.1)\%$ [162] from Belle, where $\lambda_{CP} = (q/p)_d(\overline{A}_{CP}/A_{CP})$ is defined for a *CP*-even final state (the sensitivity to the overall sign of

[16] Two old unpublished CDF2 measurements [156, 157] are also omitted from our averages, Table 15 and Fig. 3.

Table 15: Time-dependent measurements included in the Δm_d average. The results obtained from multi-dimensional fits involving also the B^0 (and B^+) lifetimes as free parameter(s) [81, 83, 84] have been converted into one-dimensional measurements of Δm_d. All the measurements have then been adjusted to a common set of physics parameters before being combined.

Experiment and Ref.	Method rec.	Method tag	Δm_d in ps^{-1} before adjustment	Δm_d in ps^{-1} after adjustment
ALEPH [138]	ℓ	Q_{jet}	$0.404\pm0.045\pm0.027$	
ALEPH [138]	ℓ	ℓ	$0.452\pm0.039\pm0.044$	
ALEPH [138]	above two combined		$0.422\pm0.032\pm0.026$	$0.443\pm0.032\ ^{+0.020}_{-0.019}$
ALEPH [138]	D^*	ℓ, Q_{jet}	$0.482\pm0.044\pm0.024$	$0.482\pm0.044\pm0.024$
DELPHI [139]	ℓ	Q_{jet}	$0.493\pm0.042\pm0.027$	$0.503\pm0.042\pm0.024$
DELPHI [139]	$\pi^*\ell$	Q_{jet}	$0.499\pm0.053\pm0.015$	$0.501\pm0.053\pm0.015$
DELPHI [139]	ℓ	ℓ	$0.480\pm0.040\pm0.051$	$0.499\pm0.040\ ^{+0.042}_{-0.040}$
DELPHI [139]	D^*	Q_{jet}	$0.523\pm0.072\pm0.043$	$0.518\pm0.072\pm0.043$
DELPHI [140]	vtx	comb	$0.531\pm0.025\pm0.007$	$0.526\pm0.025\pm0.006$
L3 [141]	ℓ	ℓ	$0.458\pm0.046\pm0.032$	$0.468\pm0.046\pm0.028$
L3 [141]	ℓ	Q_{jet}	$0.427\pm0.044\pm0.044$	$0.441\pm0.044\pm0.042$
L3 [141]	ℓ	ℓ(IP)	$0.462\pm0.063\pm0.053$	$0.473\pm0.063\ ^{+0.045}_{-0.044}$
OPAL [142]	ℓ	ℓ	$0.430\pm0.043\ ^{+0.028}_{-0.030}$	$0.469\pm0.043\ ^{+0.017}_{-0.016}$
OPAL [143]	ℓ	Q_{jet}	$0.444\pm0.029\ ^{+0.020}_{-0.017}$	$0.482\pm0.029\pm0.013$
OPAL [144]	$D^*\ell$	Q_{jet}	$0.539\pm0.060\pm0.024$	$0.544\pm0.060\pm0.023$
OPAL [144]	D^*	ℓ	$0.567\pm0.089\ ^{+0.029}_{-0.023}$	$0.571\pm0.089\ ^{+0.028}_{-0.022}$
OPAL [72]	$\pi^*\ell$	Q_{jet}	$0.497\pm0.024\pm0.025$	$0.496\pm0.024\pm0.025$
CDF1 [145]	$D\ell$	SST	$0.471\ ^{+0.078}_{-0.068}\ ^{+0.033}_{-0.034}$	$0.470\ ^{+0.078}_{-0.068}\ ^{+0.033}_{-0.034}$
CDF1 [146]	μ	μ	$0.503\pm0.064\pm0.071$	$0.515\pm0.064\pm0.070$
CDF1 [147]	ℓ	ℓ, Q_{jet}	$0.500\pm0.052\pm0.043$	$0.549\pm0.052\pm0.036$
CDF1 [148]	$D^*\ell$	ℓ	$0.516\pm0.099\ ^{+0.029}_{-0.035}$	$0.523\pm0.099\ ^{+0.028}_{-0.035}$
D0 [149]	$D^{(*)}\mu$	OST	$0.506\pm0.020\pm0.016$	$0.506\pm0.020\pm0.016$
BABAR [150]	B^0	ℓ, K, NN	$0.516\pm0.016\pm0.010$	$0.521\pm0.016\pm0.008$
BABAR [151]	ℓ	ℓ	$0.493\pm0.012\pm0.009$	$0.487\pm0.012\pm0.006$
BABAR [83]	$D^*\ell\nu$(part)	ℓ	$0.511\pm0.007\pm0.007$	$0.513\pm0.007\pm0.007$
BABAR [81]	$D^*\ell\nu$	ℓ, K, NN	$0.492\pm0.018\pm0.014$	$0.493\pm0.018\pm0.013$
Belle [152]	$D^*\pi$(part)	ℓ	$0.509\pm0.017\pm0.020$	$0.514\pm0.017\pm0.019$
Belle [6]	ℓ	ℓ	$0.503\pm0.008\pm0.010$	$0.506\pm0.008\pm0.008$
Belle [84]	$B^0, D^*\ell\nu$	comb	$0.511\pm0.005\pm0.006$	$0.513\pm0.005\pm0.006$
LHCb [153]	B^0	OST	$0.499\pm0.032\pm0.003$	$0.499\pm0.032\pm0.003$
LHCb [154]	B^0	OST,SST	$0.516\pm0.005\pm0.003$	$0.516\pm0.005\pm0.003$
LHCb [155]	$D\mu$	OST,SST	$0.503\pm0.011\pm0.013$	$0.503\pm0.011\pm0.013$
World average (all above measurements included):				$0.510\pm0.003\pm0.002$
– ALEPH, DELPHI, L3, OPAL and CDF1 only:				$0.497\pm0.010\pm0.009$
– *BABAR* and Belle only:				$0.509\pm0.003\pm0.003$
– LHCb only:				$0.514\pm0.005\pm0.003$

Table 16: Simultaneous measurements of Δm_d and $\tau(B^0)$, and their average. The Belle analysis also measures $\tau(B^+)$ at the same time, but it is converted here into a two-dimensional measurement of Δm_d and $\tau(B^0)$, for an assumed value of $\tau(B^+)$. The first quoted error on the measurements is statistical and the second one systematic; in the case of adjusted measurements, the latter includes a contribution obtained from the variation of $\tau(B^+)$ or $\tau(B^+)/\tau(B^0)$ in the indicated range. Units are ps^{-1} for Δm_d and ps for lifetimes. The three different values of $\rho(\Delta m_d, \tau(B^0))$ correspond to the statistical, systematic and total correlation coefficients between the adjusted measurements of Δm_d and $\tau(B^0)$.

Exp. & Ref.	Measured Δm_d	Measured $\tau(B^0)$	Measured $\tau(B^+)$	Assumed $\tau(B^+)$
BABAR [81]	$0.492 \pm 0.018 \pm 0.013$	$1.523 \pm 0.024 \pm 0.022$	—	$(1.083 \pm 0.017)\tau(B^0)$
BABAR [83]	$0.511 \pm 0.007\ ^{+0.007}_{-0.006}$	$1.504 \pm 0.013\ ^{+0.018}_{-0.013}$	—	1.671 ± 0.018
Belle [84]	$0.511 \pm 0.005 \pm 0.006$	$1.534 \pm 0.008 \pm 0.010$	$1.635 \pm 0.011 \pm 0.011$	—
	Adjusted Δm_d	Adjusted $\tau(B^0)$	$\rho(\Delta m_d, B^0)$	Assumed $\tau(B^+)$
BABAR [81]	$0.492 \pm 0.018 \pm 0.013$	$1.523 \pm 0.024 \pm 0.022$	-0.22 $+0.71$ $+0.16$	$(1.076 \pm 0.004)\tau(B^0)$
BABAR [83]	$0.512 \pm 0.007 \pm 0.007$	$1.506 \pm 0.013 \pm 0.018$	$+0.01$ -0.85 -0.48	1.638 ± 0.004
Belle [84]	$0.511 \pm 0.005 \pm 0.006$	$1.535 \pm 0.008 \pm 0.011$	-0.27 -0.14 -0.19	1.638 ± 0.004
Average	$0.509 \pm 0.004 \pm 0.004$	$1.527 \pm 0.006 \pm 0.008$	-0.19 -0.25 -0.23	1.638 ± 0.004

