Equation (15) of Grossman, Nir, and Perez [PRL 103, 071602 (2009)] is

$$\left(1-\left|\frac{q}{p}\right|^4\right)^2\left[\frac{1+\left(\frac{y}{x}\right)^4\tan^2\phi}{\sin^2\phi}\right] \;=\; 16\left(\frac{y}{x}\right)^2\left|\frac{q}{p}\right|^4 \;+\; \left[1+\left(\frac{y}{x}\right)^2\right]^2\left(1-\left|\frac{q}{p}\right|^4\right)^2 .$$

Defining $z\equiv|q/p|^4$ and $r\equiv(y/x)^2$ gives

$$(1-z)^2\left(\frac{1+r^2\tan^2\phi}{\sin^2\phi}\right) \;=\; 16rz \;+\; (1+r)^2(1-z)^2 \tag{1}$$

$$(1-z)^2\left[\frac{1+r^2\tan^2\phi}{\sin^2\phi} \;-\; (1+r)^2\right] \;=\; 16rz\,.$$

This can be written in the standard quadratic form $\alpha z^2+\beta z+\gamma=0$, where

$$\begin{aligned}\alpha &= \left[\frac{1+r^2\tan^2\phi}{\sin^2\phi} \;-\; (1+r)^2\right]\\ \beta &= -16r-2\alpha\\ \gamma &= \alpha\,.\end{aligned}$$

Thus $z=(-\beta\pm\sqrt{\beta^2-4\alpha\gamma})/2\alpha$, and $|q/p|=z^{1/4}$. Note that there is a two-fold ambiguity in $z$ arising from the quadratic equation. We find that usually the solution with the positive sign is the correct one, but not always.

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From Eq. (1) we obtain

$$\frac{(1-z)^2}{16rz+(1+r)^2(1-z)^2} \;=\; \frac{\sin^2\phi}{1+r^2\tan^2\phi}\,.$$

Defining the left-hand side as $\xi$ gives

$$\begin{aligned}\xi \;+\; \xi r^2\tan^2\phi &= \sin^2\phi\\ \xi\cos^2\phi \;+\; \xi r^2\sin^2\phi &= \sin^2\phi\cos^2\phi\\ \xi-\xi\sin^2\phi \;+\; \xi r^2\sin^2\phi &= \sin^2\phi-\sin^4\phi\,.\end{aligned}$$

Defining $u\equiv\sin^2\phi$, we can write this in the quadratic form $\alpha u^2+\beta u+\gamma=0$, where

$$\begin{aligned}\alpha &= 1\\ \beta &= \xi r^2-\xi-1\\ \gamma &= \xi\,.\end{aligned}$$

Thus $u=(-\beta-\sqrt{\beta^2-4\alpha\gamma})/2\alpha$, and $\phi=\sin^{-1}(\pm\sqrt{u})$. Note that there is a sign ambiguity in $\phi$ that cannot be resolved by this method. In principle there is a two-fold ambiguity in $u$ arising from the quadratic equation; however, we find that the solution with the negative sign is always the correct one.