

Given the theoretical parameters

$$x_{12} \equiv \frac{2|M_{12}|}{\Gamma} \quad y_{12} \equiv \frac{|\Gamma_{12}|}{\Gamma} \quad \phi_{12} \equiv \text{Arg} \left( \frac{M_{12}}{\Gamma_{12}} \right),$$

we express the experimental parameters  $(x, y, |q/p|, \phi)$  in terms of  $(x_{12}, y_{12}, \phi_{12})$  as follows. Solving the eigenvalue problem gives [see Eqs. (10)(11) of Kagan and Sokoloff, PRD 80, 076008 (2009)]:

$$(x - iy)^2 = x_{12}^2 - y_{12}^2 - i 2x_{12}y_{12} \cos \phi_{12} \quad (1)$$

$$\frac{q}{p} = -\frac{\Gamma(x - iy)}{2(M_{12} - i\Gamma_{12}/2)} = -\frac{x - iy}{e^{i\phi_{\Gamma_{12}}}(x_{12} e^{i\phi_{12}} - iy_{12})}. \quad (2)$$

Squaring Eq. (2) and using Eq. (1) gives

$$\begin{aligned} \left(\frac{q}{p}\right)^2 &= \frac{(x - iy)^2}{e^{i2\phi_{\Gamma_{12}}}(x_{12} e^{i\phi_{12}} - iy_{12})^2} = \frac{x_{12}^2 - y_{12}^2 - i 2x_{12}y_{12} \cos \phi_{12}}{e^{i2\phi_{\Gamma_{12}}}(x_{12}^2 e^{i2\phi_{12}} - y_{12}^2 - i 2x_{12}y_{12} e^{i\phi_{12}})} \\ \Rightarrow \left|\frac{q}{p}\right|^4 &= \frac{(x_{12}^2 - y_{12}^2)^2 + 4x_{12}^2 y_{12}^2 \cos^2 \phi_{12}}{|x_{12}^2 \cos 2\phi_{12} - y_{12}^2 + 2x_{12}y_{12} \sin \phi_{12} + i(x_{12}^2 \sin 2\phi_{12} - 2x_{12}y_{12} \cos \phi_{12})|^2} \\ &= \frac{x_{12}^4 + y_{12}^4 - 2x_{12}^2 y_{12}^2 (1 - 2 \cos^2 \phi_{12})}{(x_{12}^2 \cos 2\phi_{12} - y_{12}^2 + 2x_{12}y_{12} \sin \phi_{12})^2 + (x_{12}^2 \sin 2\phi_{12} - 2x_{12}y_{12} \cos \phi_{12})^2} \\ &= \frac{x_{12}^4 + y_{12}^4 + 2x_{12}^2 y_{12}^2 \cos 2\phi_{12}}{(x_{12}^2 + y_{12}^2)^2 + 4x_{12}y_{12} (x_{12}y_{12} \sin^2 \phi_{12} - x_{12}^2 \sin \phi_{12} - y_{12}^2 \sin \phi_{12})} \end{aligned}$$

Taking the modulus squared of Eq. (2) gives

$$\begin{aligned} \left|\frac{q}{p}\right|^2 &= \frac{x^2 + y^2}{x_{12}^2 + y_{12}^2 + i x_{12}y_{12}(e^{i\phi_{12}} - e^{-i\phi_{12}})} \\ \Rightarrow x^2 + y^2 &= \left|\frac{q}{p}\right|^2 (x_{12}^2 + y_{12}^2 - 2x_{12}y_{12} \sin \phi_{12}) \quad (3) \end{aligned}$$

But matching the real and imaginary parts of Eq. (1) implies

$$x^2 - y^2 = x_{12}^2 - y_{12}^2, \quad (4)$$

so combining Eqs. (3) and (4) gives

$$x^2 = \frac{1}{2} \left[ x_{12}^2 \left( \left|\frac{q}{p}\right|^2 + 1 \right) + y_{12}^2 \left( \left|\frac{q}{p}\right|^2 - 1 \right) - 2x_{12}y_{12} \left|\frac{q}{p}\right|^2 \sin \phi_{12} \right] \quad (5)$$

$$y^2 = \frac{1}{2} \left[ x_{12}^2 \left( \left|\frac{q}{p}\right|^2 - 1 \right) + y_{12}^2 \left( \left|\frac{q}{p}\right|^2 + 1 \right) - 2x_{12}y_{12} \left|\frac{q}{p}\right|^2 \sin \phi_{12} \right]. \quad (6)$$

Assuming  $\Gamma_{12}$  is real, squaring Eq. (2) gives

$$\begin{aligned} \left(\frac{q}{p}\right)^2 &= \frac{(x - iy)^2}{(x_{12} e^{i\phi_{12}} - iy_{12})^2} = \frac{x_{12}^2 - y_{12}^2 - i 2x_{12}y_{12} \cos \phi_{12}}{x_{12}^2 e^{i2\phi_{12}} - y_{12}^2 - i 2x_{12}y_{12} e^{i\phi_{12}}} \\ &= \frac{x_{12}^2 - y_{12}^2 - i 2x_{12}y_{12} \cos \phi_{12}}{x_{12}^2 \cos 2\phi_{12} - y_{12}^2 + 2x_{12}y_{12} \sin \phi_{12} + i(x_{12}^2 \sin 2\phi_{12} - 2x_{12}y_{12} \cos \phi_{12})}. \end{aligned}$$

This equation has the form

$$\left(\frac{q}{p}\right)^2 = \frac{a + ib}{c + id} = \frac{(ac + bd) + i(bc - ad)}{c^2 + d^2},$$

and thus

$$\begin{aligned} \phi &= \text{Arg} \left[ \frac{q}{p} \right] = \left( \frac{1}{2} \right) \text{Arg} \left[ \left( \frac{q}{p} \right)^2 \right] \\ &= \left( \frac{1}{2} \right) \tan^{-1} \left( \frac{bc - ad}{ac + bd} \right). \end{aligned}$$

To calculate  $\phi$  we make the substitutions

$$a = x_{12}^2 - y_{12}^2$$

$$b = -2x_{12}y_{12} \cos \phi_{12}$$

$$c = x_{12}^2 \cos 2\phi_{12} - y_{12}^2 + 2x_{12}y_{12} \sin \phi_{12}$$

$$d = x_{12}^2 \sin 2\phi_{12} - 2x_{12}y_{12} \cos \phi_{12}.$$