sign(Reλ_{CP})$\Delta\Gamma_d/\Gamma_d$ comes from the use of B^0 decays to *CP* final states). In addition, the D0 collaboration have recently extracted a value of $\Delta\Gamma_d/\Gamma_d = (0.50 \pm 1.38)\%$ [163] from their measurements of the same-sign dimuon charge asymmetry, under the interpretation that the observed asymmetries are due to *CP* violation in neutral B-meson mixing and interference. More recently LHCb has obtained $\Delta\Gamma_d/\Gamma_d = (-0.044 \pm 0.025 \pm 0.011)\%$ [86] by comparing measurements of the $B^0 \to J/\psi K^{*0}$ and $B^0 \to J/\psi K^0_S$ decays, following the method of Ref. [164]. Assuming Re$\lambda_{CP} > 0$, as expected from the global fits of the Unitarity Triangle within the Standard Model [165], a combination of these five results (after adjusting the DELPHI and *BABAR* ones to $1/\Gamma_d = \tau(B^0) = 1.519 \pm 0.005$ ps) yields

$$\Delta\Gamma_d/\Gamma_d = 0.001 \pm 0.010\,. \tag{44}$$

Assuming $\Delta\Gamma_d = 0$ and using $1/\Gamma_d = \tau(B^0) = 1.519 \pm 0.005$ ps, the Δm_d and χ_d results are combined through Eq. (43) to yield the world average

$$\Delta m_d = 0.510 \pm 0.003 \text{ ps}^{-1}\,, \tag{45}$$

or, equivalently,

$$x_d = 0.774 \pm 0.006 \quad \text{and} \quad \chi_d = 0.1874 \pm 0.0018\,. \tag{46}$$

Figure 3 compares the Δm_d values obtained by the different experiments.

The B^0 mixing averages given in Eqs. (45) and (46) and the b-hadron fractions of Table 5 have been obtained in a fully consistent way, taking into account the fact that the fractions are computed using the χ_d value of Eq. (46) and that many individual measurements of Δm_d at high energy depend on the assumed values for the b-hadron fractions. Furthermore, this set of averages is consistent with the lifetime averages of Sec. 3.2.

It should be noted that the most recent (and precise) analyses at the asymmetric B factories measure Δm_d as a result of a multi-dimensional fit. Two *BABAR* analyses [81,83], based on fully

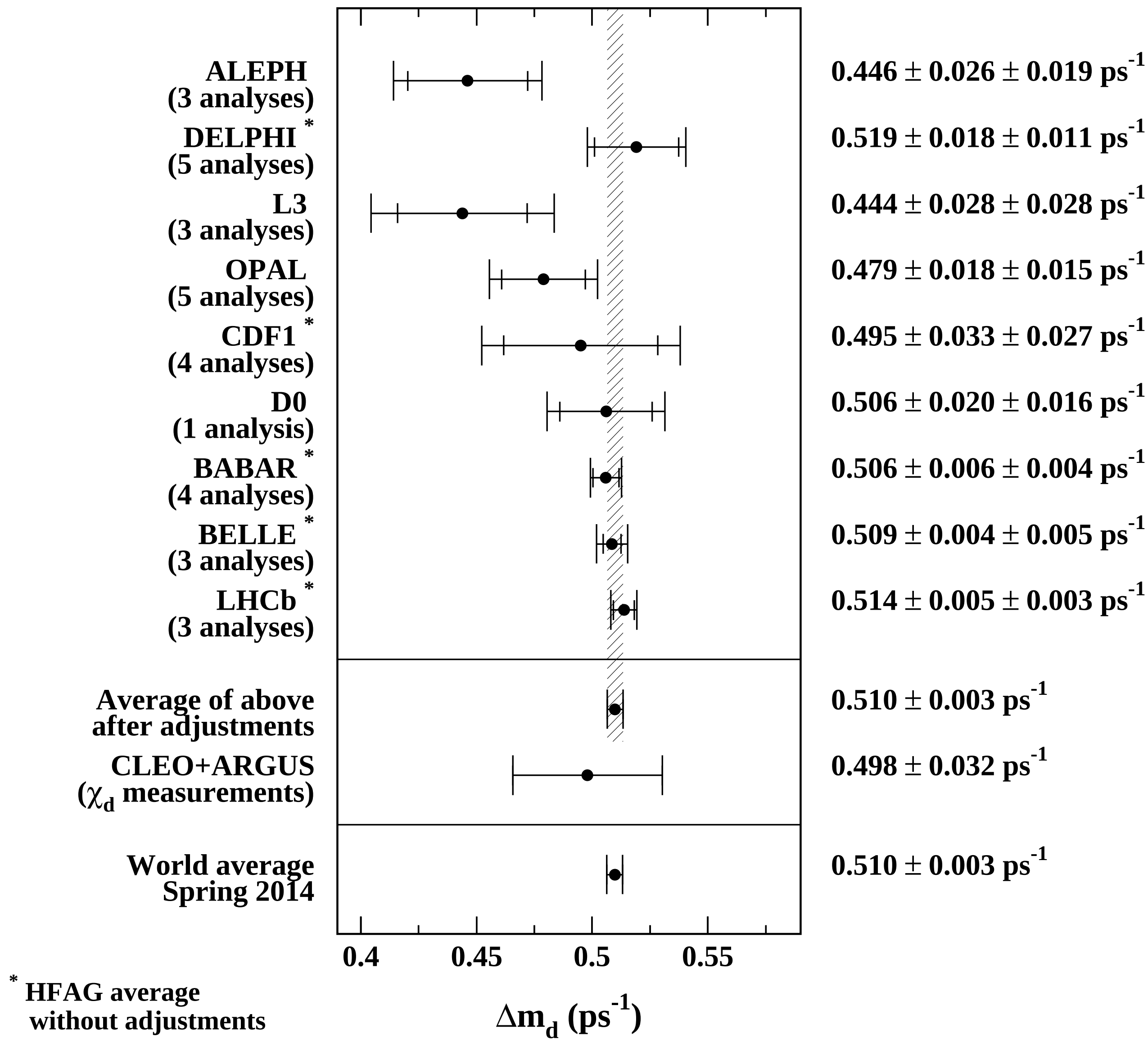

Figure 3: The B^0–$\overline{B}^0$ oscillation frequency Δm_d as measured by the different experiments. The averages quoted for ALEPH, L3 and OPAL are taken from the original publications, while the ones for DELPHI, CDF, *BABAR*, Belle and LHCb have been computed from the individual results listed in Table 15 without performing any adjustments. The time-integrated measurements of χ_d from the symmetric B factory experiments ARGUS and CLEO have been converted to a Δm_d value using $\tau(B^0) = 1.519 \pm 0.005$ ps. The two global averages have been obtained after adjustments of all the individual Δm_d results of Table 15 (see text).

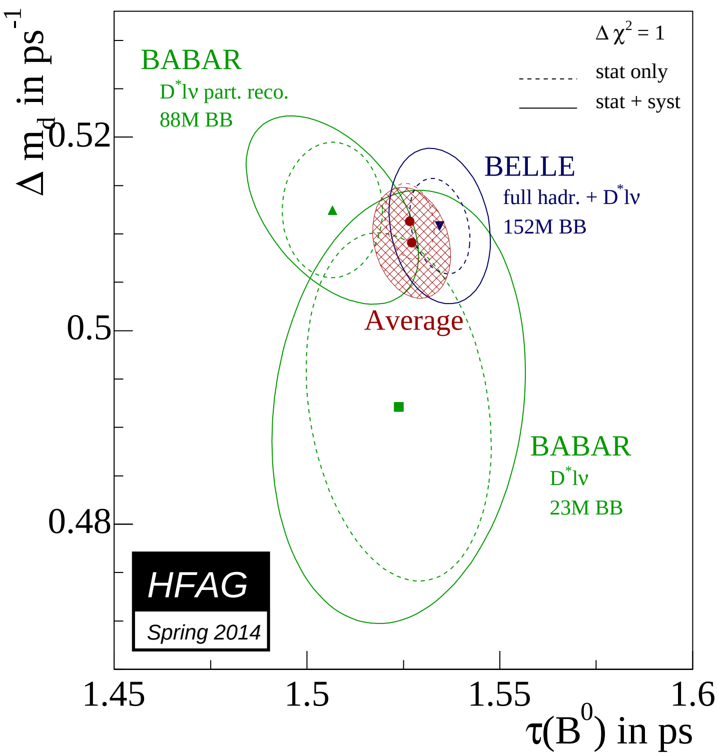

Figure 4: Simultaneous measurements of Δm_d and $\tau(B^0)$ [81, 83, 84], after adjustment to a common set of parameters (see text). Statistical and total uncertainties are represented as dashed and solid contours respectively. The average of the three measurements is indicated by a hatched ellipse.

and partially reconstructed $B^0 \to D^*\ell\nu$ decays respectively, extract simultaneously Δm_d and $\tau(B^0)$ while the latest Belle analysis [84], based on fully reconstructed hadronic B^0 decays and $B^0 \to D^*\ell\nu$ decays, extracts simultaneously Δm_d, $\tau(B^0)$ and $\tau(B^+)$. The measurements of Δm_d and $\tau(B^0)$ of these three analyses are displayed in Table 16 and in Fig. 4. Their two-dimensional average, taking into account all statistical and systematic correlations, and expressed at $\tau(B^+) = 1.638 \pm 0.004$ ps, is

$$\left.\begin{array}{r}\Delta m_d = 0.509 \pm 0.006 \text{ ps}^{-1} \\ \tau(B^0) = 1.527 \pm 0.010 \text{ ps}\end{array}\right\} \text{ with a total correlation of } -0.23. \tag{47}$$

3.3.2 B_s^0 mixing parameters $\Delta\Gamma_s$ and Δm_s

Definitions and an introduction to $\Delta\Gamma_s$ have been given in Sec. 3.2.4. Neglecting *CP* violation, the mass eigenstates are also *CP* eigenstates, with the short-lived state being *CP*-even and the long-lived state being *CP*-odd.

The best sensitivity to $\Delta\Gamma_s$ is currently achieved by the recent time-dependent measurements of the $B_s^0 \to J/\psi\phi$ (or more generally $B_s^0 \to J/\psi K^+K^-$) decay rates performed at CDF [166], D0 [167], ATLAS [168], CMS [169] and LHCb [109], where the *CP*-even and *CP*-odd amplitudes are statistically separated through a full angular analysis (see last two columns of Table 21). In addition, LHCb [109] has analyzed $B_s^0 \to J/\psi\pi^+\pi^-$ decays, for which no

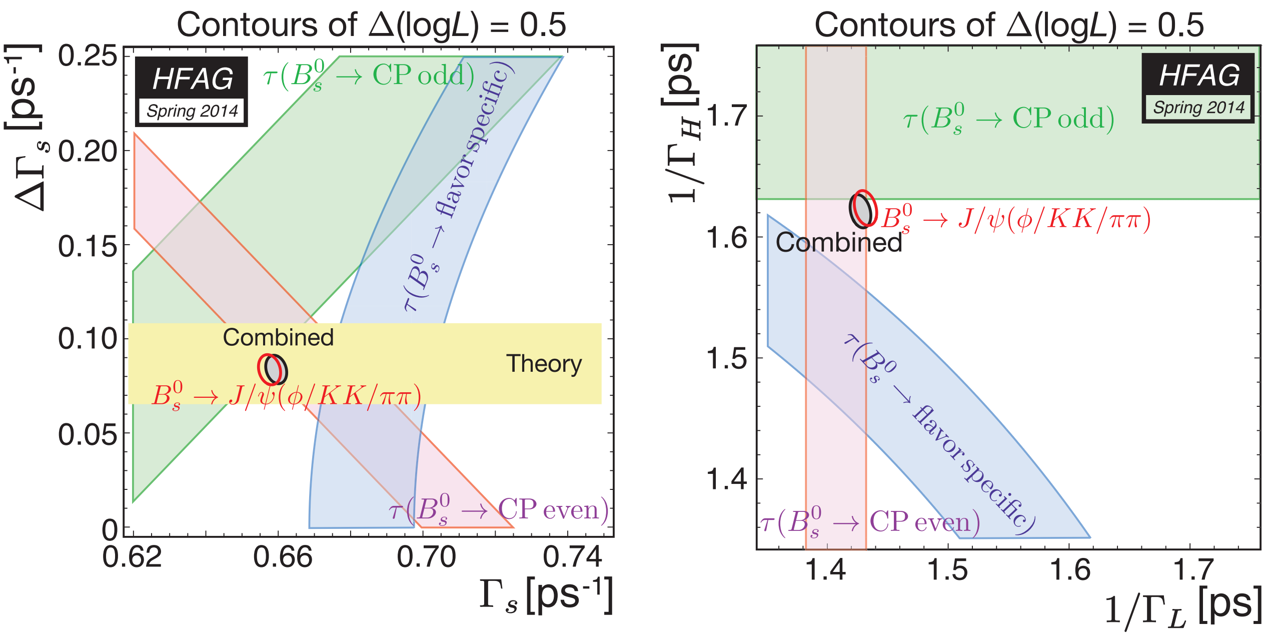

Figure 5: Contours of $\Delta \ln L = 0.5$ (39% CL for the enclosed 2D regions, 68% CL for the bands) shown in the $(\Gamma_s,\ \Delta\Gamma_s)$ plane on the left and in the $(1/\Gamma_{\rm L},\ 1/\Gamma_{\rm H})$ plane on the right. The average of all the $B_s^0 \to J/\psi\,\phi$, $J/\psi\,K^+K^-$ and $J/\psi\,\pi^+\pi^-$ results is shown as the red contour, and the constraints given by the effective lifetime measurements of B_s^0 to flavour-specific final states, $B_s^0 \to CP$-odd and $B_s^0 \to CP$-even are shown as the blue, green and purple bands, respectively. The average taking all constraints into account is shown as the gray-filled contour. The yellow band is a theory prediction $\Delta\Gamma_s = 0.087 \pm 0.021$ ps^{-1} [91] that assumes no new physics in B_s^0 mixing.

angular analysis is needed. WIth the exception of the CMS analysis, these studies use both untagged and tagged B_s^0 candidates and are optimized for the measurement of the CP-violating phase $\phi_s^{c\bar{c}s}$, defined later in Sec. 3.3.4. The LHCb collaboration analyzed the $B_s^0 \to J/\psi\,K^+K^-$ decay, considering that the K^+K^- system can be in a P-wave or S-wave state, and measured the dependence of the strong phase difference between the P-wave and S-wave amplitudes as a function of the K^+K^- invariant mass [93]. This allowed, for the first time, the unambiguous determination of the sign of $\Delta\Gamma_s$, which was found to be positive at the $4.7\,\sigma$ level and the following averages present only the $\Delta\Gamma_s > 0$ solutions.

The available results [109, 166–169] are combined, taking into account, in each analysis, the correlation between $\Delta\Gamma_s$ and Γ_s. The results, displayed as the red contours labelled "$B_s^0 \to J/\psi\,(\phi/KK/\pi\pi)$ measurements" in the plots of Fig. 5, are given in the first column of numbers of Table 17.

An alternative approach, which is directly sensitive to first order in $\Delta\Gamma_s/\Gamma_s$, is to determine the effective lifetime of untagged B_s^0 candidates decaying to CP eigenstates; measurements exist for $B_s^0 \to K^+K^-$ [106, 107][17], $B_s^0 \to D_s^+D_s^-$ [100], $B_s^0 \to J/\psi\,f_0(980)$, $J/\psi\,\pi^+\pi^-$ [108, 109], and $B_s^0 \to J/\psi\,K_{\rm S}^0$ [110]. The precise extraction of $1/\Gamma_s$ and $\Delta\Gamma_s$ from such measurements, discussed in detail in Ref. [170], requires additional information in the form of theoretical assumptions or

[17] An old unpublished measurement of the $B_s^0 \to K^+K^-$ effective lifetime by CDF [114] is no longer considered.

Table 17: Averages of $\Delta\Gamma_s$, Γ_s and related quantities, obtained from $B^0_s \to J/\psi\,\phi$, $J/\psi\,K^+K^-$, $J/\psi\,\pi^+\pi^-$ alone (first column), adding the constraints from the effective lifetimes measured in pure *CP* modes $B^0_s \to K^+K^-$, $D^+_s D^-_s$, $J/\psi\,f_0(980)$, $J/\psi\,K^0_{\rm S}$ (second column), and adding the constraint from the effective lifetime measured in flavour-specific modes $B^0_s \to D^-_s \ell^+ \nu X$, $D^-_s\pi^+$, $D^-_s D^+$ (third column, recommended world averages).

	$B^0_s \to J/\psi\,hh$ modes only	$B^0_s \to J/\psi\,hh$ modes + pure *CP* modes	$B^0_s \to J/\psi\,hh$ modes + pure *CP* modes + flavour-specific modes
$\Delta\Gamma_s$	$+0.083 \pm 0.008$ ps^{-1}	$+0.086 \pm 0.008$ ps^{-1}	$+0.084 \pm 0.008$ ps^{-1}
Γ_s	0.6574 ± 0.0031 ps^{-1}	0.6575 ± 0.0030 ps^{-1}	0.6591 ± 0.0029 ps^{-1}
$1/\Gamma_s$	1.521 ± 0.007 ps	1.521 ± 0.007 ps	1.517 ± 0.007 ps
$1/\Gamma_{\rm L}$	1.431 ± 0.009 ps	1.428 ± 0.009 ps	1.427 ± 0.009 ps
$1/\Gamma_{\rm H}$	1.623 ± 0.014 ps	1.627 ± 0.014 ps	1.620 ± 0.014 ps
$\Delta\Gamma_s/\Gamma_s$	$+0.126 \pm 0.012$	$+0.131 \pm 0.012$	$+0.127 \pm 0.012$

external inputs on weak phases and hadronic parameters. If f designates a final state in which both B^0_s and $\overline{B}^0_s$ can decay, the ratio of the effective B^0_s lifetime decaying to f relative to the mean B^0_s lifetime is [170][18]

$$\frac{\tau_{\rm single}(B^0_s \to f)}{\tau(B^0_s)} = \frac{1}{1-y_s^2}\left[\frac{1-2A^{\Delta\Gamma}_f y_s + y_s^2}{1-A^{\Delta\Gamma}_f y_s}\right] , \tag{48}$$

where

$$A^{\Delta\Gamma}_f = -\frac{2{\rm Re}(\lambda_f)}{1+|\lambda_f|^2} . \tag{49}$$

To include the measurements of the effective $B^0_s \to K^+K^-$ (*CP*-even), $B^0_s \to D^+_s D^-_s$ (*CP*-even), $B^0_s \to J/\psi\,f_0(980)$ (*CP*-odd) and $B^0_s \to J/\psi\,K^0_{\rm S}$ (*CP*-odd) lifetimes as constraints in the $\Delta\Gamma_s$ fit, we neglect sub-leading penguin contributions and possible direct *CP* violation. Explicitly, in Eq. (49), we set $A^{\Delta\Gamma}_{CP\text{-even}} = \cos\phi^{c\bar{c}s}_s$ and $A^{\Delta\Gamma}_{CP\text{-odd}} = -\cos\phi^{c\bar{c}s}_s$. Given the small value of $\phi^{c\bar{c}s}_s$, we have, to first order in y_s:

$$\tau_{\rm single}(B^0_s \to CP\text{-even}) \approx \frac{1}{\Gamma_{\rm L}}\left(1+\frac{(\phi^{c\bar{c}s}_s)^2 y_s}{2}\right) , \tag{50}$$

$$\tau_{\rm single}(B^0_s \to CP\text{-odd}) \approx \frac{1}{\Gamma_{\rm H}}\left(1-\frac{(\phi^{c\bar{c}s}_s)^2 y_s}{2}\right) . \tag{51}$$

The numerical inputs are taken from Eqs. (29) and (30)[19] and the resulting averages, combined with the $B^0_s \to J/\psi\,hh$ information, are indicated in the second column of numbers of Table 17. These averages assume $\phi^{c\bar{c}s}_s = 0$, which is the central value of the $\phi^{c\bar{c}s}_s$ average presented in Sec. 3.3.4.

[18] The definition of $A^{\Delta\Gamma}_f$ given in Eq. (49) has the sign opposite to that given in Ref. [170].

[19] The LHCb effective lifetime measurement obtained with $B^0_s \to J/\psi\,\pi^+\pi^-$ decays [109] is first removed from the average of Eq. (30), yielding $\tau_{\rm single}(B^0_s \to CP\text{-odd}) = 1.721 \pm 0.090$ ps, because this information is already contained in the $B^0_s \to J/\psi\,hh$ analysis.

Information on $\Delta\Gamma_s$ can also be obtained from the study of the proper time distribution of untagged samples of flavour-specific B_s^0 decays [111], where the flavour (*i.e.* B_s^0 or $\overline{B}_s^0$) at the time of decay can be determined by the decay products. In such decays, *e.g.* semileptonic B_s^0 decays, there is an equal mix of the heavy and light mass eigenstates at time zero. The proper time distribution is then a superposition of two exponential functions with decay constants $\Gamma_{\rm L,H} = \Gamma_s \pm \Delta\Gamma_s/2$. This provides sensitivity to both $1/\Gamma_s$ and $(\Delta\Gamma_s/\Gamma_s)^2$. Ignoring $\Delta\Gamma_s$ and fitting for a single exponential leads to an estimate of Γ_s with a relative bias proportional to $(\Delta\Gamma_s/\Gamma_s)^2$, as shown in Eq. (25). Including the constraint from the world-average flavour-specific B_s^0 lifetime, given in Eq. (26), leads to the results shown in the last column of Table 17. These world averages are displayed as the gray contours labelled "Combined" in the plots of Fig. 5. They correspond to the lifetime averages $1/\Gamma_s = 1.517 \pm 0.007$ ps, $1/\Gamma_{\rm L} = 1.427 \pm 0.009$ ps, $1/\Gamma_{\rm H} = 1.620 \pm 0.014$ ps, and to the decay-width difference

$$\Delta\Gamma_s = +0.084 \pm 0.008 \text{ ps}^{-1} \quad \text{and} \quad \Delta\Gamma_{\rm s}/\Gamma_{\rm s} = +0.127 \pm 0.012\,, \tag{52}$$

which is in good agreement with the Standard Model prediction $\Delta\Gamma_s = 0.087 \pm 0.021$ ps^{-1} [91].

Independent estimates of $\Delta\Gamma_s/\Gamma_s$ obtained from measurements of the $B_s^0 \to D_s^{(*)+} D_s^{(*)-}$ branching fraction [105, 171–173] have not been used[20], since they are based on the questionable [91] assumption that these decays account for all *CP*-even final states. The results of early lifetime analyses attempting to measure $\Delta\Gamma_s/\Gamma_s$ [62, 69, 96, 102] have not been used either.

The strength of B_s^0 mixing is known to be large since more than 20 years. Indeed the time-integrated measurements of $\overline{\chi}$ (see Sec. 3.1.3), when compared to our knowledge of χ_d and the b-hadron fractions, indicated that χ_s should be close to its maximal possible value of 1/2. Many searches of the time dependence of this mixing were performed by ALEPH [174], CDF (Run I) [175], DELPHI [96, 102, 140, 176], OPAL [177, 178] and SLD [179–181], but did not have enough statistical power and proper time resolution to resolve the small period of the B_s^0 oscillations.

B_s^0 oscillations have been observed for the first time in 2006 by the CDF collaboration [182], based on samples of flavour-tagged hadronic and semileptonic B_s^0 decays (in flavour-specific final states), partially or fully reconstructed in 1 fb^{-1} of data collected during Tevatron's Run II. This was shortly followed by an independent evidence obtained by the D0 collaboration with 2.4 fb^{-1} of data [183]. More recently the LHCb collaboration obtained the most precise results using fully reconstructed $B_s^0 \to D_s^- \pi^+$ and $B_s^0 \to D_s^- \pi^+\pi^-\pi^+$ decays at the LHC [184, 185]. LHCb has also observed B_s^0 oscillations with $B_s^0 \to J/\psi K^+K^-$ decays [109] and with semileptonic $B_s^0 \to D_s^- \mu^+ X$ decays [155]. The measurements of Δm_s are summarized in Table 18.

A simple average of the CDF and LHCb results[21], taking into account the correlated systematic uncertainties between the three LHCb measurements, yields

$$\Delta m_s = 17.761 \pm 0.021 \pm 0.007 \text{ ps}^{-1} = 17.761 \pm 0.022 \text{ ps}^{-1} \tag{53}$$

and is illustrated in Figure 6. Multiplying this result with the mean B_s^0 lifetime of Eq. (33), $1/\Gamma_s = 1.517 \pm 0.007$ ps, yields

$$x_s = \frac{\Delta m_s}{\Gamma_s} = 26.95 \pm 0.12\,. \tag{54}$$

[20] A new average is being prepared.

[21] We do not include the old unpublished D0 [183] result in the average.

Table 18: Measurements of Δm_s.

Experiment	Method		Data set	Δm_s (ps^{-1})	Ref.
CDF2	$D_s^{(*)-}\ell^+\nu$, $D_s^{(*)-}\pi^+$, $D_s^-\rho^+$		1 fb^{-1}	$17.77 \pm 0.10 \pm 0.07$	[182]
D0	$D_s^-\ell^+X$, $D_s^-\pi^+X$		2.4 fb^{-1}	$18.53 \pm 0.93 \pm 0.30$	[183][u]
LHCb	$D_s^-\pi^+$, $D_s^-\pi^+\pi^-\pi^+$	2010	0.034 fb^{-1}	$17.63 \pm 0.11 \pm 0.02$	[184]
LHCb	$J/\psi K^+K^-$	2011	1.0 fb^{-1}	$17.77 \pm 0.10 \pm 0.01$	[109]
LHCb	$D_s^-\mu^+X$	2011	1.0 fb^{-1}	$17.93 \pm 0.22 \pm 0.15$	[155]
LHCb	$D_s^-\pi^+$	2011	1.0 fb^{-1}	$17.768 \pm 0.023 \pm 0.006$	[185]
Average of CDF and LHCb measurements				$17.761 \pm 0.021 \pm 0.007$	

[u] Unpublished.

With $2y_s = \Delta\Gamma_s/\Gamma_s = +0.127 \pm 0.012$ (see Eq. (52)) and under the assumption of no *CP* violation in B_s^0 mixing, this corresponds to

$$\chi_s = \frac{x_s^2 + y_s^2}{2(x_s^2+1)} = 0.499315 \pm 0.000006\,. \tag{55}$$

The ratio of the B^0 and B_s^0 oscillation frequencies, obtained from Eqs. (45) and (53),

$$\frac{\Delta m_d}{\Delta m_s} = 0.02870 \pm 0.00020\,, \tag{56}$$

can be used to extract the following ratio of CKM matrix elements,

$$\left|\frac{V_{td}}{V_{ts}}\right| = \xi\sqrt{\frac{\Delta m_d}{\Delta m_s}\frac{m(B_s^0)}{m(B^0)}} = 0.2166 \pm 0.0007 \pm 0.0108\,, \tag{57}$$

where the first quoted error is from experimental uncertainties (with the masses $m(B_s^0)$ and $m(B^0)$ taken from Ref. [13]), and where the second quoted error is from theoretical uncertainties in the estimation of the SU(3) flavour-symmetry breaking factor $\xi = 1.268 \pm 0.063$ obtained from unquenched lattice QCD calculations [186].

3.3.3 *CP* violation in B^0 and B_s^0 mixing

Evidence for *CP* violation in B^0 mixing has been searched for, both with flavour-specific and inclusive B^0 decays, in samples where the initial flavour state is tagged. In the case of semileptonic (or other flavour-specific) decays, where the final state tag is also available, the following asymmetry

$$\mathcal{A}_{\rm SL}^d = \frac{N(\overline{B}^0(t)\to\ell^+\nu_\ell X) - N(B^0(t)\to\ell^-\overline{\nu}_\ell X)}{N(\overline{B}^0(t)\to\ell^+\nu_\ell X) + N(B^0(t)\to\ell^-\overline{\nu}_\ell X)} = \frac{|p/q|_d^2 - |q/p|_d^2}{|p/q|_d^2 + |q/p|_d^2} \tag{58}$$

has been measured, either in time-integrated analyses at CLEO [160, 187], CDF [188][22] and D0 [163,190], or in time-dependent analyses at OPAL [143], ALEPH [191], *BABAR* [161,192,193]

[22] We do not include the unpublished measurement of Ref. [189] in our average.

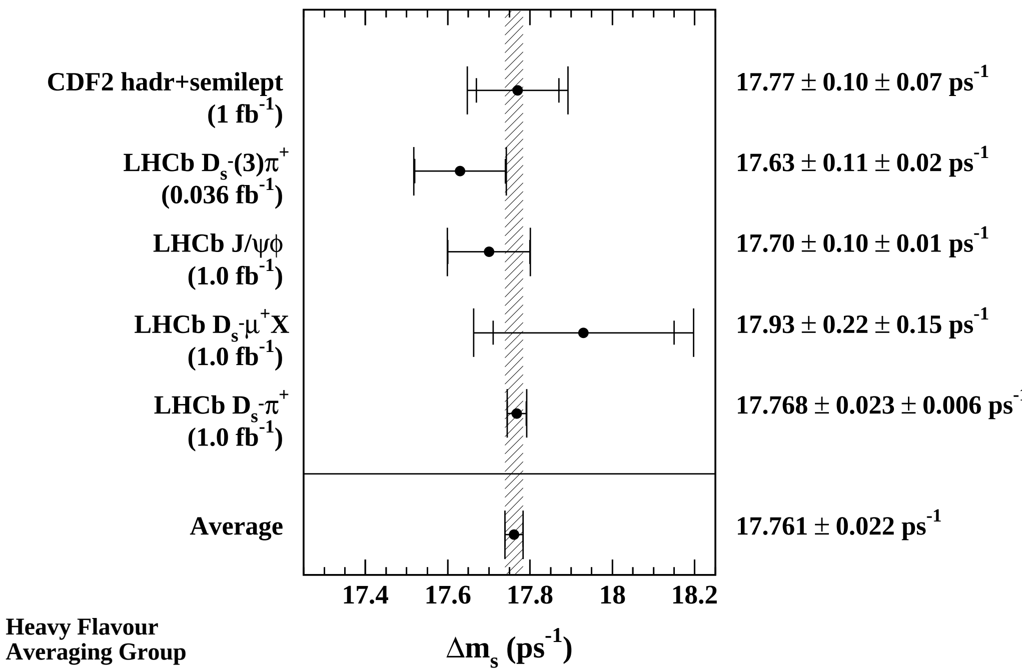

Figure 6: Published measurements of Δm_s, together with their average.

and Belle [194]. In the inclusive case, also investigated and published at ALEPH [191] and OPAL [71], no final state tag is used, and the asymmetry [195]

$$\frac{N(B^0(t) \to \text{all}) - N(\overline{B}^0(t) \to \text{all})}{N(B^0(t) \to \text{all}) + N(\overline{B}^0(t) \to \text{all})} \simeq \mathcal{A}^d_{\rm SL} \left[\frac{\Delta m_d}{2\Gamma_d} \sin(\Delta m_d\, t) - \sin^2\left(\frac{\Delta m_d\, t}{2}\right)\right] \qquad (59)$$

must be measured as a function of the proper time to extract information on CP violation. Table 19 summarized the different measurements: in all cases asymmetries compatible with zero have been found, with a precision limited by the available statistics.

A simple average of all measurements performed at B factories [160,161,187,192–194] yields $\mathcal{A}^d_{\rm SL} = +0.0002 \pm 0.0032$; adding also the D0 measurement obtained with reconstructed B^0 decays [190] yields

$$\mathcal{A}^d_{\rm SL} = +0.0023 \pm 0.0026 \quad \Longleftrightarrow \quad |q/p|_d = 0.9989 \pm 0.0013\,, \qquad (60)$$

where the relation between $\mathcal{A}^d_{\rm SL}$ and $|q/p|_d$ is given in Eq. (58). The latest dimuon D0 analysis [163] separates the B^0 and B^0_s contributions by exploiting the dependence on the muon impact parameter cut; combining the $\mathcal{A}^d_{\rm SL}$ result quoted by D0 with the above B^0 average of Eq. (60) yields $\mathcal{A}^d_{\rm SL} = -0.0000 \pm 0.0023$.

All the other B^0 analyses performed at high energy, either at LEP or at the Tevatron, did not separate the contributions from the B^0 and B^0_s mesons. Under the assumption of no CP violation in B^0_s mixing, a number of these analyses [44, 71, 143, 191] quote a measurement of

Table 19: Measurements[23] of *CP* violation in B^0 mixing and their average in terms of both $\mathcal{A}_{\rm SL}^d$ and $|q/p|_d$. The individual results are listed as quoted in the original publications, or converted[27] to an $\mathcal{A}_{\rm SL}^d$ value. When two errors are quoted, the first one is statistical and the second one systematic. The ALEPH and OPAL results assume no *CP* violation in B_s^0 mixing.

Exp. & Ref.	Method	Measured $\mathcal{A}_{\rm SL}^d$	Measured $\vert q/p\vert_d$
CLEO [160]	partial hadronic rec.	$+0.017\ \pm0.070\ \pm0.014$	
CLEO [187]	dileptons	$+0.013\ \pm0.050\ \pm0.005$	
CLEO [187]	average of above two	$+0.014\ \pm0.041\ \pm0.006$	
BABAR [161]	full hadronic rec.		$1.029\ \pm0.013\ \pm0.011$
BABAR [192]	dileptons		$0.9992\ \pm0.0027\ \pm0.0019$
BABAR [193]	part. rec. $D^* X \ell\nu$	$+0.0006\pm0.0017^{+0.0038}_{-0.0032}$	$0.99971\pm0.00084\pm0.00175$
Belle [194]	dileptons	$-0.0011\pm0.0079\pm0.0085$	$1.0005\ \pm0.0040\ \pm0.0043$
Average of above 6 B factory results		$+0.0002\pm0.0032$ (tot)	0.9999 ± 0.0016 (tot)
D0 [190]	$B^0\to D^{(*)-}\mu+X$	$+0.0068\pm0.0045\pm0.0014$	
Average of above 7 pure B^0 results		$+0.0023\pm0.0026$ (tot)	0.9989 ± 0.0013 (tot)
D0 [163]	dimuons	-0.0062 ± 0.0043 (tot)	
Average of above 8 direct measurements		-0.0000 ± 0.0023 (tot)	1.0000 ± 0.0011 (tot)
OPAL [143]	leptons	$+0.008\ \pm0.028\ \pm0.012$	
OPAL [71]	inclusive (Eq. (59))	$+0.005\ \pm0.055\ \pm0.013$	
ALEPH [191]	leptons	$-0.037\ \pm0.032\ \pm0.007$	
ALEPH [191]	inclusive (Eq. (59))	$+0.016\ \pm0.034\ \pm0.009$	
ALEPH [191]	average of above two	$-0.013\ \pm0.026$ (tot)	
Average of above 13 results		-0.0001 ± 0.0022 (tot)	1.0001 ± 0.0011 (tot)
Best fit value from 2D combination of $\mathcal{A}_{\rm SL}^d$ and $\mathcal{A}_{\rm SL}^s$ results (see Eq. (63))		-0.0009 ± 0.0021 (tot)	1.0005 ± 0.0011 (tot)

$\mathcal{A}_{\rm SL}^d$ or $|q/p|_d$ for the B^0 meson. Including also these results[23] in the previous average leads to $\mathcal{A}_{\rm SL}^d = -0.0001\pm0.0022$ under the assumption $\mathcal{A}_{\rm SL}^s = 0$. The latter assumption makes sense within the Standard Model, since $\mathcal{A}_{\rm SL}^s$ is predicted to be much smaller than $\mathcal{A}_{\rm SL}^d$ [91], but may not be suitable in presence of New Physics.

The following constraints on a combination of $\mathcal{A}_{\rm SL}^d$ and $\mathcal{A}_{\rm SL}^s$ (or equivalently $|q/p|_d$ and $|q/p|_s$) have been obtained by the Tevatron experiments, using inclusive semileptonic decays of b hadrons:

$$\frac{1}{4}\left(f'_d\chi_d\mathcal{A}_{\rm SL}^d + f'_s\chi_s\mathcal{A}_{\rm SL}^s\right) = +0.0015\pm0.0038({\rm stat})\pm0.0020({\rm syst}) \quad \text{CDF1 [188]}, \quad (61)$$

$$\mathcal{A}_{\rm SL}^b = \frac{f'_d\chi_d\mathcal{A}_{\rm SL}^d + f'_s\chi_s\mathcal{A}_{\rm SL}^s}{f'_d\chi_d + f'_s\chi_s} = -0.00496\pm0.00153({\rm stat})\pm0.00072({\rm syst}) \quad \text{D0 [163]}. \quad (62)$$

While the imprecise CDF1 result is compatible with no *CP* violation[24], the D0 result of Eq. (62), obtained by measuring the charge asymmetry of like-sign dimuons, differs by 2.8 standard

[23] A low-statistics result published by CDF using the Run I data [188] and an unpublished result by CDF using Run II data [189] are not included in our averages, nor in Table 19.

[24] A measurement from CDF2, $\mathcal{A}_{\rm SL}^b = +0.0080\pm0.0090({\rm stat})\pm0.0068({\rm syst})$ [189], more precise than the D0

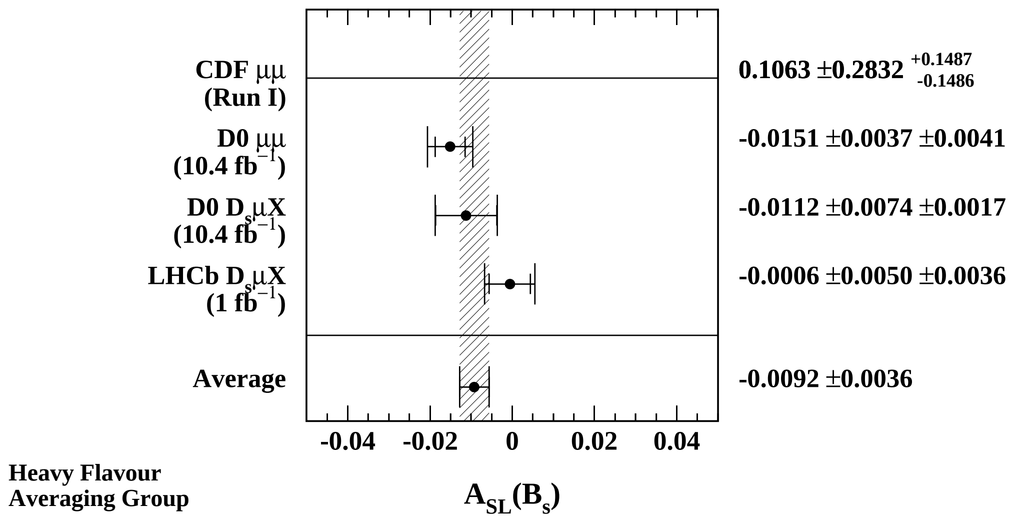

Figure 7: Measurements of $\mathcal{A}^s_{\rm SL}$, derived from CDF [188][24], D0 [163, 196] and LHCb [197] analyses, adjusted to the pure B^0 average of $\mathcal{A}^d_{\rm SL}$. The combined value of $\mathcal{A}^s_{\rm SL}$ is also shown.

deviations from the Standard Model expectation of $\mathcal{A}^b_{\rm SL}({\rm SM}) = (-2.3 \pm 0.4) \times 10^{-4}$ [91, 163]. With a more sophisticated analysis in bins of the muon impact parameters, D0 conclude that the overall deviation of the their measurements from the SM is at the level of $3.6\,\sigma$.

Using the average $\mathcal{A}^d_{\rm SL} = +0.0023{\pm}0.0026$ of Eq. (60), obtained from pure B^0 measurements, the two results of Eqs. (61) and (62) are turned[25] into the measurements of $\mathcal{A}^s_{\rm SL}$ displayed in the top part of Fig. 7. Taking into account the uncertainties on the b-hadron fractions and mixing parameters, the value derived from the D0 analysis does not show evidence of *CP* violation in the B^0_s system. In addition, the third and fourth lines of Fig. 7 show direct determination of $\mathcal{A}^s_{\rm SL}$ obtained by D0 [196] and LHCb [197] by measuring the time-integrated charge asymmetry of untagged $B^0_s \to D_s\mu X$ decays. The four results of Fig. 7 are combined to yield $\mathcal{A}^s_{\rm SL} = -0.0092 \pm 0.0027({\rm stat}) \pm 0.0023({\rm syst}) = -0.0092 \pm 0.0036$ or, equivalently through Eq. (58), $|q/p|_s = 1.0046 \pm 0.0014({\rm stat}) \pm 0.0012({\rm syst}) = 1.0046 \pm 0.0018$. The quoted systematic errors include experimental systematics as well as the correlated dependence on external parameters.

As mentioned above, the D0 like-sign dimuon analysis investigates the dependence of the charge asymmetry as a function of the muon impact parameters. Interpreting the observed asymmetries in terms of *CP* violation in B meson mixing and interference, and using the mixing parameters and the world b-hadron fractions of Ref. [198], the D0 collaboration extracts [163] values for $\mathcal{A}^d_{\rm SL}$ and $\mathcal{A}^s_{\rm SL}$ and their correlationr coefficient[26], as shown in the first line of Table 20. However, the individual contributions to the total quoted errors from this analysis and from the external inputs are not given, so the adjustment of these results to different or more recent

measurement, is also compatible with no *CP* violation, but since it is unpublished since 2007 we no longer include it in our averages, nor in Fig. 7.

[25]For simplicity, we set $f'_q = f_q$.

[26]They also extract at the same time a value for $\Delta\Gamma_d/\Gamma_d$ (see Sec. 3.3.1).

Table 20: Direct measurements of *CP* violation in B_s^0 and B^0 mixing, together with their two-dimensional average. Only total errors are quoted.

Exp. & Ref.	Method	Measured $\mathcal{A}_{\rm SL}^s$	Measured $\mathcal{A}_{\rm SL}^d$	$\rho(\mathcal{A}_{\rm SL}^s, \mathcal{A}_{\rm SL}^d)$
D0 [196]	$B_s^0 \to D_s \mu X$	-0.0112 ± 0.0076		
LHCb [197]	$B_s^0 \to D_s \mu X$	-0.0006 ± 0.0062		
Average of above B_s^0 results		-0.0048 ± 0.0048		
Average of B^0 results (Eq. (60))			$+0.0023 \pm 0.0026$	
D0 [163]	dimuons	-0.0082 ± 0.0099	-0.0062 ± 0.0043	-0.61
Average of all above		-0.0077 ± 0.0042	-0.0009 ± 0.0021	-0.195

values of the external inputs cannot (easily) be done. Using a two-dimensional fit, these values are combined with the pure B^0 average of Eq. (60) and with the results from the $B_s^0 \to D_s \mu X$ analyses [196, 197], assumed to be independent and also shown in Table 20. The result, shown graphically in Fig. 8, is

$$\mathcal{A}_{\rm SL}^d = -0.0009 \pm 0.0021 \quad \Longleftrightarrow \quad |q/p|_d = 1.0005 \pm 0.0011\,, \tag{63}$$

$$\mathcal{A}_{\rm SL}^s = -0.0077 \pm 0.0042 \quad \Longleftrightarrow \quad |q/p|_s = 1.0039 \pm 0.0021\,, \tag{64}$$

$$\rho(\mathcal{A}_{\rm SL}^d, \mathcal{A}_{\rm SL}^s) = -0.195\,. \tag{65}$$

The average of Fig. 7 ignores the impact parameter study of D0. The average of Eq. (64) ignores the CDF1 result, which has a very large uncertainty anyway. We choose the results of Eqs. (63), (64), and (65) as our final averages[27], since they better incorporate the available published data.

The above averages show no evidence of *CP* violation in B^0 and B_s^0 mixing. They deviate by $1.5\,\sigma$ from the very small predictions of the Standard Model, ${\mathcal{A}_{\rm SL}^d}^{\rm SM} = -(4.1 \pm 0.6) \times 10^{-4}$ and ${\mathcal{A}_{\rm SL}^s}^{\rm SM} = +(1.9 \pm 0.3) \times 10^{-5}$ [91]. Given the current size of the experimental uncertainties, there is still significant room for a possible New Physics contribution, especially in the B_s^0 system. In this respect, the deviation of the D0 dimuon asymmetry [163] from expectation has generated a lot of excitement, however recent results from D0 and LHCb have not yet settled the issue, and more experimental data (especially from LHCb) is awaited eagerly.

At the more fundamental level, *CP* violation in B_s^0 mixing[28] is caused by the weak phase difference

$$\phi_{12} = \arg\left[-M_{12}/\Gamma_{12}\right], \tag{66}$$

where M_{12} and Γ_{12} are the off-diagonal elements of the mass and decay matrices of the $B_s^0 - \overline{B}{}_s^0$ system. This is related to the observed decay-width difference through the relation

$$\Delta\Gamma_s = 2|\Gamma_{12}|\cos\phi_{12} + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right), \tag{67}$$

[27] Early analyses and (perhaps hence) the PDG use the complex parameter $\epsilon_B = (p-q)/(p+q)$; if *CP* violation in the mixing in small, $\mathcal{A}_{\rm SL}^d \cong 4{\rm Re}(\epsilon_B)/(1+|\epsilon_B|^2)$ and the averages of Eqs. (60) and (63) correspond to ${\rm Re}(\epsilon_B)/(1+|\epsilon_B|^2) = +0.0006 \pm 0.0007$ and -0.0002 ± 0.0005, respectively.

[28] Of course, a similar formalism exists for the B^0 system; for simplicity we omit here the subscript s for ϕ_{12}, M_{12} and Γ_{12}.

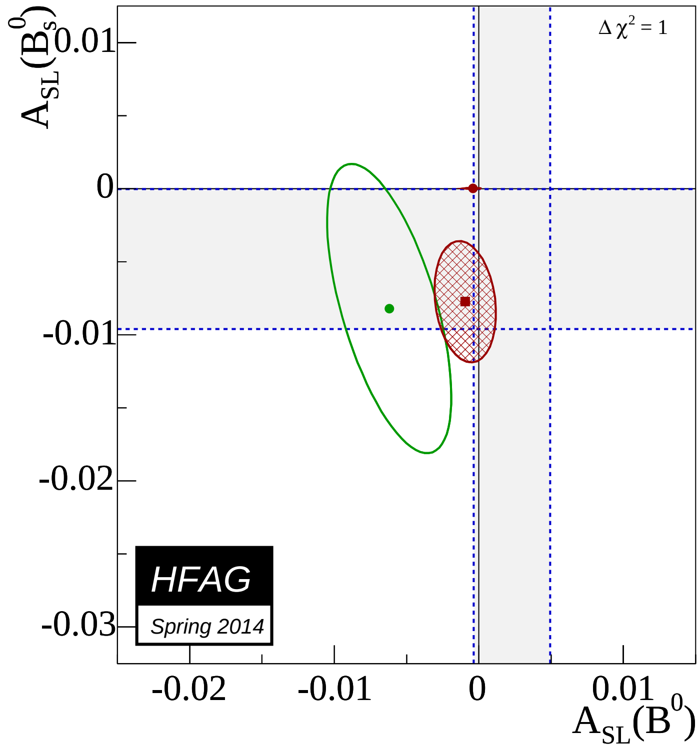

Figure 8: Direct measurements of $\mathcal{A}_{\rm SL}^s$ and $\mathcal{A}_{\rm SL}^d$ listed in Table 20 (B^0 average as the vertical band, B_s^0 average as the horizontal band, D0 dimuon result as the green ellipse), together with their two-dimensional average (red hatched ellipse). The red point close to $(0,0)$ is the Standard Model prediction of Ref. [91] with error bars multiplied by 10. The prediction and the experimental average deviate from each other by $1.5\,\sigma$.

where quadratic (or higher-order) terms in the small quantity $|\Gamma_{12}/M_{12}| \sim \mathcal{O}(m_b^2/m_t^2)$ can be neglected. The SM prediction for this phase is tiny, $\phi_{12}^{\rm SM} = 0.0038 \pm 0.0010$ [91]; however, new physics in B_s^0 mixing could change this observed phase to

$$\phi_{12} = \phi_{12}^{\rm SM} + \phi_{12}^{\rm NP} \,. \tag{68}$$

The B_s^0 semileptonic asymmetry can be expressed as [199]

$$\mathcal{A}_{\rm SL}^s = {\rm Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right) = \frac{\Delta\Gamma_s}{\Delta m_s}\tan\phi_{12} + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right) \,. \tag{69}$$

Using this relation, the current knowledge of $\mathcal{A}_{\rm SL}^s$, $\Delta\Gamma_s$ and Δm_s, given in Eqs. (64), (52), and (53) respectively, yield an experimental determination of ϕ_{12},

$$\tan\phi_{12} = \mathcal{A}_{\rm SL}^s \frac{\Delta m_s}{\Delta\Gamma_s} = -1.6 \pm 0.9 \,, \tag{70}$$

which only represents a very weak constraint at present.

Table 21: Direct experimental measurements of $\phi_s^{c\bar{c}s}$, $\Delta\Gamma_s$ and Γ_s using $B_s^0 \to J/\psi\,\phi$, $B_s^0 \to J/\psi\,K^+K^-$ and $B_s^0 \to J/\psi\,\pi^+\pi^-$ decays. Only the solution with $\Delta\Gamma_s > 0$ is shown, since the two-fold ambiguity has been resolved in Ref. [93]. The first error is due to statistics, the second one to systematics. The last line gives our average.

Exp.	Mode	Ref.	$\phi_s^{c\bar{c}s}$	$\Delta\Gamma_s$ (ps^{-1})	Γ_s (ps^{-1})
CDF	$J/\psi\,\phi$	[166]	$[-0.60,\, 0.12]$, 68% CL	$0.068 \pm 0.026 \pm 0.009$	$0.654 \pm 0.008 \pm 0.004$
D0	$J/\psi\,\phi$	[167]	$-0.55^{+0.38}_{-0.36}$	$0.163^{+0.065}_{-0.064}$	$0.693^{+0.018}_{-0.017}$
ATLAS	$J/\psi\,\phi$	[168][p]	$+0.12 \pm 0.25 \pm 0.11$	$0.053 \pm 0.021 \pm 0.009$	$0.677 \pm 0.007 \pm 0.003$
LHCb	$J/\psi\,KK$	[109]	$+0.07 \pm 0.09 \pm 0.01$	$0.100 \pm 0.016 \pm 0.003$	$0.663 \pm 0.005 \pm 0.006$
LHCb	$J/\psi\,\pi\pi$	[109]	$-0.014^{+0.17}_{-0.16} \pm 0.01$	—	—
LHCb	combined	[109]	$+0.01 \pm 0.07 \pm 0.01$	$0.106 \pm 0.011 \pm 0.007$	$0.661 \pm 0.004 \pm 0.006$
Combined			$+0.00 \pm 0.07$	$+0.091 \pm 0.010$	0.6574 ± 0.0031

[p] Preliminary.

3.3.4 Mixing-induced *CP* violation in B_s^0 decays

CP violation induced by $B_s^0 - \overline{B}_s^0$ mixing has been a field of very active study and fast experimental progress in the past couple of years. Similarly to what has happened at the B factories a decade ago, when the B^0 mixing-induced phase 2β was measured, the Tevatron and LHC experiments are now obtaining point estimates of the B_s^0 mixing-induced phase $\phi_s^{c\bar{c}s}$. This *CP*-violating phase is defined as the weak phase difference between the $B_s^0 - \overline{B}_s^0$ mixing amplitude and the $b \to c\bar{c}s$ decay amplitude.

The golden mode for such studies is $B_s^0 \to J/\psi\,\phi$, followed by $J/\psi \to \mu^+\mu^-$ and $\phi \to K^+K^-$, for which a full angular analysis of the decay products is performed to separate statistically the *CP*-even and *CP*-odd contributions in the final state. As already mentioned in Sec. 3.3.2, CDF [166], D0 [167], ATLAS [168] and LHCb [109] have used both untagged and tagged $B_s^0 \to J/\psi\,\phi$ (and $B_s^0 \to J/\psi\,K^+K^-$) events for the measurement of $\phi_s^{c\bar{c}s}$. In addition, LHCb [109] is using $B_s^0 \to J/\psi\,\pi^+\pi^-$ events (including $B_s^0 \to J/\psi\,f_0(980)$), without the need for an angular analysis since the $J/\psi\,\pi^+\pi^-$ final state has been shown to be almost *CP* pure with a *CP*-odd fraction larger than 0.977 at 95% CL [200].

All CDF, D0 and ATLAS analyses provide two mirror solutions related by the transformation $(\Delta\Gamma_s, \phi_s) \to (-\Delta\Gamma_s, \pi - \phi_s)$. However, the LHCb analysis of $B_s^0 \to J/\psi\,K^+K^-$ resolves this ambiguity and rules out the solution with negative $\Delta\Gamma_s$ [93]. Therefore, in what follows we only consider the solution with $\Delta\Gamma_s > 0$.

We perform a combination of the CDF [166], D0 [167], ATLAS [168] and LHCb [109] results summarized in Table 21. This is done by adding the two-dimensional log profile-likelihood scans of $\Delta\Gamma_s$ and $\phi_s^{c\bar{c}s}$ from the four $B_s^0 \to J/\psi\,\phi$ ($B_s^0 \to J/\psi\,K^+K^-$) analyses and a one-dimensional log profile-likelihood of $\phi_s^{c\bar{c}s}$ from the $B_s^0 \to J/\psi\,\pi^+\pi^-$ analysis, and then maximizing the combined likelihood with respect to $\Delta\Gamma_s$ and $\phi_s^{c\bar{c}s}$. We obtain the individual and combined contours shown in Fig. 9 (left). Profiling the likelihood in each of the $\Delta\Gamma_s$ and ϕ_s dimensions, we find, as summarized in Table 21:

$$\Delta\Gamma_s = +0.091 \pm 0.010 \text{ ps}^{-1}\,, \tag{71}$$
$$\phi_s^{c\bar{c}s} = +0.00 \pm 0.07\,. \tag{72}$$

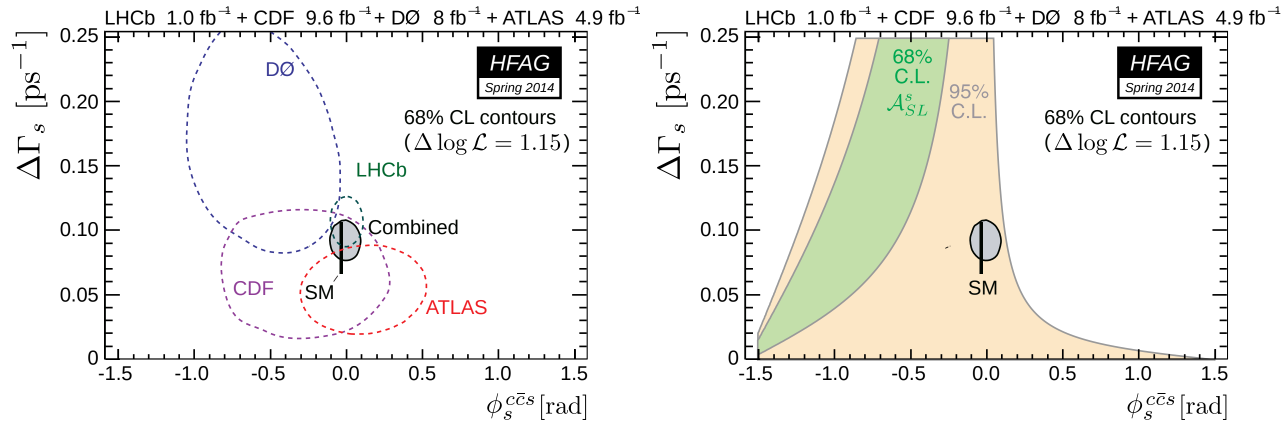

Figure 9: Left: 68% CL regions in B_s^0 width difference $\Delta\Gamma_s$ and weak phase $\phi_s^{c\bar{c}s}$ obtained from individual and combined CDF [166], D0 [167], ATLAS [168] and LHCb [109] likelihoods of $B_s^0 \to J/\psi\,\phi$, $B_s^0 \to J/\psi\,KK$ and $B_s^0 \to J/\psi\,\pi\pi$. Right: same combined contour compared with the 68% CL (green) and 95% CL (yellow) regions allowed by the measurements of $\mathcal{A}_{\rm SL}^s$ and Δm_s. The expectation within the Standard Model [91, 165] is shown as the black rectangle.

In the Standard Model and ignoring sub-leading penguin contributions, $\phi_s^{c\bar{c}s}$ is expected to be equal to $-2\beta_s$, where $\beta_s = \arg\left[-\left(V_{ts}V_{tb}^*\right)/\left(V_{cs}V_{cb}^*\right)\right]$ is a phase analogous to the angle β of the usual CKM unitarity triangle (aside from a sign change). An indirect determination via global fits to experimental data gives [165]

$$(\phi_s^{c\bar{c}s})^{\rm SM} = -2\beta_s = -0.0367^{+0.0014}_{-0.0013}\,. \tag{73}$$

The average value of $\phi_s^{c\bar{c}s}$ from Eq. (72) is consistent with this Standard Model expectation.

New physics could contribute to $\phi_s^{c\bar{c}s}$. Assuming that new physics only enters in M_{12} (rather than in Γ_{12}), one can write [91]

$$\phi_s^{c\bar{c}s} = -2\beta_s + \phi_{12}^{\rm NP}\,, \tag{74}$$

where the new physics phase $\phi_{12}^{\rm NP}$ is the same as that appearing in Eq. (68). In this case

$$\phi_{12} = \phi_{12}^{\rm SM} + 2\beta_s + \phi_s^{c\bar{c}s} \tag{75}$$

and Eq. (69) then provides a relation between $\Delta\Gamma_s$ and $\phi_s^{c\bar{c}s}$, based on the measured values of $\mathcal{A}_{\rm SL}^s$ and Δm_s (Eqs. (64) and (53)) as well as the expectations for $\phi_{12}^{\rm SM}$ and $-2\beta_s$. The allowed region in the ($\Delta\Gamma_s$, $\phi_s^{c\bar{c}s}$) plane is shown in Fig. 9 (right), where it is compared both with the direct measurement of $\Delta\Gamma_s$ and $\phi_s^{c\bar{c}s}$, and with the Standard Model expectations. No inconsistency is observed between all these data.

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